Raphael da Silva Ornellas

Essays on Learning in Macroeconomics

Tese de Doutorado

DEPARTAMENTO DE ECONOMIA
Programa de Pós-Graduação em Economia

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Pontifícia Universidade Católica do Rio de Janeiro

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Advisor : Prof. Carlos Viana de Carvalho
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To my beloved parents, Cristóvão and Romilda.

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Abstract

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This thesis is composed of two chapters which applies the learning methodology in macroeconomics issues. In the first one, we assess how countries choose their exchange rate regime, specifically whether to fix or to float. When choosing the arrangement, the policymaker cares about growth and inflation; however, he lacks full knowledge of how the regimes impact these variables. Instead, the policymaker learns it from his own experience as well as from the other's. We find no evidence of the policymaker considering inflation in his judgment. In addition, the model evidences intuitive connections between the choices and some associated economic variables as well as fits well the realized data. In the second chapter, we propose a model in which the household relies on two sources of information to predict future inflation. In each period he can either lean into the professional's views or forecast inflation with a learning model. The model fits well the realized data of household inflation expectations for the US and Brazil economies. Within the latter, the learning mechanism plays a major role in modeling the household expectations in an economy with a history of hyperinflation. We also find a positive correlation between news on inflation assimilated by households and the probabilities with which they rely on professional's view, in addition to shedding some light on how demographic groups shape their inflation expectations. Finally, we find evidence that central bank transparency improves inflation predictability for households.

Keywords

Learning; Escolha de regimes cambiais; Expectativas de inflação;

Resumo

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Esta tese é composta por dois capítulos que abordam a metodologia de learning em questões macroeconômicas. No primeiro, estudamos como os países escolhem regimes cambiais, especificamente fixo ou flutuante. Ao decidir sobre o regime, o formulador de política considera crescimento econômico e inflação, entretanto, ele desconhece como arranjos impactam essas variáveis. Em vez disso, o formulador de política aprende esse relacionamento a partir de sua própria experiência, bem como a de outros países. O modelo evidencia conexões intuitivas entre as escolhas e variáveis econômicas relacionadas, além de ter um bom ajuste aos dados realizados. No segundo capítulo, propomos um modelo no qual o consumidor pode se pautar em duas alternativas para projetar a inflação futura. Em cada período, ele pode tanto utilizar a visão dos profissionais, ou estimar via um modelo de learning. O modelo se ajusta bem aos dados para as economias dos Estados Unidos e do Brasil. Em relação ao último, evidenciamos a importante regra que o mecanismo de learning possui na modelagem das expectativas de inflação dos consumidores de uma economia com histórico de inflação. Também encontramos evidência de uma correlação positiva entre notícias de inflação assimiladas pelos consumidores e as probabilidades com as quais eles se inclinam à visão dos profissionais, além de analisarmos como grupos demográficos formam expectativas de inflação. Por fim, encontramos indícios que a transparência do banco central aumenta a capacidade dos consumidores de preverem a inflação.

Palavras–chave

Learning; Exchange Rate Regime Choices; Inflation Expectations;

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Two roads diverged in a yellow wood,
And sorry I could not travel both
And be one traveler, long I stood
And looked down one as far as I could
To where it bent in the undergrowth;

Then took the other, as just as fair,
And having perhaps the better claim,
Because it was grassy and wanted wear;
Though as for that the passing there
Had worn them really about the same,

And both that morning equally lay
In leaves no step had trodden black.
Oh, I kept the first for another day!
Yet knowing how way leads on to way,
I doubted if I should ever come back.

I shall be telling this with a sigh
Somewhere ages and ages hence:
Two roads diverged in a wood, and I—
I took the one less traveled by,
And that has made all the difference.

Robert Frost, *Mountain Interval.*

1
Accounting for the Choice of Exchange Rate Regimes

1.1
Introduction

How to choose the appropriate exchange rate regime is a relevant question in international economics which remains unsolved. The arrangements' impact on key economic variables is theoretically complex, which might explain a large part of this open question. For instance, when it comes to growth, a fixed regime reduces relative price volatility, stimulating trade and investment, which causes a positive effect on growth. Nevertheless, the regime is inefficient to isolate the economy from external shocks. Along with price rigidity, the lack of outer shock adjustment contributes to price distortion and misallocation, causing high output volatility and low economic growth. Moreover, the regime is also associated with greater susceptibility to financial and currency crisis. With respect to inflation, its relationship with the regimes is more consensual.[1] For example, when credible, the fixed regime has a proeminent impact on private sector inflation's expectations, since it acts as a nominal anchor. So, the public is more prone to believe that the monetary authority is committed to a sustained monetary policy, ruling out discretionary policies which may be inflationary. Thus, to some extent, a more fixed exchange rate regime is linked to less inflation uncertainty, reinforcing a supportive environment for inflation dynamics. On the other hand, a macroeconomic framework encompassing floating exchange rate is also a conductive environment to keep low inflation, mainly within emergent markets.

Perhaps resulting from the theme complexity, the literature assessing the relationship between the arrangements and key economic variables reports ambiguous results. Baxter e Stockman (1989) find little impact of exchange rate regime on some macroeconomic variables when comparing countries in two periods under distinct regimes. However, Mundell (1997) concludes that real economic growth performed better under fixed exchange rate, as well as Moreno (2003). On the other hand, Ghosh et al. (1997) suggest that pegged exchange rate can lead to lower inflation and slower productivity growth.

The works of Levy-Yeyati e Sturzenegger (2003) and Husain et al. (2005) must be highlighted in addressing a causal relationship between exchange rate regime and economic variables. The former finds that a more fixed

[1]See Ghosh et al. (1997), Bleaney (1999), Engel (2009) and Clerc et al. (2011).

exchange rate regime is associated with slower growth and greater output volatility for developing countries, but with insignificant impact in industrial economies. The latter evidences pegs as responsible for relatively low inflation in developing economies, with little exposure to international capital markets, while floats are associated with higher growth in advanced economies. Levy Yeyati et al. (2010) consider three theoretical approaches to explain arrangement choices: the optimal currency area theory, the financial view, and the political view. By using a multinomial logit, the authors find that policymakers consider some variables related to these theories when choosing the exchange rate regime. Other papers covering regimes' choices are Alesina e Wagner (2006) and Von Hagen e Zhou (2009).

As a consequence, this lack of consensus has also impacted policy purposes. In the last fifteen years, the International Monetary Fund (IMF) has addressed studies with different guidance for the exchange rate. According to the IMF[2], in the early 1990s, countries seeking economic stabilization should choose peg their currency to a strong one, often the dollar, to import monetary credibility from the foreign currency issuer. Nonetheless, growing cases of capital account crisis took place in emerging countries in the late 1990s, collapsing their currencies and showing fragility of fixed exchange rate regime as a guidance for economic stabilization.

Therefore, the IMF revisited the role of the exchange rate with Mussa et al. (2000) and proposed hard pegs (e.g. currency boards, monetary unions) or free floats to mitigate the probability of balance of payment crisis. Nevertheless, this bipolar prescription had a short life, since the Argentinian economy succumbed in 2002, with its peso hard-pegged to the dollar. This time around, the IMF reviewed again which regime is the most appropriate and, as exposed by Rogoff et al. (2003), suggested the use of freely floating exchange rate to avoid financial crises in an increasingly integrated world.

Finally, the last IMF study proposed a more flexible view of exchange rate regime choice. In work of Ghosh et al. (2011), the fund suggests that choices on exchange rate regime must follow particular economic challenges of each country.[3] Thus, the idea of "one-size-fits-all" exchange rate regime was ruled out. In addition, the work reinforced that more fixed regimes are associated with low nominal output volatility and deeper trade integration, enhancing economic growth; whilst more flexible regimes are related to smoother external adjustment and lower susceptibility to financial crises.

[2]See Caramazza e Aziz (1998).

[3]However, the IMF warns that countries with weak macroeconomic fundamentals could suffer from potential financial crisis if a pegged regime is adopted.

This uncertainty relating arrangements and key economic variables might be a driving force for the herd of time varying beliefs on whether fixed or floating regime is suitable for the economies. Figure 1.1 reports the choices and the unconditional growth and inflation. As we can see, an increasing number of countries leaning into the floating exchange rate regime follows the end of Bretton Woods System, as expected. At the same period, increasing inflation took place in the world economy, suggesting a positive relationship between floating regime and inflation. By 1980s and early 1990s, the rising ratio of countries adopting floating was not accompanied by a further increase in world inflation. On the other hand, a falling ratio in the mid-1990s was followed by declining world inflation. Regarding economic growth, we are unable to verify any pattern with the currency arrangement.

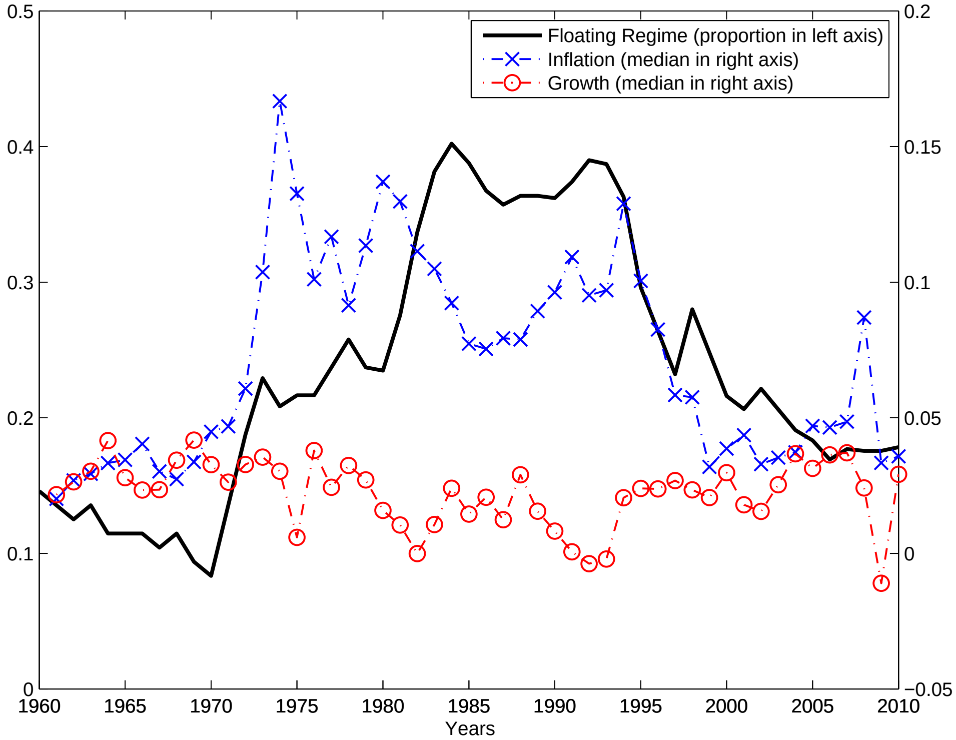

Figure 1.1: Dynamic of floating regime, inflation, and growth

To assess the underlying uncertainty within the countries' choice of exchange rate regime, we adapt from Buera et al. (2011) and propose a model able to take into account the belief changes over time. Specifically, we assume the policymaker lacking the knowledge of how each regime impacts growth and inflation. Instead, he learns it from own experience as well as from the others, giving rise to a time-varying perceived relationship between the arrangements and the economic variables. Moreover, we restrict the policymaker's choice to internal economic variables in addition to also incur switching costs.

We estimate the model with Bayesian techniques with annual data ranging from 1970 to 2010 for 91 countries. We find no evidence of the policymaker considering inflation in his choice. A possible explanation can be extracted from the Brazilian case, which relied upon fixed exchange rate to strongly disinflate the economy but then adopted a standard macroeconomic framework, encompassing floating regime, to ensure a low inflation environment. So, it seems that hard pegs are used for major shifts in the inflation rate, as in hyperinflation case, while conventional monetary tools are claimed for marginal changes in prices, with the latter efficiency being strengthened by a floating regime. In addition, the results also suggest that policymaker's decision on currency arrangements is indeed constrained by internal economic variables. For instance, high international reserves encourage fixed regime, an intuitive result. Lastly, the model fits well the observed regimes' choices, specifically, the herd of them observed along time. As a drawback, we note that the model can suffer from potential overparameterization problem, which we try to mitigate using priors within the Bayesian scheme.

Besides this introduction, the work proceeds as follows: Section 2 examines the data on exchange rate regime used in the paper. Section 3 describes the model. Section 4 reports the estimation methodology and discuss the results. Finally, section 5 concludes the paper.

1.2 Data on Exchange Rate Regime Dynamic

The IMF's Annual Report on Exchange Rate Arrangements and Exchange Restrictions officially classifies the countries according to their exchange rate regimes. The report asks members to self-declare their arrangement as fixed, limited flexibility, managed floating or independently floating. However, it seems that there is a difference between the arrangement officially reported and the one actually prevailing, as shown by Reinhart e Rogoff (2004) and Levy-Yeyati e Sturzenegger (2005). To overcome this issue, we use the reclassification made by Reinhart e Rogoff (2004) as data for exchange rate arrangements.

According to Reinhart e Rogoff (2004), the history of exchange rate policy may look very different when using official rates. Thus, the authors develop a system using a database on market-determined parallel exchange rates and reinterpret the history of exchange rate arrangements. As a result, they find evidence that exchange rate arrangements may be quite important for growth, trade and inflation.

The finest Reinhart and Rogoff's reclassification sorts the regimes between 1 and 15, from more fixed to more floating, obeying the following criteria:

1. No separate legal tender;
2. Pre announced peg or currency board arrangement;
3. Pre announced horizontal band that is narrower than or equal to +/-2%;
4. De facto peg;
5. Pre announced crawling peg;
6. Pre announced crawling band that is narrower than or equal to +/-2%;
7. De factor crawling peg;
8. De facto crawling band that is narrower than or equal to +/-2%;
9. Pre announced crawling band that is wider than or equal to +/-2%;
10. De facto crawling band that is narrower than or equal to +/-5%;
11. Moving band that is narrower than or equal to +/-2% (i.e., allows for both appreciation and depreciation over time);
12. Managed floating;
13. Freely floating;
14. Freely falling;
15. Dual market in which parallel market data is missing.[4]

[4]Cases in which one or more of the exchange rates is market-determined.

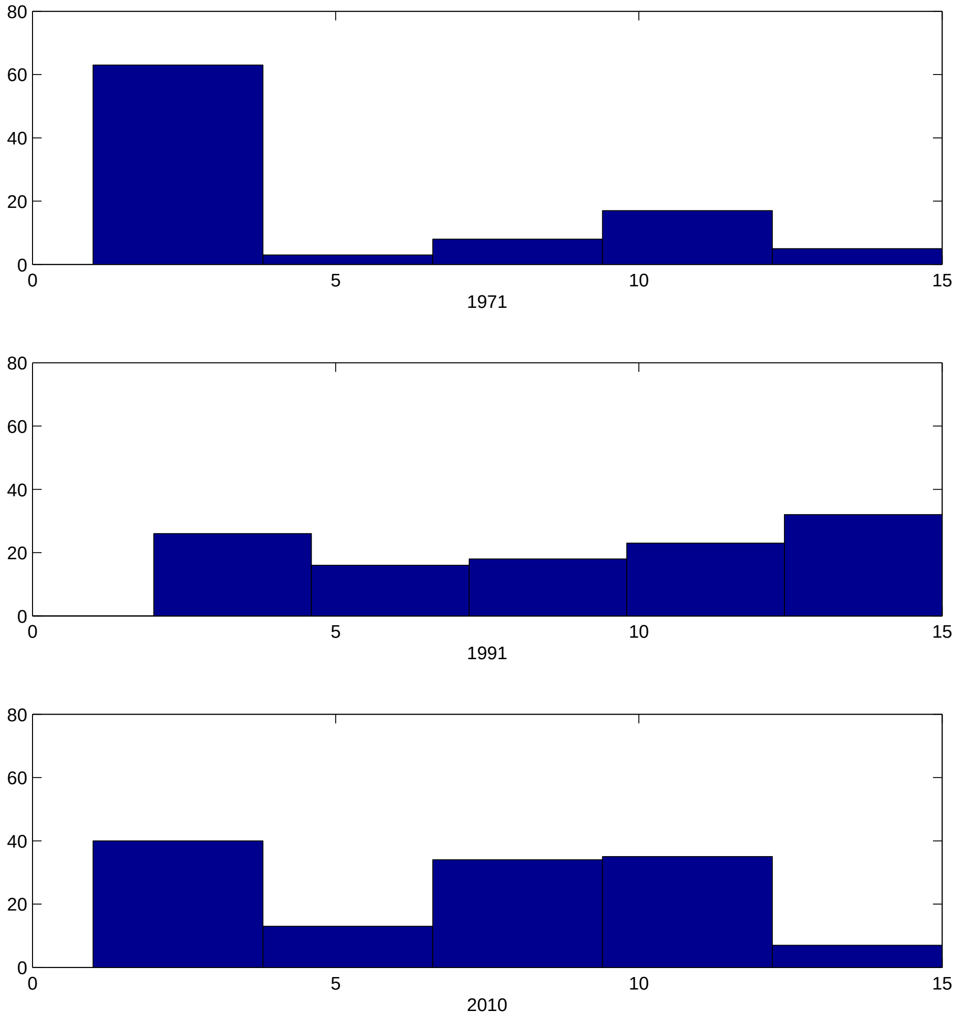

Figure 1.2: Distribution of the fine regimes

Figure 1.2 reports the histogram of regimes in three different years within the sample. Except for beginning of the sample, which still reflects the end of Bretton Woods, it seems that the arrangements are well distributed throughout the finest reclassification.

To adapt the data into the model, we select a cohort which splits the regimes as fixed and floating. So, the regimes indexed between 1 and 10, inclusive, are defined as fixed regime, while the complementary is assumed as floating regime. The Figure 1.3 describes the dynamic of growth and inflation when assuming this dual scheme regime. For the period analyzed, countries under floating regime had mean growth of 1.36% and median inflation of 16.14%, while countries under fixed regime had mean growth of 2.54% and median inflation of 6.22%. Once again, the relation between exchange rate arrangement and inflation seems less uncertain.

Figure 1.4 reports the proportion of countries adopting floating exchange rate regime within their correspondent income level. We can note similar dynamic for each one, except for the upper middle level between 1980's and late 1990's. The significant flow of gross capital into the emerging countries during this period might be made the fixed regimes more costly to maintain, since a possible increasing perception of sudden reversals occurrence.

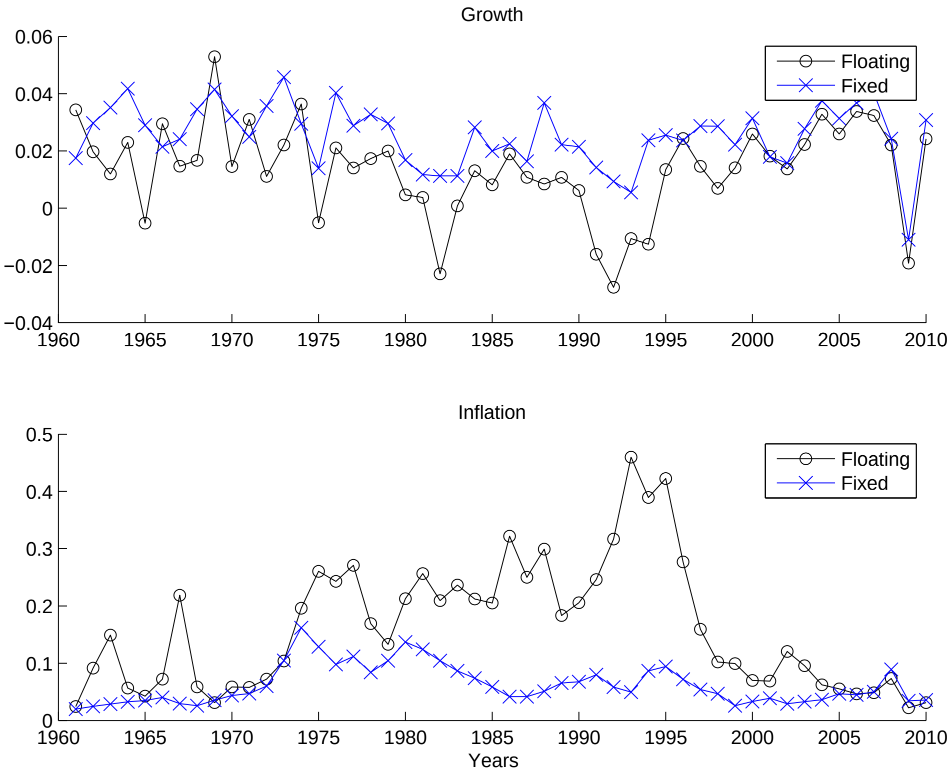

Figure 1.3: Growth and Inflation by Regime

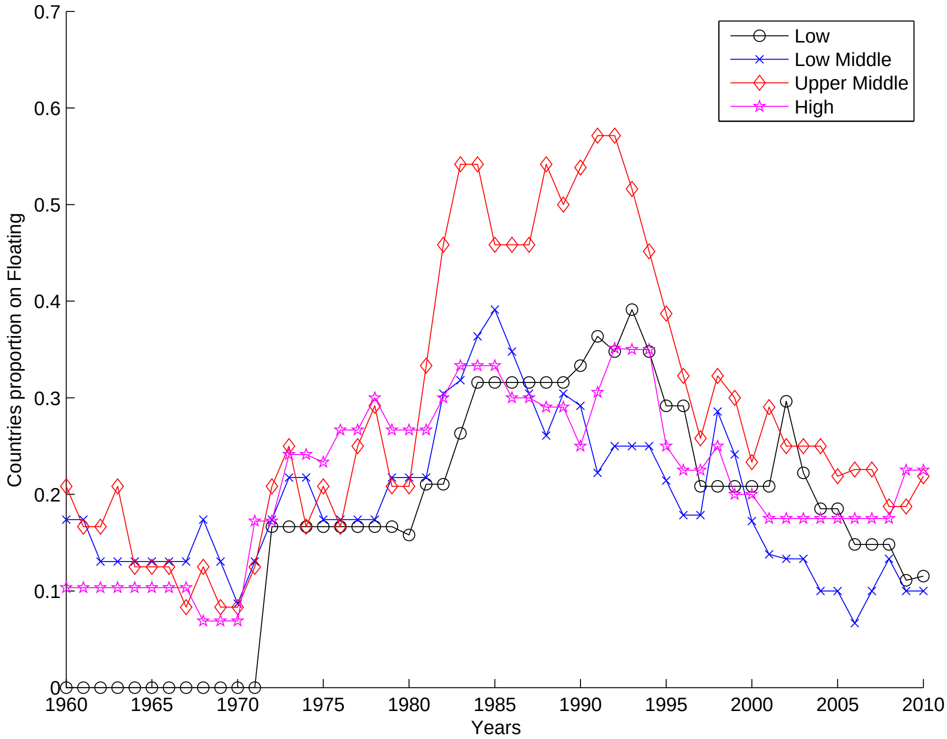

Figure 1.4: Floating Regime by Income

1.3 Model

We adapt from Buera et al. (2011) and propose an economy in which the policymaker does not know exactly how the exchange rate regime (henceforth, ERR) impacts growth and inflation. Instead, he is in constant learning, whenever new data is available to him. In fact, the policymaker observes the recent history of ERR impacting growth and inflation of every country and then shapes his belief of the impact on his country. The data arrives with uncertainty, which is directly proportional to the country's distance from where the information comes from. For instance, a Brazilian policymaker would be less skeptical about information relating ERR and growth (and inflation) coming from Argentine than from China. Moreover, a known cost of adopting the float ERR is also considered, which arises from restrictions on related economic variables. For instance, with other things the same, managing the currency demands a high level of international reserves, so, having it decreases the cost of floating regime.

Specifically, the policymaker is in power for one period and must choose between floating and fixed ERR. Let $\theta_{n,t}$ be a period t indicator variable that equals one if the policymaker of country n chooses floating and equals zero otherwise. With the purpose of analyzing individual driving forces on choices, we rely upon three specifications for policy function. In the first one, the policymaker cares only about growth, while in the second one, the policymaker cares only about inflation. Finally, in the more general specification, the policymaker cares about both variables, weighing each of them into the function. Specifically, the general period t objective function of the policymaker of country n is

$$E_{n,t-1}\left[\begin{array}{c}\mathbb{I}_{(m\in\{1,3\})}y_{n,t}+a\mathbb{I}_{(m\in\{2,3\})}\pi_{n,t}-\theta_{n,t}K_{n,t}...\\-\varphi^{F}\left(\theta_{n,t}-\theta_{n,t-1}\right)\theta_{n,t}+\varphi^{P}\left(\theta_{n,t}-\theta_{n,t-1}\right)\left(1-\theta_{n,t}\right)\end{array}\right] \tag{1-1}$$

where $\mathbb{I}_{(m\in\{j,k\})}$ is an indicator function for the j-th and k-th specification of the policy function, $y_{n,t}$ is the period t growth rate of the per capita GDP of country n, a is the relative weight on inflation, $\pi_{n,t}$ is the period t inflation rate of country n, $K_{n,t}$ is the period t floating ERR cost of country n, φ^F is the switching cost from fixed to floating regime and φ^P is the switching cost from floating to fixed regime. So, the policymaker considers growth and inflation, either separately or weighted in according to the specification, but always considering the cost in adopting the floating regime and the cost of switching between the arrangements.

As aforementioned, the regime's choices are restricted by domestic variables. For instance, with other things the same, it is unlikely to pursue a fixed regime indefinitely with scarce international reserves. Therefore, low international reserves must be an encouraging factor toward a floating regime. To deal with this, we assume a cost of adopting the floating regime, defined by $K_{n,t}$, which is a linear function of a country specific term f_n , a period t vector $\Pi_{n,t}$ of observable economic variables related to regimes' choice (such as international reserves, external debt, gdp per capita) plus an error term $k_{n,t}$. That is,

$$K_{n,t}\equiv f_n+\xi^{'}\Pi_{n,t}+k_{n,t}, \tag{1-2}$$

where $k_{n,t}\overset{\text{i.i.d}}{\sim}N\left(0,\varrho_n^2\right)$.

We follow Buera et al. (2011) and assume each policymaker as having a perceived relation between growth and regimes. Additionally, we also propose a similar policymaker's beliefs on inflation and regimes. Specifically, for each country n, we have

$$X_{n,t}=\mathbb{B}_{n,t}\Theta_{n,t}+\epsilon_{n,t}, \tag{1-3}$$

where $X_{n,t}=\left[\begin{array}{c}y_{n,t}\\\pi_{n,t}\end{array}\right]$, $\mathbb{B}_{n,t}=\left[\begin{array}{cc}\beta^{y,F}_{n,t}&\beta^{y,P}_{n,t}\\\beta^{\pi,F}_{n,t}&\beta^{\pi,P}_{t}\pi^{*}_{t-1}\end{array}\right]$,$\Theta_{n,t}=\left[\begin{array}{c}\theta_t\\1-\theta_t\end{array}\right]$, $\epsilon_{n,t}=$

$\begin{bmatrix} \varepsilon^{y}_{n,t} \\ \varepsilon^{\pi}_{n,t} \end{bmatrix}$, $\beta^{y,F}_{n,t}$ and $\beta^{y,P}_{n,t}$ are the period t perceived economic growth under floating and fixed ERR, respectively, $\beta^{\pi,F}_{n,t}$ is the period t perceived inflation under floating ERR , $\beta^{\pi,P}_{t}$ is the period t proportion in which the inflation pegs to a target inflation under fixed ERR, π^{*}_{t} is the period t target inflation in which we assume being the period $t-1$ US inflation, $\varepsilon^{y}_{n,t} \overset{\text{i.i.d}}{\sim} N\left(0, \Sigma^{y}_{n,t}\right)$ and $\varepsilon^{\pi}_{n,t} \overset{\text{i.i.d}}{\sim} N\left(0, \Sigma^{\pi}_{n,t}\right)$. So, in each period t, the policymaker of country n has different beliefs ($\beta^{y,F}_{n,t}$ and $\beta^{y,P}_{n,t}$) about how regimes impact growth and he believes that inflation will be $\beta^{\pi,F}_{n,t}$ if he adopts the floating regime and a proportion $\beta^{\pi,F}_{t}$ of the last period US inflation if he adopts the fixed regime. We rely on this last feature to avoid additional (in fact, n) parameters within the model, since it already may suffer from possible overparameterization problem. Furthermore, when a policymaker pegs its currency, somehow he is trying to import monetary credibility from a country with low inflation, as the United States. So, usually, the currencies are pegged to the American Dollar. Thus, we see as appropriate to assume the country inflation moving toward the US inflation when the fixed regime is adopted.

The uncertainty arises since the policymaker lacks knowledge of real values of $\beta^{y,P}_{n,t}$, $\beta^{y,F}_{n,t}$, $\beta^{\pi,F}_{n,t}$ and $\beta^{\pi,P}_{t}$. Instead, he makes inference about their distributions whenever new information arrives in each period. Particularly, at the end of period $t-1$, the policymaker observes the relation between ERR choices and growth and inflation from all countries and updates his beliefs about $\beta^{y,P}_{n,t}$, $\beta^{y,F}_{n,t}$, $\beta^{\pi,F}_{n,t}$ and $\beta^{\pi,P}_{t}$. Lastly, at the beginning of period t, he observes the realization of the floating cost and, with knowledge of φ^{F} and φ^{P}, he chooses the ERR to foster. The learning procedure is obtained by application of recursive estimation of weighted least squares in which we endow the policymakers with $\hat{\Sigma}$, the covariance matrix of a reduced form model regarding growth, inflation, and exchange rate regimes. See Appendix for details.

The general problem of the policymaker is to maximize (1-1) constrained to (1-2) and (1-3), obeying the beliefs dynamic of $\mathbb{B}_{n,t}$. Solving it, we have the following optimal policy:

$$\theta_{n,t} = \mathbb{I}_{\left[E_{n,t-1}\left(\beta^{y,F}_{n,t} + a\beta^{\pi,F}_{n,t}\right) - \varphi^{F}(1-\theta_{n,t-1}) > E_{n,t-1}\left(\beta^{y,P}_{n,t} + a\beta^{\pi,P}_{t}\pi^{*}_{t}\right) + K_{n,t} - \varphi^{P}\theta_{n,t-1}\right]},$$

where $\mathbb{I}_{[\cdot]}$ is the indicator function.

Thus, if the last regime was floating, the policymaker chooses to keep it if the difference between the expected growth rate and inflation under floating is greater than this difference under fixed regime plus the costs of floating ERR and the switching toward fixed regime. Additionally, if the last regime was fixed, the policymakers choose to switch to floating if the difference between the expected growth rate and inflation under floating minus the switching cost to floating is greater than this difference under fixed regime plus the costs of floating ERR.

1.4 Inference and Results

1.4.1 Estimation

The model is estimated by using Bayesian techniques.[5] Thus, we are interested in the posterior density of $\alpha \equiv \left[\left\{\hat{\beta}^{y,P}_{n,0}\right\}_n, \left\{\hat{\beta}^{y,F}_{n,0}\right\}_n, \left\{\hat{\beta}^{\pi,F}_{n,0}\right\}_n, \hat{\beta}^{\pi,F}_0, \{\nu^y_n\}_n, \{\nu^\pi_n\}_n, \nu^{\pi,P}, \{\varrho_n\}_n, \xi, \varphi^P, \varphi^F, a, \gamma_y, \gamma_\pi\right]$. Defining $D_t = \left[y_t, \hat{\pi}_t, \theta_t, \Pi_t\right] \equiv \left[\{y_{n,t}\}_n, \{\hat{\pi}_{n,t}\}_n, \{\theta_{n,t}\}_n, \{\Pi_{n,t}\}_n\right]$ and $D^T \equiv \{D_t\}^T_{t=1}$, the application of Bayes' rule results in $p\left(\alpha|D^T\right) \propto \mathcal{P}\left(D^T|\alpha\right) \cdot p\left(\alpha\right)$, where $p\left(\alpha|D^T\right)$ is the posterior density of α, $\mathcal{P}\left(D^T|\alpha\right)$ is likelihood function and $p\left(\alpha\right)$ is the prior density of α.

Exploring the optimal policy[6], we obtain the following likelihood function:

$$\mathcal{P}\left(\theta_{n,t}|\Pi_{n,t}, D^{t-1}, \alpha\right) =$$
$$\left[\Phi\left(\frac{\hat{\beta}^F_{n,t-1} - \hat{\beta}^P_{n,t-1} - f_n + a\left(\beta^{\pi,F}_{n,t} - \beta^{\pi,P}_t \pi^*_t\right) - \xi' \Pi_{n,t} - \varphi^F\left(1-\theta_{n,t-1}\right) + \varphi^P \theta_{n,t-1}}{\varrho_n}\right)\right]^{\theta_{n,t}} \cdots$$
$$\cdots \left[1 - \Phi\left(\frac{\hat{\beta}^F_{n,t-1} - \hat{\beta}^P_{n,t-1} - f_n + a\left(\beta^{\pi,F}_{n,t} - \beta^{\pi,P}_t \pi^*_t\right) - \xi' \Pi_{n,t} - \varphi^F\left(1-\theta_{n,t-1}\right) + \varphi^P \theta_{n,t-1}}{\varrho_n}\right)\right]^{1-\theta_{n,t}},$$

where $\Phi\left(\cdot\right)$ is the standard Gaussian cumulative distribution.

In addition, we use informative priors to mitigate the overparameterization problem of the model. Thus, we assume the parameters as having the following prior distributions:

[5]The model with no learning can be viewed as standard Probit model. However, in advance, the estimated parameters strengthening the learning mechanism are statistically significant. Moreover, the incidental parameters problem can arise in estimation of binary nonlinear models with fixed effects, such as the proposed model either encompassing or not the learning structure.

[6]See appendix for details.

$$
\begin{aligned}
\hat{\beta}^{P}_{n,0} &\sim N\left(\bar{\beta}^{P}_{0}, \omega^{2}_{\beta}\right), & n = 1, ..., N,\\
\hat{\beta}^{F}_{n,0} &\sim N\left(\bar{\beta}^{F}_{0}, \omega^{2}_{\beta}\right), & n = 1, ..., N,\\
\nu^{y}_{n} &\sim \mathrm{IG}\left(s_{\nu}, d_{\nu}\right), & n = 1, ..., N,\\
\nu^{\pi}_{n} &\sim \mathrm{IG}\left(s_{\nu}, d_{\nu}\right), & n = 1, ..., N,\\
\nu^{\pi,P} &\sim \mathrm{IG}\left(s_{\nu}, d_{\nu}\right), & \\
\varrho_{n} &\sim \mathrm{IG}\left(s_{\varrho}, d_{\varrho}\right), & n = 1, ..., N,\\
f_{n} &\sim N\left(\bar{f}, \omega^{2}_{f}\right) & \\
a &\sim \text{Uniform}, & \\
\xi &\sim \text{Uniform}, & \\
\gamma_{k} &\sim \text{Uniform}, & k \in \{y,\ \pi\}.
\end{aligned}
$$

We set $\bar{\beta}^{P}_{0} = 3.05\%$ and $\bar{\beta}^{F}_{0} = 2.19\%$ according to pre-sample average data; $\omega^{2}_{\beta} = 0.03$ to adopt a skeptical view on $\beta_{n,0}$; $s_{\nu} = d_{\nu} = 0.26$ according to Buera et al. (2011); $s_{\varrho} = d_{\varrho} = 0.01$ to avoid the model to fit the data using a large variance in $K_{n,t}$; $\bar{f} = 0$ to prevent a prior view of how floating cost impacts choices, on average; and $\omega^{2}_{f} = 0.02$ to also adopt a skeptical view on f_{n}. Finally, we use flat priors for α, ξ and γ_{k}, so every value is equally likely to all countries.

Finally, the estimate for $\hat{\alpha}$ is obtained by maximization of the posterior of the model. Specifically, we have

$$
\hat{\alpha} = \arg\max_{\alpha} \left\{ \prod_{j \in J} p\left(\alpha_{j}\right) \prod_{n=1}^{N} \left[\mathcal{P}\left(\theta_{n,1} | \Pi_{n,1}, \alpha\right) \cdot \prod_{t=2}^{T} \mathcal{P}\left(\theta_{n,t} | \Pi_{n,t}, D^{t-1}, \alpha\right) \right] \right\},
$$

where α_{j} accounts for the individuals parameters that compose α, J is the set of all estimated parameters in the model, $p\left(\alpha_{j}\right)$ is the prior density of parameter α_{j} according to above definitions and $\mathcal{P}\left(\cdot\right)$ is the likelihood function.

We note that the posterior is not necessarily strictly concave. Thus, it may exist maximization issues in the optimization procedure. To handle this question, the optimization is performed starting from some different initial points. Finally, the estimates are the ones that return the maximum posterior among all maximizers.

Finally, the model is estimated with annual data, ranging from 1970 to 2010 and covering 91 countries. Because of missing data, the total observation is 2314. As briefly discussed early, the possible overparameterization problem is mitigated by using priors in the estimation procedure. Data on inflation, growth, and other economic variables are extracted from World Bank, while data on exchange rate choices are extracted from Reinhart e Rogoff (2004).

1.4.2
Reduced-form regressions

Before going to structural model results, we run unbalanced panel regressions to further examine the relationship between ERR, growth, and inflation. Specifically, we find partial correlations that floating ERR is negatively correlated with growth and positively correlated with inflation, even after controlling for other economic variables.

Furthermore, interestingly for our purposes is to capture some reduced-form evidence of learning in the data. To that end, we also run unbalanced panel regressions and show that the choice of country to adopt floating ERR is correlated with the fraction of neighboring countries following these policies and their past growth and inflation performances.

Specifically, we consider the following model:

$$\theta_{n,t} = \phi_0 + \phi_1\theta_{n,t-1} + \phi_2\tilde{E}_{n,t-1}\left(\bar{y}_t|\theta = 1\right) + \phi_3\tilde{E}_{n,t-1}\left(\bar{y}_t|\theta = 0\right) + ...$$
$$... + \phi_4\tilde{E}_{n,t-1}\left(\bar{\pi}_t|\theta = 1\right) + \phi_5\tilde{E}_{n,t-1}\left(\bar{\pi}_t|\theta = 0\right) + \Phi C_{n,t},$$

where $\tilde{E}_{n,t}\left(\bar{x}|\theta\right) \equiv \frac{\sum_{s=1}^{\tau}\sum_{j:\theta_{j,t-s}=\theta}\exp\left(\frac{-d_{nj}}{\delta}\right)x_{n,t-s}}{\sum_{s=1}^{\tau}\sum_{j:\theta_{j,t-s}=\theta}\exp\left(\frac{-d_{nj}}{\delta}\right)}$ captures some effects of countries beliefs on their choices and $C_{n,t}$ accounts for economic variables (reserves, trade, debt and GDP) as controls; we set $\delta = 2500$, which fixes the effective neighborhood of the median country, defined as $\sum_{j\neq i}\exp\left(\frac{-d_{nj}}{\delta}\right)$, to be 20 countries and we set $\tau = 3$, allowing three years of historical data in shaping the countries beliefs. For robustness purposes, we also run the model setting $\delta = 500$ and $\tau = 6$ and the results do not change significantly.

As our theory argues, we expect $\phi_1 > 0$ as a consequence of persistent belief of the Bayesian learning. Moreover, we also expect that countries are more prone to adopt floating ERR in periods in which their neighbors which adopted the same have higher growth and lower inflation, that is $\phi_2 > 0$ and $\phi_4 < 0$. Finally, we expect countries less prone to adopt floating ERR in periods in which their neighbors which adopted the fixed regime have higher growth and lower inflation, that is $\phi_3 < 0$ and $\phi_5 > 0$. As we can verify in Table 1.1, we find statistical significance in the reduced-form model evidencing that countries tend to adopt floating regime when neighbors do the same and have greater growth and lower inflation, and when neighbors chose to fix and have lower growth and higher inflation.

	(I)	**(II)**	**(III)**	**(IV)**
ϕ_0	0.0514 (0.0292)	0.0517 (0.0292)	0.0482 (0.0399)	0.0484 (0.0399)
ϕ_1	0.8522*** (0.0129)	0.8522*** (0.0129)	0.8510*** (0.0130)	0.8510*** (0.0130)
ϕ_2	1.3553** (0.3619)	1.3537** (0.3685)	1.0268* (0.4836)	1.0257* (0.4842)
ϕ_3	−2.6279** (0.7143)	−2.6335** (0.7438)	−1.8740 (1.1062)	−1.8740 (1.1082)
ϕ_4	−0.0028 (0.0022)	−0.0028 (0.0022)	−0.0050 (0.0029)	−0.0049 (0.0029)
ϕ_5	0.0078*** (0.0022)	0.0078*** (0.0022)	0.0089** (0.0029)	0.0089* (0.0029)
Reserves	−0.0967 (0.0216)	−0.0966 (0.0216)	−0.0939 (0.0215)	−0.0939 (0.0215)
Trade	0.7171 (0.3889)	0.7195 (0.3891)	0.7473 (0.3861)	0.7480 (0.3860)
Debt Service	0.0007 (0.0011)	0.0007 (0.0009)	0.0008 (0.0011)	0.00010 (0.0010)
GDP per capita	3.2488* (1.6307)	3.2495* (1.6302)	3.2106* (1.6487)	3.2030* (1.6480)
δ	2500	500	2500	500
τ	3	3	6	6

*,**and *** mean significant at 5%, 1% and 0.1% respectively.

Table 1.1: Reduced form results with “learning”

1.4.3 Semi-structural Models

Now we turn to the results of the semi-structural models. Table 1.2 reports the point estimates of the parameters with standard errors in parenthesis. As we can see, we find no statistical significance for the parameter weighting the inflation on policy function, besides having a positive sign. So, the policymaker would give more value to high inflation, a non-intuitive result. One possible explanation lies in the fact that we use data containing only countries with inflation lower than 50% per year[7], ruling out countries which suffered from hyperinflation, those in which the most would take advantage of choosing the exchange rate regime when fostering low inflation. Moreover, in last decades, other nominal anchors are adopted by countries in controlling inflation, such as an inflation target.

The results also report that prior belief of the cross-country correlation decreases with geographic distance, since the coefficient λ is positive. This has an intuitive interpretation: in the learning process, the policymaker gives more weight to observed relationship between regimes and growth in closer countries. For example, a Brazilian policymaker is less uncertain about how this relationship in Argentina could be related to his country than this relationship in China.

Regarding floating costs, the results suggest that the larger reserves,Regarding floating costs, the results suggest that the larger reserves, the higher the cost of floating exchange rate regime. In other words, the policymaker tends to choose a fixed regime when the reserves are high. An intuitive result, since high reserves help to lessen the probability of currency attackIn fact, managing exchange rate demands a high level of foreign currency., generating an auspicious environment for fixed regime. This finding is consistent with the early reported IMF's concerns about the arrangements.

[7]We do that because of technical issues. Very high inflation cause misbehavior in the optimization procedure, specifically, the $\hat{\Sigma}$ acquires unstable values.

	Description	Specification I	Spec. II	Spec. III
Inflation Weighting				
a	weight on inflation	0.0115 (0.0074)	-	1
Prior Correlation				
γ_y	geographic distance (growth)	0.0082 (0.0039)	0.0516 (0.0178)	-
γ_π	geographic distance (inflation)	0.0022 (0.039)	-	2.5771 (0.6916)
Floating Costs				
ξ_1	reserves over GDP	0.0203 (0.0098)	0.1366 (0.0524)	0.1664 (0.0855)
ξ_2	debt services over GDP	-0.0005 (0.0001)	-0.0026 (0.0048)	-0.0021 (0.0032)
ξ_3	relative GDP per capita	0.7727 (0.5222)	-0.3156 (0.2246)	-0.5487 (0.2310)
ξ_4	trade over GDP	-0.0101 (0.0029)	0.0012 (0.0936)	0.0576 (0.0528)
Switching Costs				
φ^P	cost from floating to fixed regime	0.0096 (0.0034)	0.0324 (0.0445)	0.0483 (0.0310)
φ^F	cost from fixed to floating regime	0.0042 (0.0034)	0.0420 (0.0377)	0.0542 (0.0330)

Table 1.2: Estimation Results

With respect to debt services, the model reports a slight evidence this variable encouraging floating regime, with the interpretation relying on credibility issues. Besides the reserves, the policymaker can also use the interest rate as a tool to peg his currency to another. However, if the country lacks credibility in maintaining the fixed regime, the international market will request increasing interest rate to offset an expected depreciation. But, existing internal issues limits the increase in interest rate, such as unemployment.[8] So, a country with high debt must stay prone to choose a floating regime. A similar interpretation is pointed by Bleaney e Ozkan (2011), in which the authors claim that the perceived likelihood of using an "escape clause" in pegged regime raises its adoption cost.

The results evidence a negative impact of GDP per capita on floating choice. Views related to political economy are more appropriate to interpret it. Governments fostering low inflation, but also showing low institutional

[8]For example, the Brazil undervaluation in 1999.

credibility, may adopt fixed regime to tame inflationary expectations, unlike countries with high institutional quality. It follows that the formers are more prone to foster a fixed regime, while the latter may embrace a floating regime. These results are in agreement with Levy Yeyati et al. (2010). Regarding trade, we find a negative estimate, evidencing that trade discourages floating regime, as expected. When it comes to the switching cost, we see evidence of positive cost for both, from fixed to floating and from floating to fixed, but only the latter is statistically significant.

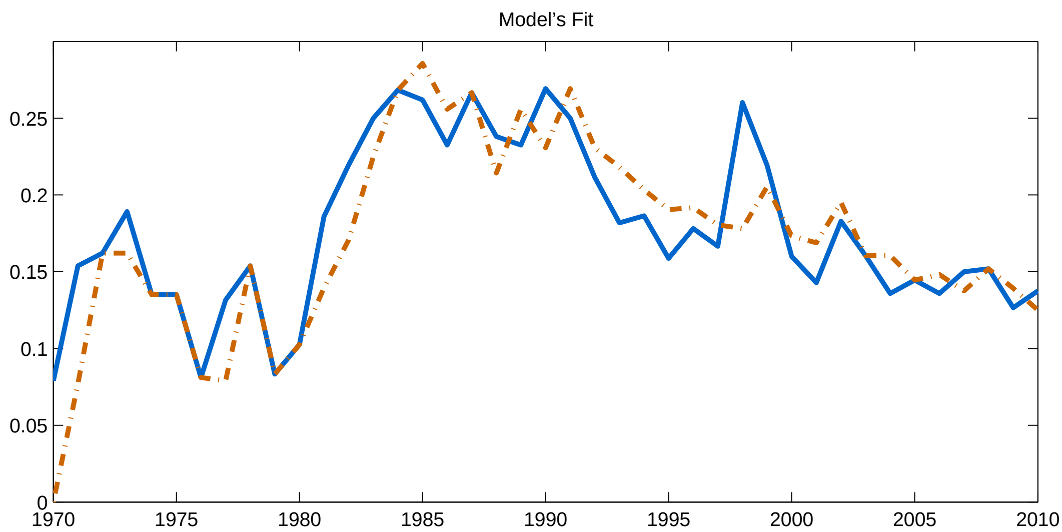

Figure 1.5: Model's Fit

Figure 1.5 reports the model's ability in fitting the observed data. The predicted series corresponds to the one-step-ahead prediction with no shock to the floating ERR. As we can see, the predicted data is able to match fairly well the observed data. In addition, the model predicts 96.4% of the observed policy choices. However, these results must be interpreted with caveats. Additional analysis is requested to assess potential overparameterization problem, since a large number of estimated parameters in the sample. Future works encompass out-of-sample forecasting and a better understanding of learning rule in prediction. Lastly, we also turn off the learning mechanism[9] and the estimates do not change significantly, as well as the prediction rate, which falls marginally to 95.9%.

However, when predicting policy switches, the learning mechanism gain a prominent rule. While the no learning model is unable to predict any regime switch within a one-year window, on the other hand, when encompassing the

[9]Specifically, we let $\Psi_t = \Psi_0$ and $P_t = P_0$, $\forall t$.

learning mechanism, the model forecasts 14% of the switches. So, the learning dynamic is essential to predict policy switches

Lastly, figures 1.6 and 1.7 report the model generated beliefs for growth and inflation, respectively, for each country according to their choices, with the bold line accounting for median values. As we can see in the former, in the median, the growth belief if fixed is greater than growth belief if floating, with both decreasing along years. In the latter, a falling tendency in inflation if fixed is observed, while the opposite movement is verified in inflation if floating.

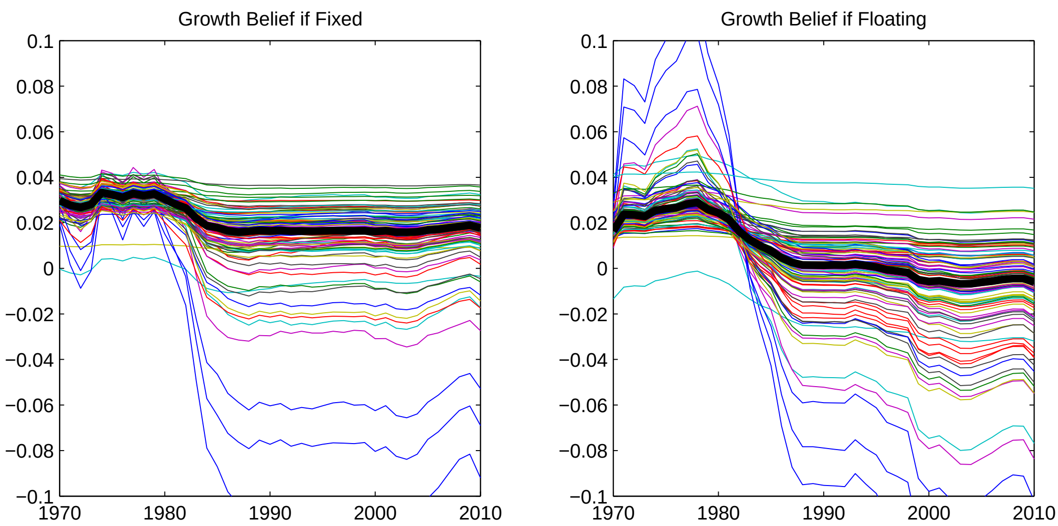

Figure 1.6: Growth Beliefs

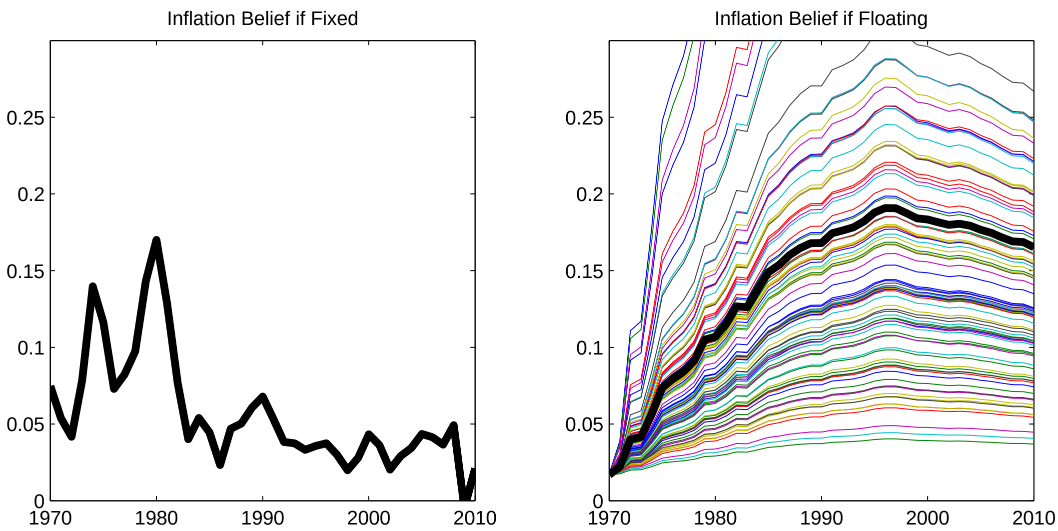

Figure 1.7: Inflation Beliefs

1.5 Conclusion

The paper analyzes how countries choose their exchange rate regime, whether to fix or to float. We explore the lack of consensus on it and propose a model in which the policymaker learns how the regimes impact growth and inflation. Additional to the learning process, relevant economic variables related to an open economy are also considered on choices, as well as costs of switching between regimes.

The model was estimated by Bayesian techniques and we find results suggesting intuitive connections between economic variables and choices. Specifically, the results evidence that higher reserves encourage fixed regime, while higher debt service and GDP per capita encourage floating regime. Moreover, we find evidence of positive switching costs, but statistically significant only from floating to pegged. Lastly, we find no evidence that the policymaker considers inflation in his choice. In fact, fixed regime usually is used to strongly disinflate the economy, while conventional monetary tools are requested to maintain an environment with low inflation. Since, for computational purposes, we have discarded hyperinflation cases, the data encompass only the latter scenario. Finally, as a drawback, the model can suffer from potential overparameterization problem, which we tried to mitigate using priors within the Bayesian scheme.

2
A Model For Household Inflation Expectations

2.1
Introduction

The paper models how households form inflation expectations. To do so, we suggest an environment in which, when forecasting future inflation, the household relies on two informational sources. On one hand, he can derive his inflation prediction from the expectations of professionals, which could be assimilated from economic news reported by the media. On the other, the inflation predicted by a typical person will be one to be replicated by a simple econometric model encompassing a learning structure. As we will see, the learning mechanism plays a major role when discounting past data to predict future inflation, mainly in countries with a history of hyperinflation, just as the Brazilian case. The switches between the alternatives are time-varying, based on characteristics of the sources, such as the historical success in matching past inflation, and on characteristics of the individuals, such as the capacity in assimilating news. Because of the individual characteristics, not all households are guided by the same informational source. So, the mean household inflation expectations is a weighted average considering the ratio of households guided by each informational sources.

Inflation expectations assume a key role in monetary policy, mainly in those countries in which the central bank is committed to price stability. The intuition is straightforward and follows the New Keynesian theory: when planning his optimal consumption choices, the consumer must consider the expected economic conditions that will prevail, including future prices. Thus, inflation expectations influence actual consumption and, as a consequence, the price index to be controlled by the monetary authority. So, how agents form inflation expectations becomes an important matter for policymaker's issues.

Since the work of Muth (1961) and Lucas (1972), the benchmark theoretical model of expectations' formation in economics has been the rational expectations theory, which advocates agent expectations being the true statistical conditional expectation of the variables in the economy. However, this demand for complex knowledge of the world has generated several criticisms on theory, mainly focused on its inadequacy in accounting for real process of economic forecasting. In that way, some works find evidence against rational expectations hypothesis when compared with survey expectations.[1] Therefore, as shown by Bernanke (2007), while the rational expectations theory is helpful for some specific analysis, it might be less effective within an environment in which people know imperfectly the structure of the economy which constantly evolves over time and the private sector lacks full understanding of the policymaker behavior. The Bernanke's criticism is even more valuable when dealing with ordinary people shaping expectations, as in our case.

As a consequence, in the last decade, an increasing number of studies addressing how agents form expectations has gained ground in the literature. Many of them are based on informational frictions.[2] In this way, Sims (2003) elaborates the rational inattention theory, in which "to pay attention" incurs a cost to the agent, since he has a finite capacity to process all information that surrounds him. Thus, this scarcity on cognitive capacities forces the agent to select what is worth for him to pay more, or less, attention to. In addition, Mankiw e Reis (2002) propose a sticky information expectations encompassing a slow dispersal of information among the people regarding macroeconomic conditions. While Mankiw e Reis (2002)'s model accounts for expectations' formation of a general agent, Carroll (2003) is the first one to suggest how household forms expectations on inflation. The author proposes an epidemiological expectations model in which a fixed proportion of households forms inflation expectations when observing the views of professionals reported by the media, while the complementary proportion uses his last predicted value. The work of Lanne et al. (2009) also must be highlighted, in which the authors slightly modify Carroll's model, allowing the household to replicate the last realized inflation instead of professional's view as an alternative to last predicted value when predicting future inflation.

Another growing branch in the literature of expectations' formation is the learning theory, based on the principle of cognitive consistency theory.[3] The theory assumes the agent behaving as an econometrician, so economic variables are forecast by using of time-series econometric techniques. One of the interesting features in the learning procedure is the possibility to adjust the agent's information set and parameters according to his knowledge of the economy. For instance, Branch e Evans (2006), when modeling professional inflation expectations, use an adaptive learning procedure in which inflation and output are present in the agent's information set. In advance, we use

[1] See Pesaran e Weale (2006) for discussion on that.

[2] See e.g. Mankiw e Reis (2002); Carroll (2003); Sims (2003); Reis (2006), Reis (2006) and Lanne et al. (2009).

[3] See Evans e Honkapohja (2001) for an overview of learning on economics and e.g. Branch (2004), Branch (2007); Branch e Evans (2006); Orphanides e Williams (2008) and Easaw e Mossay (2015) for applications of learning on expectations formation.

only inflation in the information set of the household when partially modeling his inflation expectations. Moreover, Weber (2007) finds that professional uses higher constant gain parameter in learning algorithm than a household. Thus, as expected, it seems that the process of updating information is less costly for the former than for the latter.

As contributing to literature, the proposed model brings together features of both processes of expectations' formation when modeling the household inflation expectations. First, we assume the household with informational constraints. As we will see, the household derives utility from the observable historical success, as it being a potential indicator of future success in predicting the inflation. Nevertheless, he may still be guided by the source having the worst past success. This is possible since the model also allows non-observable to affect what source to follow. For instance, the household may rely on the econometric model even if the professional has better historical success, because he may not have paid attention to the news. Thus, assimilated news should be a potential driving force in leaning into the professional's views, but not the only, as in Carroll (2003). However, note that assimilating news from professional's view does not necessarily imply to be guided by professional expectations. The household may still be inclined to follow the econometric model forecast, once again, since it may have better historical success.

Second, the household behaves as an econometrician when not relying on professional's views. In the existing literature of how household forms inflation expectations, when not following the professionals, the household usually replicates his previous forecast, or he assumes the last realized inflation. In our case, the household forecast inflation by using a simple linear relation whenever new data of inflation is available. We model it by adaptive learning with constant gain, which has a predominant role in discounting past inflation when forecasting the future one, being crucial in modeling expectations in an economy which has suffered from hyperinflation, as the Brazilian case. In the same vein, Malmendier et al. (2016) have also emphasized central implications of learning from experience when modeling expectations. According to the authors, the expectations are history-dependent and heterogeneous, with young people placing more weight on recent data than the older ones. So, this latter feature is widely suitable for our purposes.

We also analyze the partial correlation between news absorbed by households and the formation of inflation expectations. In our model, the spreading news is one of the channels in which the household assimilates the professional's views of future inflation.[4] So, with other things the same,

[4]The way in which media influences the people's views about economic aggregate is well

more news would imply more weight for professional expectations in household expectations. Thus, we use realized data for the proportion of households assimilating news of inflation and assess its co-movement with the model generated data related to the probabilities with which the household chooses the professional's view. In fact, we find that household absorbing news has a positive correlation with his choice for the professional inflation expectations. Carroll (2003) and Pfajfar e Santoro (2013) analyze the same implication by considering the impact of news over an observable gap between the prediction of inflation by household and by professional.

The remainder of the paper is organized as follows. Section 2 describes the model; Section 3 assesses the model's fit to the data - emphasizing the major role of learning mechanism when modeling inflation expectations in an economy with history of high inflation - and discusses the correlation between news and the formation of household inflation expectations; Section 4 analyzes the link between central bank transparency and the formation of inflation expectations; and, finally, Section 5 concludes the paper.

2.2
Economic Environment

We propose an environment in which the household forms expectations about future inflation based on two alternative sources: from professional forecasting and from an econometric model forecasting. The professional source reports the inflation expectations of experts and the household can access it by various means, including news reported by the media. A similar way of household absorbing information of inflation is found in Carroll (2003), Easaw et al. (2010) and Pfajfar et al. (2013). As an alternative to the professional source, the household predicts future inflation on his own, which we assume to follow a simple econometric model, encompassing an adaptive learning structure. So, the household updates his inflation forecast as soon as new data on inflation is available. Branch et al. (2006) use a resembling learning structure, but with wider information set, to analyze how professional shapes inflation expectations.

The switches between the alternatives take into account observables related to the sources, such as historical success in matching past inflation, and non-observables related to characteristics of each household, such as propensity to assimilate the news from media.[5] Because of individual characteristics, in each period, not all households are guided by the same informational source.

documented in Blinder and Krueguer (2004)

[5]Another means would be word to mouth. In this case, one household would absorb the news, or even predict as professional, and pass through to others.

Thus, the final household inflation expectations is a weighted mean considering the likelihood of the household relying on each alternative of information.[6] In contrast to Carroll (2003), the household considers the (time-varying) observable historical success of alternatives when choosing, so the assimilated news do not assume the unique role that dictates the source to follow.

Specifically, let y_t be the information source chosen by the household in period t, let $j \in \{P, M\}$ denote the alternative source of information with P for "professional" source and M for "econometric model" source, let X_t be a vector with period t historical success in match past inflation of alternatives[7] and let a_t be a vector of unobservable variables affecting households' tastes of each alternative. For example, the vector a may contain the news assimilated by household, the only channel in which the household chooses the professional's views in Carroll (2003). In addition, consider a situation in which, in each period t, the household obtains information of future inflation from professional expectations with probability $\mathcal{P}\left(y_t = P|X_t, a_t\right)$, while he obtains information of future inflation from his own forecast, which we assume to follow an econometric model, with probability $\mathcal{P}\left(y_t = M|X_t, a_t\right)$. Then, the period t population mean of household inflation expectations given the information set in $t-1$ is

$$\pi^H_{t|t-1} = \mathcal{P}\left(y_t = P|X_t, a_t\right)\pi^P_{t|t-1} + \mathcal{P}\left(y_t = M|X_t, a_t\right)\pi^M_{t|t-1}, \tag{2-1}$$

where $\pi^P_{t|t-1}$ is the professional inflation expectation for period t given the information set in $t-1$ and $\pi^M_{t|t-1}$ is the econometric model inflation forecasting for period t given the information set in $t-1$.

Unlike Carroll (2003), we allow the probability of relying on the information sources to vary along time. For instance, the household must be more prone to lean into the professional source since it successfully matched the last inflations, or still he may be more likely to absorb news in some period. In turn, the absorbing news may be related to non-observable data, such as increasing household reading skills, decreasing newspaper price, more spreading media information and so on. In addition, the non-observable data can also include the tones of news reported in media, since not only the amount of news matters for expectations formation, but also their tone, as exposed by Soroka (2006) and Hamilton (2004). An analogous analysis can be made when household relies on econometric model source.

[6]We can also assume the economy composed of a continuum of agents of measure one, in which a proportion p relies on one source, and the complement $1-p$ relies on another source.

[7]In advance, we define historical success as the negative of mean square error between the last four quarters of realized inflation and the alternative forecasts

Specifically, let $U_{t,j}$ be the period t household's utility when choosing source j that depends of the observable and non-observable variables of the alternatives. Following that, we assume

$$U_{t,j} = X_{t,j}\omega + a_t,$$

where ω is a parameter to be estimated and the others variables are as definied before.

As a rational agent, the household chooses $y_t = j \in \{P, M\}$ which maximizes his utility, that is,

$$y_t = \underset{\{P,M\}}{\text{argmax}} \left(U_{t,P},\, U_{t,M}\right).$$

Since y_t is the maximizer choice, we set $a_{t,j}$ to be Gumbel independently distributed.[8] Then, as showed by McFadden (1974),

$$\mathcal{P}\left(y_t = j | X_t\right) = \frac{\exp\left(X_{t,j}\omega\right)}{\exp\left(X_{t,P}\omega\right) + \exp\left(X_{t,M}\omega\right)}, \qquad j \in \{P, M\}. \tag{2-2}$$

So, with all else unchanged, the higher the historical success of a source, the greater the probability of household to rely on it. The parameter ω accounts for intensity of choice when the elements in X_t differs. For example, if $\omega \to \infty$ and $X_{t,P}$ differs positively from $X_{t,M}$ in an infinitesimal amount, then $\mathcal{P}\left(y_t = P|X_t\right) = 1$. On the other hand, if $\omega = 0$, then $\mathcal{P}\left(y_t = P|X_t\right) = \mathcal{P}\left(y_t = M|X_t\right) = 0.5$ whatever the difference between the elements in X_t. Somehow, the ω can also be seen as a parameter for "inattentiveness level". The equation 2-2 is usually called as conditional logit equation.

When the household does not rely on professional's views of inflation, he predicts it from his own, which we assume to follow an econometric model. This model must be as simple as possible to reflect the limited household's knowledge about the economy and also must incorporate the economy's structure changing over time, as pointed by Stock e Watson (2003), Cogley e Sargent (2005), Sims e Zha (2006) and Sargent et al. (2006). Under those circumstances, the econometric model follows a learning structure for inflation forecasting.[9] The learning literature has gained ground in modeling agents' expectations in economics. For example, Branch e Evans (2006), Easaw e Golinelli (2010) and Malmendier e Nagel (2016) model inflation expectations using a learning mechanism.

[8]The Gumbel distribution is also known as type I extreme value distribution, a particular case of the generalized extreme value distributions.

[9]Some works suggest that household keeps his last forecast when not absorbing the professional's, as Carroll (2003), Lanne et al. (2009) and Easaw et al. (2013).

A mere way to handle those features is assuming

$$\pi_t^M = b_t' x_t + \varepsilon_t,$$

where $x_t = [1 \quad \pi_{t-1}]$ and $E\varepsilon_t\varepsilon_t' = \Sigma_t$. That is, the household considers only the very recent past inflation when forecasting, with its perceived impact on actual inflation changing over time, as caught by the time-varying parameters b_t. A more complex understanding of the economy would imply a more informational vector x_t. For instance, Branch e Evans (2006) use a similar approach to modeling the professional inflation expectations, but incorporating output growth into the vector x_t.

The learning algorithm for the econometric model equation can be obtained as a special case of the Kalman Filter when applied to the estimation of the time-varying parameters. For this purpose, we assume the parameters to follow

$$b_t = b_{t-1} + \eta_t,$$

where $E\eta_t\eta_t' = \Omega_t$ and ε_t and η_t are mutually independent. The Normality of the random sequences ε_t and η_t guarantees the mean squares optimality of the filter.[10]

So, a direct application of the Kalman Filter algorithm delivers the sequence $(b_t)_t$ as follows

$$\begin{aligned}
\hat{b}_t &= \hat{b}_{t-1} + k_t\left(\pi_t - \hat{b}_{t-1}' x_t\right), \\
k_t &= \frac{(P_{t-1} + \Omega_t)\, x_t}{\Sigma_t + x_t' (P_{t-1} + \Omega_t)\, x_t}, \\
P_t &= P_{t-1} - \frac{(P_{t-1} + \Omega_t)\, x_t x_t' (P_{t-1} + \Omega_t)}{\Sigma_t + x_t' (P_{t-1} + \Omega_t)\, x_t} + \Omega_t, \\
\Omega_t &= \frac{\gamma}{1-\gamma} P_{t-1}, \\
\Sigma_t &= 1 - \gamma,
\end{aligned}$$

where $P_t = E_t\left[\left(b_t - \hat{b}_t\right)\left(b_t - \hat{b}_t\right)'\right]$ and γ is a parameter referred as "gain".

[10]A simpler way would by using a AR(1) model. However, that one does not consider structural change and, mainly, does not weight past information when shaping household's beliefs regarding future inflation. This last feature is crucial for our purpose, as we will see for Brazilian case.

If $\gamma = 0$, the model is simply a recursive formulation of ordinary least squares, known in learning literature as Recursive Least Squares (RLS). If $\gamma \in (0, 1)$, the past observations are discounted at geometric rate, and the model is known in learning literature as Constant Gain Least Squares (CGLS).

Once obtained the sequence $\left(\hat{b}_t\right)$, the forecast inflation of the econometric model is

$$\hat{\pi}^M_{t|t-1} = \hat{b}'_{t-1} x_t. \tag{2-3}$$

Therefore, the system composed of equations (2-1)-(2-3) and the professional's views of future inflation allow us to set up the household inflation expectations.

2.3 Model's Analysis

In this section, we analyze empirically the model, in addition to address the prominent role played by the learning mechanism when forecasting inflation in an economy with hyperinflation history. First of all, the analysis needs data of the household inflation expectations, professional inflation expectations and realized inflation. Then, we are able to assess the model's fit and compare it with others in literature. Besides, we also analyze the co-movement between news and the probabilities in which the household chooses the professional's views.

2.3.1 Data

For data on household inflation expectations, we rely on Michigan Survey of Consumers (henceforth, MSC), conducted by the University of Michigan and Thompson Reuters. The survey is nationally representative and interviews approximately 500 households per month. Among the questions, the respondents are asked about their expected inflation rate for the next twelve months in a numerical value. The question is as follows: "By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?"

In order to mitigate possibles outliers, we focus on median answers. The survey shows large heterogeneity between respondents. In fact, there are some households expecting extreme inflation, as we can see in Figure 2.1. A possible explanation for that lies behind the non-uniformity price changes across products. When asked about price change, the household may give a wider weight on a specific product in his consumption basket, generally, the one that most changed, as pointed by Ranyard et al. (2008). Moreover, the household may not have a adequate interpretation about the question, since the word "inflation" is not used. In accordance with Bruine de Bruin et al. (2010), some respondent can interpret it as a question about his personal expenses and report a more extreme inflation expectations. This fact is evidenced in Figure 2.2, which plots the mean and median households' answers to the survey, with the former higher than the latter for every period.

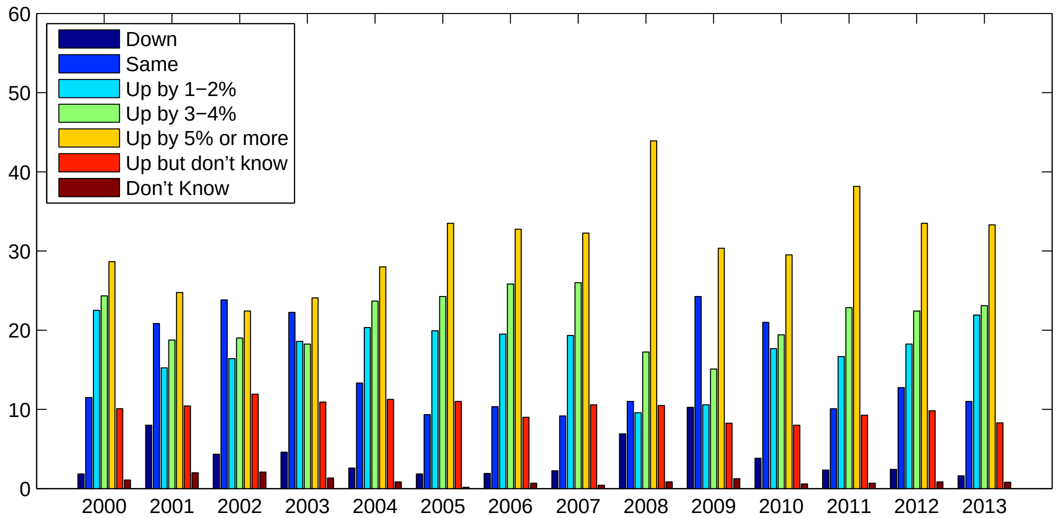

Figure 2.1: Yearly average proportions answers.

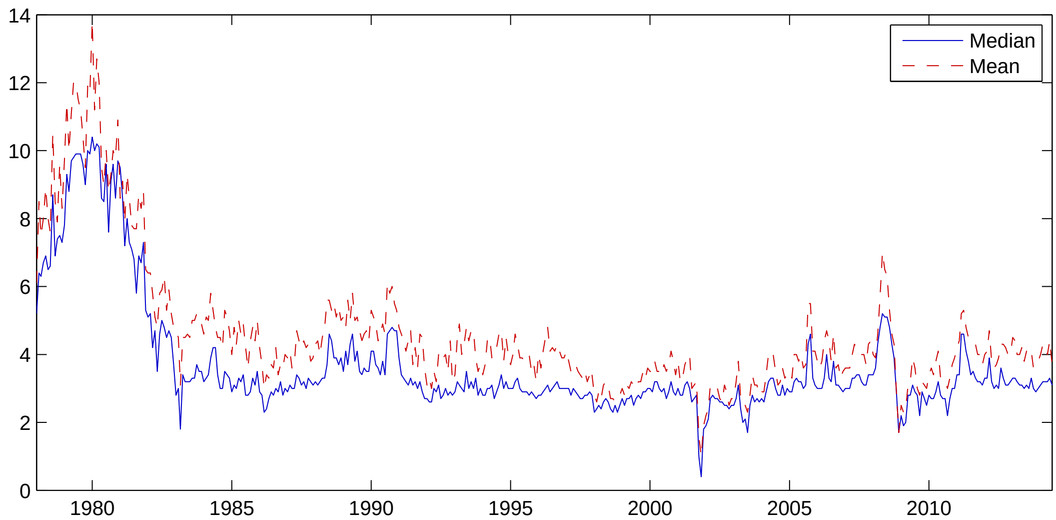

Figure 2.2: Household inflation expectations

For data on professional inflation expectations, we rely on Survey of Professional Forecasters (henceforth, SPF), conducted from 1968 to 1992 by American Statistical Association and National Bureau of Economic Research and since then by Federal Reserve Bank of Philadelphia. The survey's questionnaire is delivered quarterly to professional forecasters, asking them about future inflation, among other variables. Specifically, the respondents are questioned for their forecasts for the next four quarters on Consumer Price Index (CPI) inflation. Since mean and median answers from professionals do not present any bias, we focus on the mean one.

Lastly, for realized inflation, we use the "Consumer Price Index for All Urban Consumers: All Items" available by US Bureau of Labor Statistics. This data is built upon the average monthly change in the price for goods and services bought by urban consumers, and represents the buying habits of households.

2.3.2 Fitting

We now test the model's ability to adjust to the observed data. The fitting is analyzed by using quarterly series ranging from 1985Q1 to 2014Q1. In the learning mechanism, we use a pre-sample estimation for Kalman Filter initialization. In addition, we set $\gamma = 0.0138$ and $\omega = 0.0521$ in order to minimize the mean squared error between the model generated series and the observed household inflation expectations The calibrated constant gain parameter is similar to those found in Orphanides e Williams (2005)2008, Milani (2007) and Malmendier e Nagel (2016): 0.02, 0.0183 and 0.0180, respectively. Finally, we assume the historical success of alternative j as the negative of mean square error between the last four quarters of realized inflation and alternative forecasts, that is[11]

$$X_{t,j} = -\frac{1}{4}\sum_{s=1}^{4}\left(\pi_{t-s} - \hat{\pi}^{j}_{t-s|t-s-1}\right)^2, \qquad j \in \{P, M\}.$$

Figure 2.3 relates graphically the model's ability in fitting the observed household inflation expectations. In our point of view, the proposed model has a satisfactory success in fitting the data, highlighting the short-term movements. However, it seems to be exceptions for the years from 1987 to 1992 and 2010 onwards.

The satisfactory result is reinforced when focusing on the individuals' dynamics of series composing the generated data. Each one follows a specific and distinct path over time if compared with the household inflation

[11] We may have a potential inconsistency here. The problem arises since we assume that the household must know the last predictions of both sources, despite he used only one. However, we can think that variable as "proxying" a *perceived* historical of success. Moreover, we vary the quarters of realized inflation and the results do not change significantly.

expectations, as we note in Figure 2.3. Therefore, the model does not mimic any individual series, but it is the proposed dynamic combination of them which provides the good fit into the data.

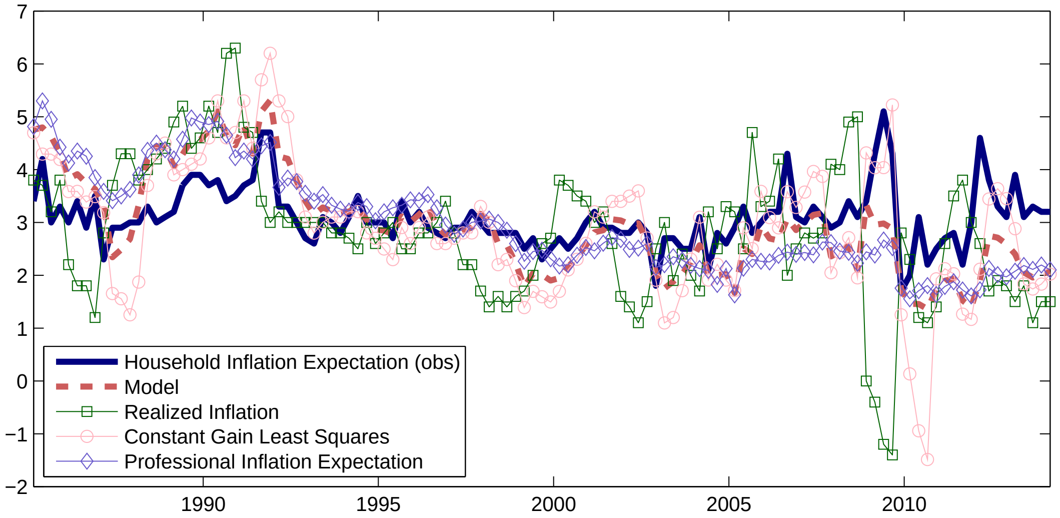

Figure 2.3: Model's fit and components

We also compare the proposed model with others. The Table 2.1 reports the mean squared error between generated and observed data, along with mean and variance of the related individual data. Focusing on those statistics, our model's generated data has a satisfactory fit to the observable data, performing above the Carroll's model, but slightly below the one proposed by Lanne et al. (2009). Moreover, once again, it is noteworthy that the proposed model is able to keep up with short-term movements of observed household inflation expectations, as we can note in Figure 2.4. In addition, despite the marginal lower performance compared to Lanne et al. (2009)'s, our model brings specific features useful to address two interesting matters: learning and inflation expectations within a hyperinflation history, and the relationship between news, household expectations and professional expectations.

	RMSE[1]	Mean	Variance
Household Inflation expectations (obs.)	0	2.9560	0.3216
Model	**0.7951**	**3.0940**	**0.8886**
Carroll (2003)	0.9424	3.1664	0.9092
Lanne et. al (2009)	0.7384	2.9119	0.6886
Recursive Least Squares	1.0026	2.9175	1.5711

[1] Relative to observable of household inflation expectations.

Table 2.1: Models' statistics (USA)

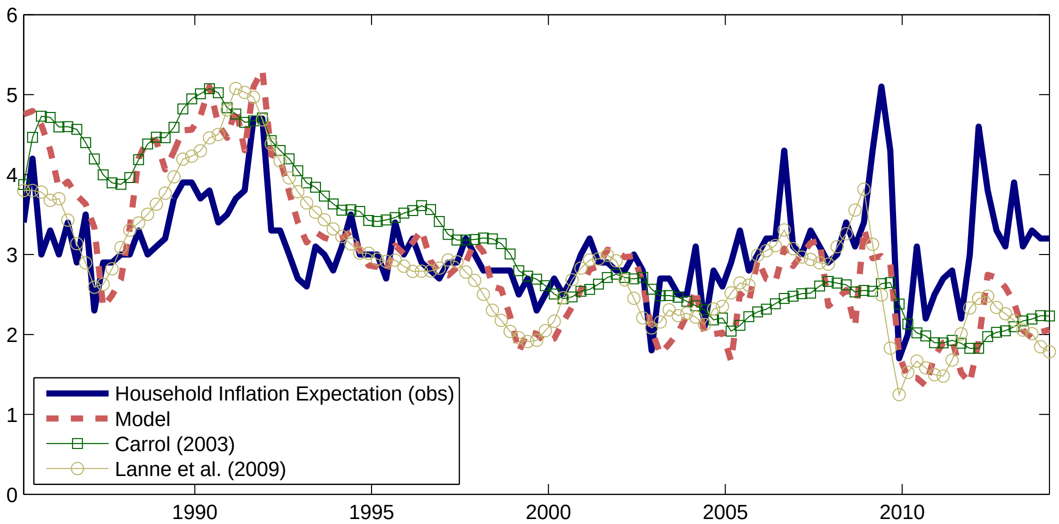

Figure 2.4: Alternatives models

2.3.3 Learning mechanism and hyperinflation history

The learning mechanism acquires major role in modeling the household inflation expectations in an economy with a record of high inflation, since it allows the consumers to discount past inflation to forecast the future one. It follows Malmendier e Nagel (2016), which argue that expectations are history-dependent and heterogeneous, with young people placing more weight on recent data than the older ones. So, individuals' personal experiences are relevant in shaping their expectations. In similar way, As a consequence to our purposes, while for the US economy, with no history of hyperinflation, the proposed model performs marginally below the one proposed by Lanne et al. (2009), for the Brazilian economy, the former overcomes significantly the latter, largely due to the adaptive learning mechanism, as we will see below.

To assess it, we use the aforementioned methodology and address the same analysis of the Brazilian economy, which has suffered from hyperinflation in the 1980's until 1995. Specifically, we explore the fact that there also exists a series of observed Brazilian consumer inflation expectations, computed by IBRE/FGV in the same vein of the Michigan Survey. The Figure 2.5a reports the realized inflation and the consumer expectations. As we can see, the consumer expectations have a positive bias, mainly if compared with the professional expectations. This may be due to a high inflation period that might cause an "inflationary memory" in the consumer expectations. If we use a simple model of adaptive expectations, such as an auto-regressive one, as an alternative to following the professional's view, a pure memory inflationary emerges for household inflation expectations, as exposed in Figure 2.5b. So, the early argued adaptive learning model acquires importance in weight past inflation when the households are forecasting the future one.

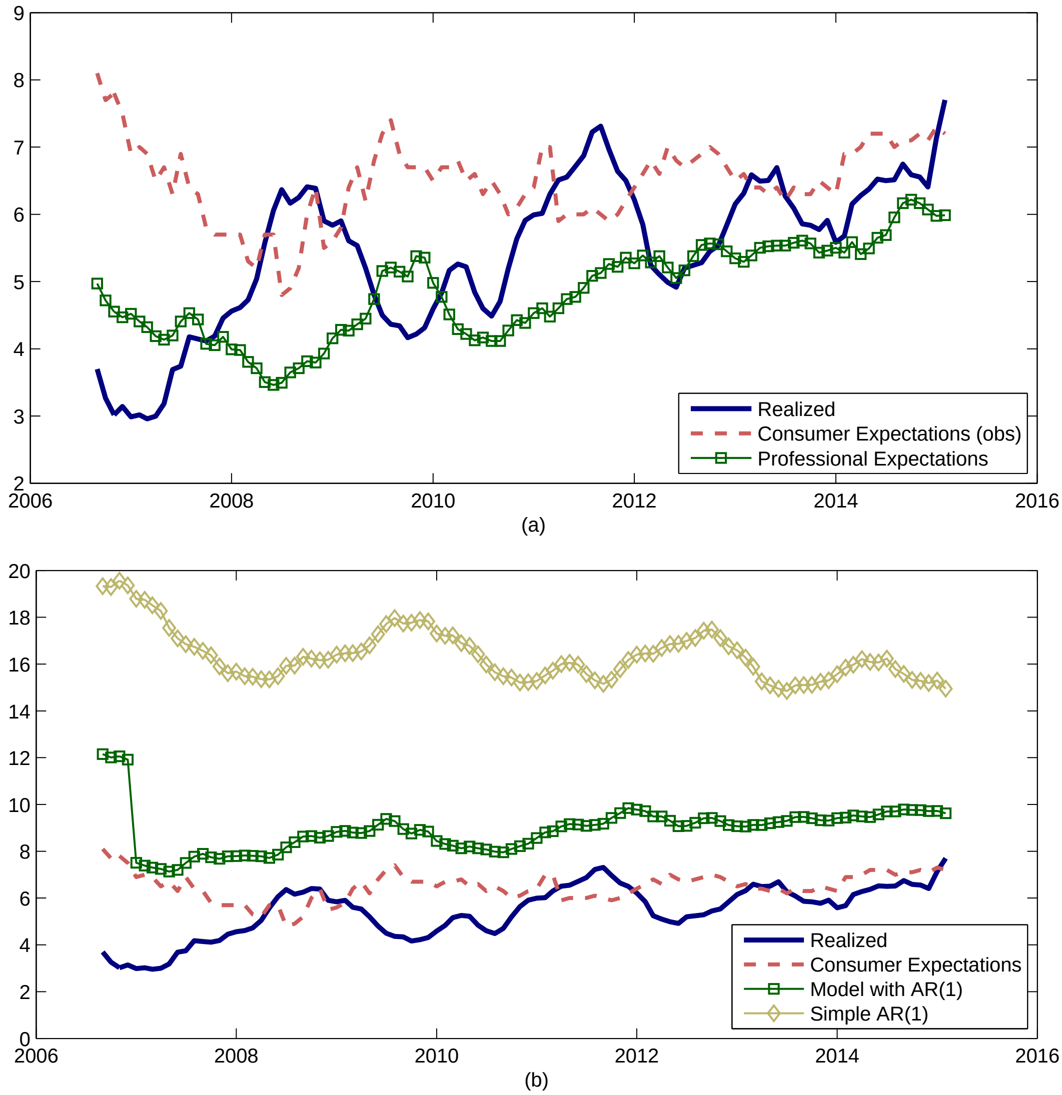

Figure 2.5: Brazilian inflation expectations

In order to estimate the adaptive learning model, we use IPCA (the official Brazilian inflation index) data ranging from January of 1981 until February of 2015. For professional expectations, we use data from Central Bank of Brazil with professional inflation expectations for twelve months ahead ranging from January of 2004 until December of 2014. The learning parameter was estimated in $\gamma^{BR} = 0.0101$ in a monthly frequency, against $\gamma^{US} = 0.0138$ in a quarterly frequency for the US, implying that Brazilian consumers use 8.2 years of inflation data while Americans use 18.1 years. We interpret it as resulting from the fact that Brazilian economy was subject to more frequent structural breaks, with the ending of the high period inflation being the most important for our results. Moreover, the parameter accounting for intensity of choice is estimated in $\omega^{BR} = 0.0013$, while for the US we estimated in $\omega^{US} = 0.0487$. So, it seems that American consumers are more sensible to the difference of historical success between the learning model forecasting and the professional expectations.

Figure 2.6 reports the model's fit compared to the Carroll's and to the Lanne et al. (2009)'s models. Once again, we are satisfied with the adjust of the model to the observed data, in which we see the learning mechanism playing a crucial role. As we can note, unlike for the US economy, the proposed model has a significantly better performance if compared with the alternatives when fitting the expectations. We see the improvement as resulting from the fact that the proposed model allows the Brazilian consumers to discount past inflation when forecasting the future one, emphasizing the importance of learning in shaping their expectations. Lastly, Table 2.2 reports some models' statistics, reinforcing the better performance of the model encompassing the learning structure when compared to the others.

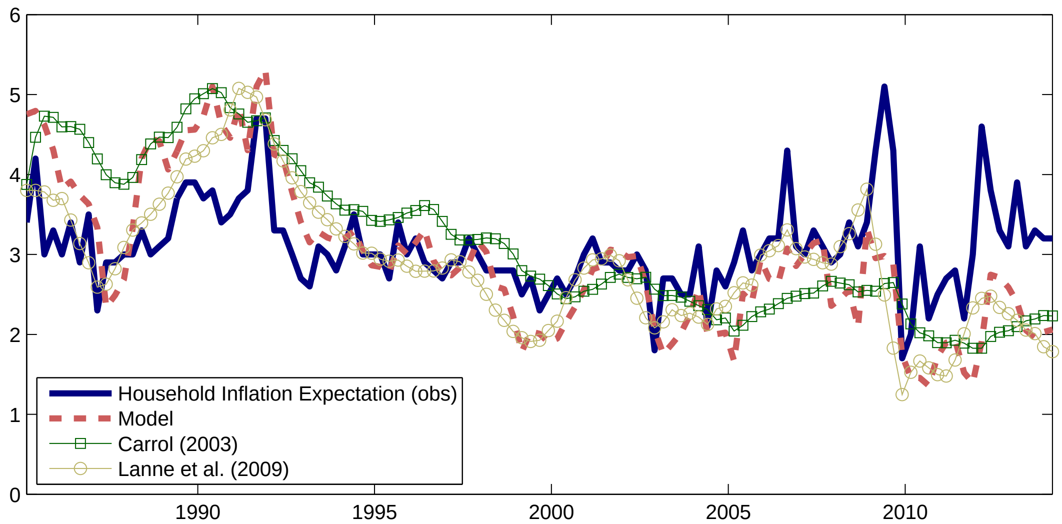

Figure 2.6: Proposed model and alternatives

	RMSE[1]	Mean	Variance
Household Inflation expectations (obs.)	0	6.4882	0.3689
Model	**0.3700**	**6.5076**	**0.8886**
Carroll (2003)	1.5738	5.0742	0.5117
Lanne et. al (2009)	1.6725	5.2626	0.9847
Recursive Least Squares	1.9733	8.2741	1.0899

[1] Relative to observable of household inflation expectations.

Table 2.2: Models' statistics (BRA)

2.3.4 News, household expectations and professional's view

According to the model, the household can assimilate professional inflation expectations when in contact with several means of communication. So, with other things the same, more news heard by households would imply more choices on professional's view. Since the model can generate the choice's probabilities, we have an intuitive and direct manner to assess the co-movement between news and the formation of household inflation expectations, something missing in the other models. Just for reminding, the alternative models assume as sticky the ratio of households leaning into the professional's view.

How the media influences the formation of public expectations is deeply analyzed by Blinder and Krueguer (2004). The authors use a specially-designed survey and find that large majority of Americans citizens wants to be well informed about economic issues, with the television and newspapers being their preferred source of information.[12] In light of these findings, Carroll (2003) and Pfajfar e Santoro (2013) examine the relationship between news and household inflation expectations using the square of the gap between the Michigan Survey of Consumers' and Survey of Professional Forecasters' predictions as a measure for "household behaving as professional". So, the greater amount of news would imply a shorter gap. However, in our view, this measure may be misspecified in capturing the impact of news on household's choice. For instance, in a country with well-anchored inflation expectations, even if the household does not choose to follow the professionals, both expectations will still be closer. Therefore, we see choice's probabilities on professional expectations as a more appropriate variable for measuring the impact of news on household expectations.

In order to circumvent this matter, we assess the co-movement of news with our generated choice's probabilities on professional expectations, instead of the gap measure. So, as in Pfajfar e Santoro (2013), we explore the fact that MSC also asks households if they have heard news about recent changes in prices. Specifically, the following question is addressed: "During the last few months, have you heard of any favorable or unfavorable in business conditions?". If so, the interviewer asks the second question, "What did you hear?", and shows a list of options regarding business conditions, including prices, that the household might have heard about. Finally, we use the proportion of the survey's respondents who had heard about inflation as the variable related to amount of news on inflation assimilated by the households.

[12]Nowadays, the Internet must be a key source of information for the general public.

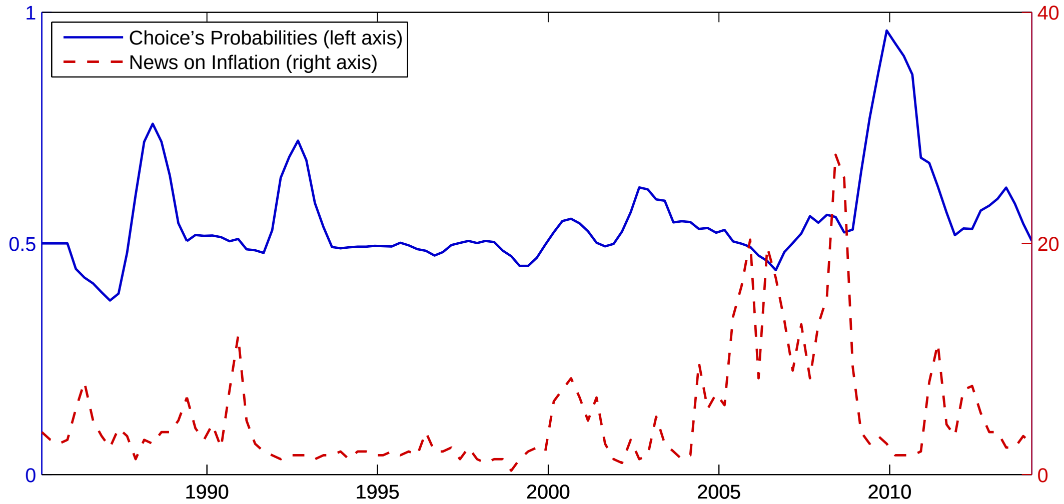

Figure 2.7: Choice's probabilities and news

Figure 2.7 reports choice's probabilities on professional expectations and news absorbed by households. On average, the household chooses professional expectations in 55% of cases, differently from the 36% of Carroll's results and from the 28% of Pfajfar e Santoro (2013)'s. A possible explanation lies on the better performance by professionals in forecasting the inflation.[13] In this sense, while in our model the household must choose between the econometric model and the professional one, in the others, the selection is between the actual professional's forecasting and the last model's forecasting, with the latter including a proportion of the last professional's forecasting. Since quarterly inflation presents persistence (Cogley e Sargent (2002)), the last model's forecasting may be more informative about actual inflation that the actual econometric model of households. Therefore, if the household considers the historical success of the alternatives in his choices, he will give a more weight on professional's forecasting in our case than in others. Indeed, we find a positive correlation of 0.26 between news heard by households and the proportion of them leaning into the professional's view.

2.3.5 Demographic groups

The data shows a disparity in the inflation expectations of distinct groups of people. For instance, the average expectations are higher the lower the income, while expectations standard deviation rises as income grows, as we can note in Table 2.3. Following Bryan e Venkat (2001), differences in inflation

[13] The root mean square error, always relative to realized inflation, are 1.35, 1.47 e 1.22 for model, MSC and SPF, respectively.

expectations among the groups might arise from shared views within each of them, instead of few extreme perceptions. The Figure 2.8 reinforces this belief, since the densities somehow assume distinct shapes between the demographic groups, with outliers occurring without exception.

	Mean	**Median**	**Standard Deviation**	**Min**	**Max**
Median Expectations	3.7098	3.1000	1.7385	1.0667	10.2000
Low Income	3.8292	3.3000	1.1947	1.8667	10.0667
Middle Income	3.4937	3.1000	1.4556	0.9333	10.1333
High Income	3.2940	2.9000	1.2997	0.3000	10.5333
Professional Expectations	2.9746	2.6283	0.9527	1.5551	6.3000
Realized	2.8137	2.8000	1.2997	-1.4000	5.2997

Table 2.3: Statistics of expectations by demographic groups

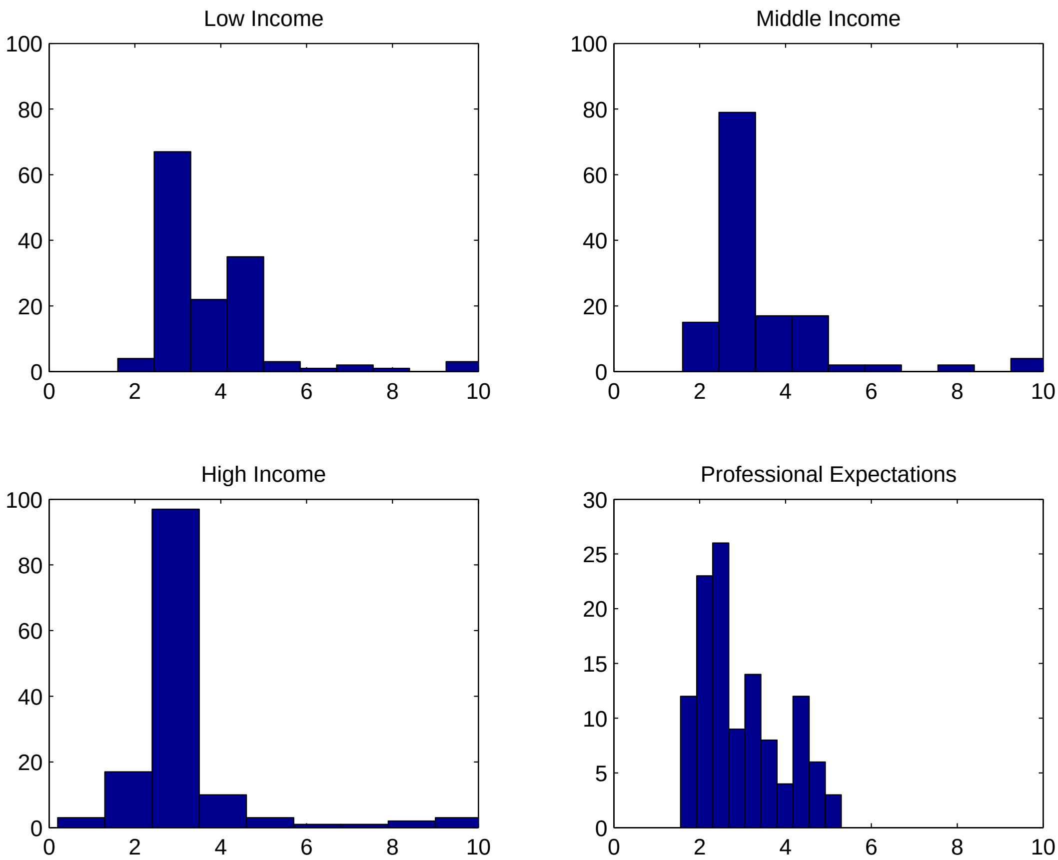

Figure 2.8: Histogram of the expectations

A possible explanation for different inflation expectations relies on the distinguished basket of goods consumed by the specific groups. As mentioned before, when asked about a price change, the household may give a wider weight on an individual product in his consumption basket, as pointed by Ranyard et al. (2008). So, as distinct income groups consume different baskets of goods, a heterogeneous perception of inflation may arises. Nevertheless, as showed by Kokoski (2000), the changing of the CPI weights according to demographic groups consumption has a minimal impact on the inflation series.

Thus, in order to shed additional light on the matter, we again run the proposed model but now considering inflation expectations by each income group. Table 2.4 reports the calibrated parameters. As we can see, the gain parameter reduces as income increases, while the contrary occurs for the intensity of choice parameter. Regarding the latter, *when* relying on the econometric model, a lower-income household discounts more the past inflation, with the top income group virtually adopting the recursive least square when learning. On the other hand, the higher-income is more attentive regarding the historical success of the alternatives, somehow in line with Burke e Manz (2014), if we consider the high correlation between income and literacy.

	γ	ω
Median Expectations	0.0140	0.0487
Low Income	0.0245	0.1273
Middle Income	0.0135	0.1431
High Income	5.2366e-009	0.6867

Table 2.4: Parameters by demographic groups

These results arise as a plausible explanation for the distinguished inflation expectations among the demographic groups. For instance, focusing on the distributions' average, as reported in Table 2.3, the more sensitiveness of the higher-income households to the past success of the alternatives is a driving force leaning them into the professional expectations, which have lower central statistics, in addition to be a more accurate option in general. In fact, the RMSE between realized inflation and forecast inflation by the learning model of low-, middle- and high-income are 1.62, 1.63 and 1.64, respectively; while the RMSE between realized inflation and professional expectations is 1.21. On the other hand, one should argue that, in a downward trend inflation, discounting past inflation more intensively would lead to a less average forecast inflation in the learning models. Indeed, it occurs, but only by marginal amounts.[14] An analogous analysis can be made for variation measures. Inspecting the Figure 2.3, we see more extreme values in the learning models, as reflecting lagged extreme values in realized inflation. Since auto-regressive models are not suitable in forecasting "shocks", when occurring those cases, the very recent past success of the learning models becomes compromised, further encouraging the higher-income household to follow the more stable professional inflation expectations.

2.4
Central Bank Transparency and Expectations

In this section, we explore the proposed model to analyze the relationship between central bank transparency and the behavior of the agent's inflation expectations. Specifically, we use the suggested environment to generate data on household inflation expectations for others countries than the U.S. To do so, we use parameters of demographic groups as calibrated in subsection 2.3.5. For countries classified as "middle income" by the IMF, we rely on calibrated parameters for Americans low income group; while for those ranked as "high income", we consider Americans middle income group. Note that, as we are interested in modeling average household, we see as more appropriate to assume this linkage. Finally, with the observable series of professional inflation expectations, we analyze if transparency of central bank correlates with the predictability of inflation by the household and professional agents.

Following Faust e Svensson (2001), we define transparency as how easily the private sector can infer the central bank intentions from "observable". In turn, central bank manages its level of transparency by providing information to the private sector about the objectives of monetary policy, the economy outlook, and the policy decisions. How the central bank speaks to the private sector is called communication, which may be through various channels, such as press conferences, minutes and regular bulletins.

Until the 1990's, the central banks were shrouded in mystery about their actions, so they communicated as little as possible about their monetary stance. However, the past decades witnessed a grown changing in central banks behavior toward the improvement of predictability of their policies, resulting in a new role for central bank transparency with communication as a key instrument for acquiring predictability of monetary policies. As exposed by

[14] The averages of forecast inflation in the learning models are 2.90, 2.92 and 2.94 for low-, middle- and high-income households, respectively; while the medians are 2.90, 2.90 and 2.92.

the New Keynesian models[15], predictability matters because expectations are relevant for the effectiveness of public policies. For example, when choosing inter-temporally the decision for consumption and investment, the private sector must consider the longer-term interest rate, which, in turn, depends on private sector's expectations about future central bank action.

In particular, most central banks have the control of inflation as objective and, since prices in the economy have some degree of stickiness, the expectations of agents take on an important role in the dynamic of inflation. So, a relevant question which emerges is if, without using of traditional monetary instruments, the central bank can behave in a way which facilitates its own goal, such as being transparent, independent or credible. For instance, the literature has found that measures of central bank independence are negatively correlated with average inflation.[16]

For transparency to affect the predictability of monetary policy, it is not sufficient what type of information the central bank releases, but also how this information is communicated to the private sector. In an extreme case, in which central bank and private sector have perfect knowledge about the economy, a minimum level of transparency guarantees perfect prediction of monetary policy, and so inflation, by all agents.[17] However, in practice, there exist some levels of asymmetric information that increase uncertainty about the monetary stance between the public and private sector. It is in these cases that transparency of central bank comes into play. Blattner et al. (2008) indicate four sources of information asymmetry. First, the private sector may not know the objectives of the central bank. Second, the private sector may not be aware of the set of relevant indicators that guides the central bank decisions. Third, the private sector may be uncertain about the central bank's interpretation and reading of the data. Finally, the private sector may not understand the operation of monetary policy.

In addition, it may exist information asymmetry within the private sector, as evidenced by Carvalho e Nechio (2014). The authors find that partial effects of unemployment on interest rate are better understood, in a sense of Taylor rule, by the household with some college degree than by household with less education. Agreeing with that, professionals may have a better understanding of monetary policy than households and, as a consequence, it can result in the better inflation prediction evidenced for the U.S. economy in footnote 13. For

[15]Good book references for new Keynesian models are Woodford (2003), Galí (2009) and Walsh (2010).

[16]See e.g. Cukierman (1992) and Eijffinger e de Haan (1996).

[17]In this case, it is sufficient the private sector to know the policy instrument used by the central bank, then the policy rule would be deduced and updated whenever new relevant information is available.

this case, transparency may have a different impact on inflation predictability depending whether professional's or household's.

A possible channel of central bank communication into the household is via news, so this section speaks to the last one. We define household in the broadest sense, encompassing every non-professional agent, including people from media. Thus, to conform to MSC data, a more transparent central bank reflects in a more news about inflation in media, with others things the same. In this way, even if the improvement of transparency has low (or none) impact on the performance of professional inflation prediction, it may improve the household inflation prediction by way of increasing absorbed news on inflation that reflects the more accurate professional inflation prediction. Figure 2.9(a) plots the U.S. yearly average of proportion of households that absorbed news on inflation reported in MSC data and the U.S. index for central bank transparency to be discussed ahead. Figure 2.9(b) plots the U.S. yearly average of proportion of households that absorbed news on inflation reported in MSC data and the U.S. yearly average realized inflation. As we can see in both figures, news and transparency seem to move together, with short-run movements of news being highly correlated to realized inflation. This last correlation was discussed in the last section. Figure 2.9(c) plots the average transparency of central banks and the average choice's probabilities toward professional views, both for countries in our sample. This figure evidences the channel in which the household improves his inflation prediction when transparency is growing.

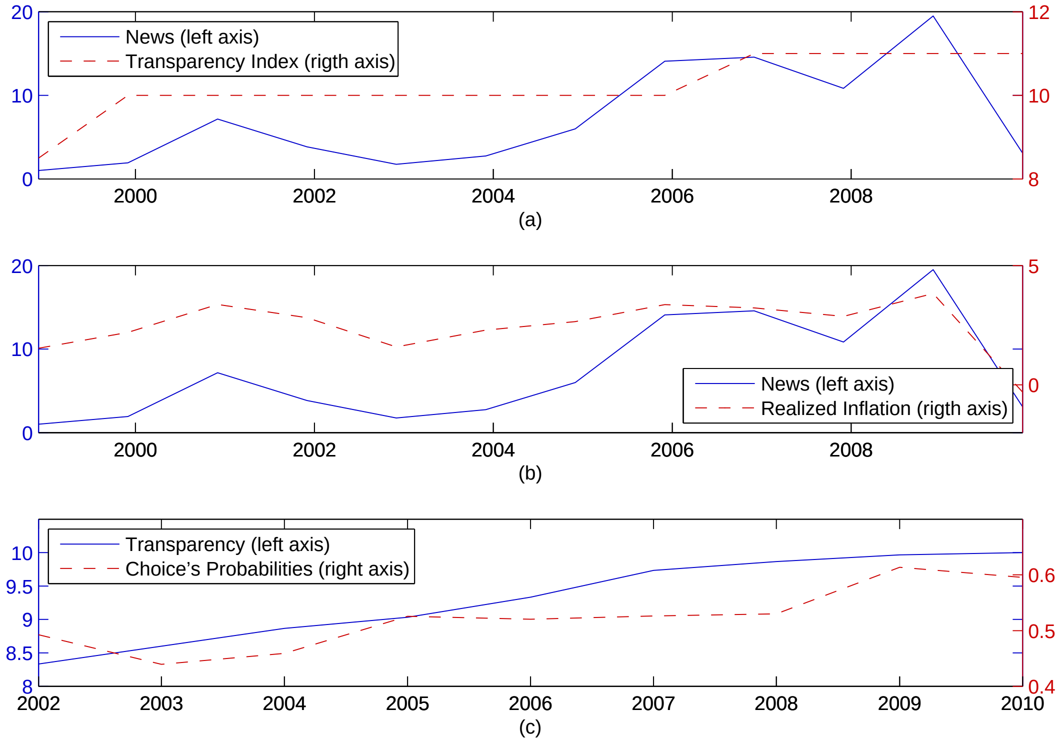

Figure 2.9: News, transparency and inflation

2.4.1 Data

We use data on transparency from Dincer e Eichengreen (2014). The authors create measures of transparency and independence for more than 100 central banks. The index of central bank transparency is built by obtaining information from the websites of central banks, where possible, and ranges from 0 to 15, according to scores for answers to fifteen questions. A question example is "Is there a formal statement of the objective(s) of monetary policy, with an explicit prioritization in case of multiple objectives?". If there is no objective(s), the index scores 0; if there are multiple objectives without prioritization, the index scores 0.5, and if there is a quantification of the primary objective, the index scores 1. In fact, the index considers how easily the private sector can get data on intentions of the central bank and how informative it is. The index for independence is constructed in a similar way.

Figure 2.10 plots the average indexes of transparency and independence built by Dincer e Eichengreen (2014) for the countries in our sample. As we can see, the average central bank seems to be more transparent and independent over time. The two series move together since the transparency is a key element for accountability of independent central banks. As the central bank independence is a mean of insulating monetary policy from short-term political pressures, the transparency becomes a mechanism enabling the public to verify if the central bank actions comply with its objectives.

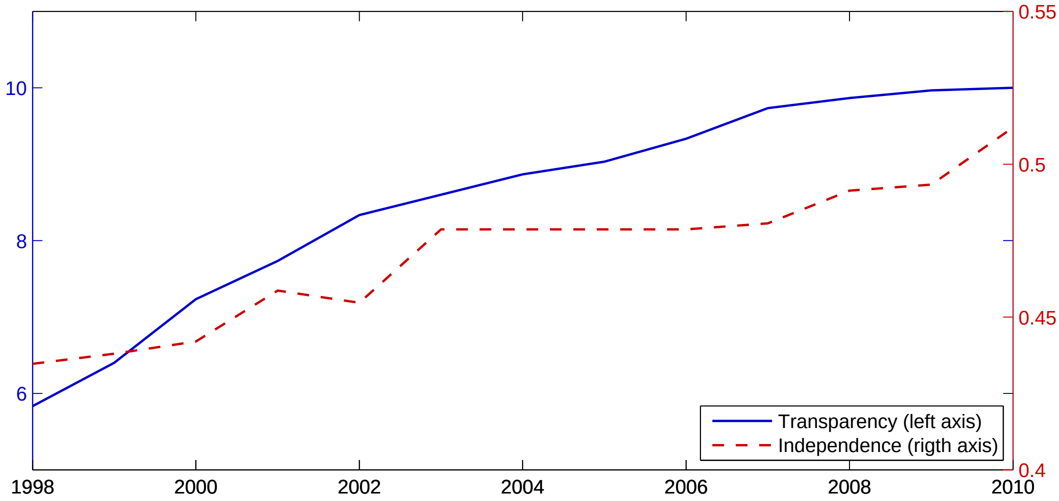

Figure 2.10: Transparency and independence indexes

We use data on professional inflation expectation from Consensus Economics, a London-based international economic survey organization that polls more than 700 economists each month to obtain their forecasts for the principal macroeconomic indicators, including inflation, over countries. We have access to monthly inflation predictions of 15 countries[18] for may-2002 until mar-2014. We construct household inflation expectation for the same fifteen countries using the American calibrated parameters of section 3. Figure 2.11 plots realized inflation (solid line), household expectation (dashed) and professional expectation (dotted).

As we can see, the expectations of agents seem to be well anchored for that countries, with the exception of Hungary and Poland, and for most countries in the period immediately after the crisis in 2008. On average, the professionals have better inflation prediction than households, with the average root mean square error of 1.86 for the former and 1.97 for the later. However, this result is reversed for Chile and India, 2.58 against 2.53 and 4.27 against 4.02, respectively.

Finally, the monthly series of realized inflation are extracted from International Financial Statistics database, built by Statistics Department of the International Monetary Fund. We use the Consumer Prices as inflation.

[18]The countries are Canada (CAN), Chile (CHI), Colombia (COL), Czech Republic (CZE), United Kingdom (GBR), Hungary (HUN), India (IND), Israel (ISR), Mexico (MEX), Norway (NOR), Peru (PER), Poland (POL), South Korea (SKO), Sweden (SWE) and Thailand (THA).

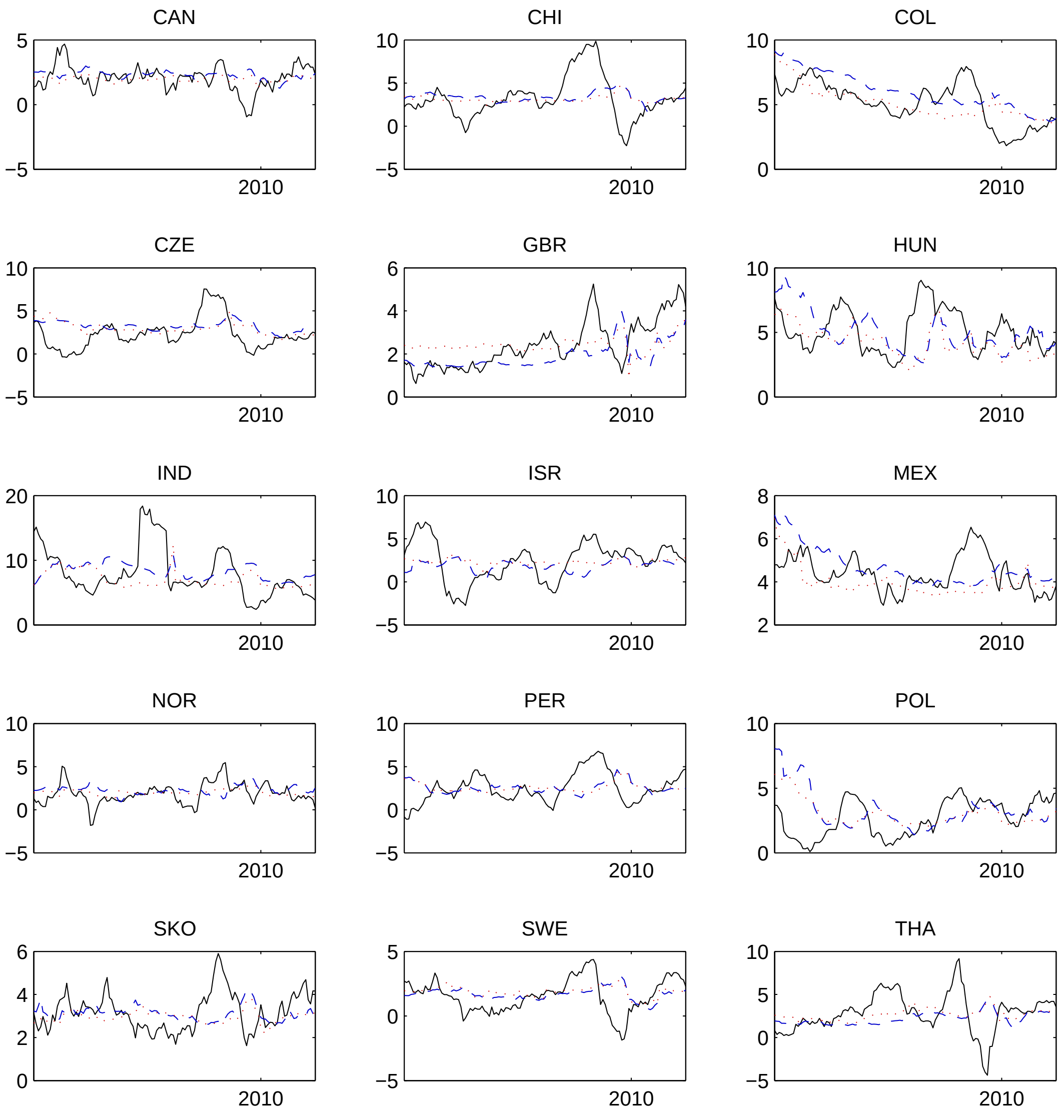

Figure 2.11: Realized inflation, household and professional expectations

2.4.2 Results

We test for some evidence if central bank transparency impacts the performance of inflation predictability of agents. An annual data is used, encompassing fifteen countries, ranging from 2002 to 2010, in a total of 135 observations. We use an annual root mean square error (RMSE) as measure for predictability and inspect how transparency affects it. Specifically, we define $RMSE^a_{n,t}$ as the year t root mean square error of agent a in country n, that is,

$$RMSE^a_{n,t} = \sqrt{\frac{1}{12} \sum_{m \in M} \left(\pi_{n,m+1} - \hat{\pi}^a_{n,m+1|n,m} \right)^2},$$

where $M \in \{$january, february, ..., december$\}$ is the set of months, m is a month from this set, $m+1$ is the month which follows m according to the calendar, a is the agent who makes the inflation prediction, $\pi_{n,m+1}$ is the realized inflation of country n in the month $m+1$ and $\hat{\pi}^{a}_{n,m+1|n,m}$ is the prediction of inflation of country n for month $m+1$ made in month m by the agent a . Note that, since we are interested in how the central bank transparency impacts the ability of the agent in to forecast the inflation, it is the timing of his set of information when predicting that is relevant in creating $RMSE^{a}_{n,t}$.

Then, we regress the following equation using a panel data model,

$$RMSE^{a}_{n,t} = \alpha_0 + \alpha_1 \cdot \text{transparency}_{n,t} + \Psi \cdot C_{n,t} + \nu_{n,t},$$

where $RMSE^{a}_{n,t}$ is how explained above, $\text{transparency}_{n,t}$ is the year t index of central bank transparency of country n, $C_{n,t}$ is a set of year t control variables of country n, $\nu_{n,t}$ is a year t term of error of country n with $E\left(\nu_n\right) = 0$, $E\left(\nu_n \nu_n^{'}\right) = \theta_n^2 I_T$, $E\left(\nu_n \nu_j^{'}\right) = 0$ for $n \neq j$, I_T is the $T \times T$ identity matrix and T is the length period.

Description	(I)	(II)	(III)	(IV)
Constant	2.088*** (0.330)	1.722*** (0.345)	2.281** (0.9862)	1.444 (1.582)
Transparency	−0.067** (0.032)	−0.083** (0.033)	−0.880 (0.106)	−0.114 (0.107)
Independence	-	1.080** (0.455)	-	2.229 (2.634)
Fixed Effect	no	no	yes	yes

Note: robust standard errors adjusted for clusters are reported in parenthesis. ** indicates significance at the 5% level.

Table 2.5: Results of transparency on household inflation predictability (household)

Description	(I)	(II)	(III)	(IV)
Constant	2.353*** (0.416)	1.883*** (0.400)	2.283 (1.361)	1.364 (1.892)
Transparency	−0.073* (0.0390)	−0.095** (0.042)	−0.066 (0.146)	−0.094 (0.151)
Independence	-	1.389** (0.586)	-	2.445 (2.900)
Fixed Effect	no	no	yes	yes

Note: robust standard errors adjusted for clusters are reported in parenthesis. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

Table 2.6: Results of transparency on professional inflation predictability (professional)

Results for household inflation predictability are reported in Table 2.5 and for professional inflation predictability in Table 2.6. Point estimates evidence central bank transparency improving the inflation predictability for both households and professionals, being statistical significant for some specifications. So, improving central bank transparency seems to enhance inflation predictability by both agents, as measured by the RMSE between expectations and realized data.

2.5 Conclusion

We propose a model of how household forms expectations about inflation. He can whether assimilate information from professional's views of future inflation or he can rely on an own model, which we assume to follow a learning mechanism. The switches vary over time according to observables related to the alternatives, and non-observables linked to the households. The model fits well to observed data for American and Brazilian economies. For the latter, we evidence the learning mechanism playing a prominent role in shaping expectations in an economy with history of hyperinflation, just as also evidenced by Malmendier e Nagel (2016).

As direct applications of the model, we find a positive correlation between inflation news heard by the households and the proportion of them relying on professional's view, reinforcing the findings of Carroll (2003) and Pfajfar e Santoro (2013). Moreover, it seems that higher-income households pay more attention regarding the historical success of the alternatives, while lower-income households discount more intensively past inflation when forecasting by the learning model. These findings provide additional explanation to distinguished inflation expectations among the demographic groups, as well documented in Bryan e Venkat (2001). Finally, we find evidence that central bank transparency enhances the inflation predictability by households and professionals.

Lastly, the results strengthen the importance in modeling heterogeneous expectations within the private sector. Some works have already done it in a DSGE framework, as Branch e McGough (2009), Del Negro e Eusepi (2011), Branch e Evans (2011) and Massaro (2013).

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A
The Learning procedure

Let we define

$$\begin{aligned} z_t &\equiv [y_{1,t},\ ...,\ y_{N,t},\ \pi_{1,t},\ \ldots, \pi_{N,t}]^{'} , \\ \Psi_t &\equiv \left[\hat{\beta}_{1,t}^{y,F},\ ..., \hat{\beta}_{N,t}^{y,F},\ \hat{\beta}_{1,t}^{y,P},\ ..., \hat{\beta}_{N,t}^{y,P}\ \hat{\beta}_{1,t}^{\pi,F}\ ..., \hat{\beta}_{N,t}^{\pi,F},\ \hat{\beta}_{t}^{\pi,P}\right]^{'} , \end{aligned}$$

and

$$X_t = \begin{bmatrix} \theta_{1,t} & \ldots & 0 & 1-\theta_{1,t} & \ldots & 0 & 0 & 0 & 0 & 0 \\ \vdots & \ddots & 0 & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \theta_{N,t} & 0 & 0 & 1-\theta_{N,t} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \theta_{1,t} & \ldots & 0 & \pi_t^* (1-\theta_{1,t}) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{N,t} & \pi_t^* (1-\theta_{N,t}) \end{bmatrix}.$$

In addition, for each period t, the policymaker believes that $\beta_t \sim N\left(\hat{\beta}_t, P_t^{-1}\right)$, with $\beta_t = [\beta_{1,t}^{y,F},\ ...,\ \beta_{N,t}^{y,F}, \beta_{1,t}^{y,P},\ ...,\ \beta_{N,t}^{y,P},\ ...,\ \beta_{1,t}^{\pi,F},\ ..., \beta_{N,t}^{\pi,F},\ ...,\ \beta_t^{\pi,P}]'$ and where $\hat{\beta}_t$ and P_t^{-1} are updated as soon as new data is available.

Using a linear recursive estimator, we have

$$\begin{aligned} z_t &= X_t \Psi_t + u_t, \\ \Psi_t &= \Psi_{t-1} + K_t (z_t - X_t \Psi_{t-1}), \end{aligned}$$

where K_t is referred as estimator gain matrix.

The current estimation error is

$$\begin{aligned} e_t &= \Psi - \Psi_t, \\ &= \Psi - \Psi_{t-1} - K_t (z_t - X_t \Psi_{t-1}), \\ &= e_{t-1} - K_t (X_t \Psi + u_t - X_t \Psi_{t-1}), \\ &= e_{t-1} - K_t X_t (\Psi - \Psi_{t-1}) - K_t u_t, \\ &= (I - K_t X_t) e_{t-1} - K_t u_t. \end{aligned}$$

We define $P_t = E\left(e_t e_t^{'}\right)$, thus

$$\begin{aligned} P_t &= E\left\{\left[\left(I - K_t X_t\right) e_{t-1} - K_t u_t\right]\left[\left(I - K_t X_t\right) e_{t-1} - K_t u_t\right]^{'}\right\}, \\ &= \left(I - K_t X_t\right) E\left(e_{t-1} e_{t-1}^{'}\right)\left(I - K_t X_t\right)^{'} - K_t E\left(u_t e_{t-1}^{'}\right)\left(I - K_t X_t\right)^{'} \\ &\qquad - \left(I - K_t X_t\right) E\left(e_{t-1} u_t^{'}\right) K_t^{'} + K_t E\left(u_t u_t^{'}\right) K_t^{'}. \end{aligned}$$

Assuming

$$\begin{aligned} E\left(u_t e_{t-1}^{'}\right) &= E\left(u_t\right) E\left(e_{t-1}^{'}\right) = 0, \\ E\left(e_{t-1} u_t^{'}\right) &= E\left(e_{t-1}\right) E\left(u_t^{'}\right) = 0, \end{aligned}$$

we have

$$P_t = \left(I - K_t X_t\right) P_{t-1}\left(I - K_t X_t\right)^{'} + K_t E\left(u_t u_t^{'}\right) K_t^{'}. \tag{A-1}$$

The derivation of optimal value for estimator gain matrix give us[1]

$$K_t = P_{t-1} X_t^{'}\left[X_t P_{t-1} X_t^{'} + E\left(u_t u_t^{'}\right)\right]^{-1}. \tag{A-2}$$

Substituting (A-2) in (A-1) and, finally, making some algebra, we have:

$$P_t = \left(P_{t-1}^{-1} + X_t^{t} E\left(u_t u_t^{'}\right) X_t\right)^{-1}$$

Thus, obeying the timing of events and applying a weighted recursive least square[2], we obtain the following optimal updating that shapes the policymaker's beliefs:

$$\begin{aligned} P_t &= \left(P_{t-1}^{-1} + X_t^{'}\hat{\Sigma}^{-1} X_t\right)^{-1}, \\ \Psi_t &= \Psi_{t-1} + P_t^{-1} X_t^{'}\hat{\Sigma}^{-1}\left(z_t - X_t \Psi_{t-1}\right). \end{aligned}$$

In period $t = 0$, we set $\hat{\beta}_0$ according to average value in pre-sample data.

[1]See Hamilton (1994) for derivation.
[2]See appendix for details.

For P_0^{-1}, we expand Buera et al. (2011) and parametrize it as

$$P_0^{-1} = \begin{bmatrix} \mathbb{P}_y & \mathbb{O}_{N\times N} & \mathbb{O}_{N\times N} & \mathbb{O}_{N\times 1} \\ \mathbb{O}_{N\times N} & \mathbb{P}_y & \mathbb{O}_{N\times N} & \mathbb{O}_{N\times 1} \\ \mathbb{O}_{N\times N} & \mathbb{O}_{N\times N} & \mathbb{P}_\pi^F & \mathbb{O}_{N\times 1} \\ \mathbb{O}_{1\times N} & \mathbb{O}_{1\times N} & \mathbb{O}_{1\times N} & \mathbb{P}_\pi^P \end{bmatrix}$$

where $\mathbb{P}_y = \left(V_y \cdot R_y^{-1} \cdot V_y\right)$, $\mathbb{P}_\pi^F = \left(V_\pi \cdot R_\pi^{-1} \cdot V_\pi\right)$, $\mathbb{P}_\pi^P = \left(\bar{\sigma}^\pi \nu^{\pi,P}\right)^2$, $V_y = \mathrm{diag}\left([\sigma_{1,1}\nu_1^y, ..., \sigma_{N,N}\nu_N^y]\right)$, $V_\pi = \mathrm{diag}\left(\left[\sigma_{N+1,N+1}\nu_1^{\pi,F}, ..., \sigma_{2N,2N}\nu_N^{\pi,F}\right]\right)$, $R_{k_{[i,j]}} = \exp\left(-z_{i,j}\gamma_k\right)$, $z_{i,j}$ is the geographic distance between countries i and j, γ_k is a parameter to be estimated, $k \in \{y, \pi\}$, $\bar{\sigma}^\pi \equiv \frac{1}{N}\sum_{l=1}^N \sigma_{N+l,N+l}$, $\sigma_{k,l}$ is the entry (k,l) of the matrix $\hat{\Sigma}$ and $\mathbb{O}_{k\times l}$ is a $k \times l$ matrix of zeros.

The format of P_0^{-1}, the prior covariance matrix, accounts for a uncorrelated impact of different ERR on growth and inflation within countries. We assume this format since we consider that uncertainty about inflation should not shapes beliefs about growth, and vice-verse. Moreover, the uncertainty of the impact of ERR on variables, parametrized by $\{\nu_n\}_n$, is different for each regime. Finally, as $z_{i,j}$ accounts for the distance between the country i and j, in the policymaker's learning process, the closer the neighbor, the less the uncertainty about how the relation between ERR and growth in this country can explain the relation in his country.

In addition, for the nested model, for each country, we run a reduced model in a form of $z_n = X_n\beta + \eta_n$, where $z_n = [y_n \; \pi_n]$ and $X_n = \begin{bmatrix} \theta_n & (1-\theta_n) \\ \theta_n & \pi^*(1-\theta_n) \end{bmatrix}$, and we obtain $\sigma_n^2 = \hat{\mathrm{var}}(\eta_n)$. We then set $\hat{\Sigma} \equiv \begin{bmatrix} \hat{\Sigma}_B & \mathbb{O}_{N\times N} \\ \mathbb{O}_{N\times N} & \hat{\Sigma}_B \end{bmatrix} \cdot \mathrm{diag}\left[\sigma^2\right]$, where $\sigma^2 = [\sigma_1^2, ..., \sigma_N^2]$, $\hat{\Sigma}_B$ is covariance matrix of a reduced model used by Buera et al. (2011) which accounts for uncertainty in freedom policies impacting growth and $\mathbb{O}_{N\times N}$ is a $N \times N$ matrix of zero. We use $\mathrm{diag}\left[\sigma^2\right]$ since some countries has no switched regimes in the sample, for which we set mean values, leadind to near singular matrix. Finally, the weighting by $\begin{bmatrix} \hat{\Sigma}_B & \mathbb{O}_{N\times N} \\ \mathbb{O}_{N\times N} & \hat{\Sigma}_B \end{bmatrix}$ is required to complete the matrix in a way that we use country intrinsic uncertainty and weight it by uncertainty of the freedom policies impacting growth, considering every country in the sample. As drawback, the same matrix is used for inflation purposes.

B
Solving the policymaker's general problem

Let $V^{*}(\cdot)$ be the value function associated to the choices and take the policy function encompassing growth and inflation. Thus, solving for each past choice, we have:

– if $\theta_{n,t-1} = 0$:

$$\begin{aligned} V^{*}(\theta_{n,t} = 1|\theta_{n,t-1} = 0) &= E_{n,t-1}\left\{\left(\beta_{n,t}^{y,F} - \beta_{n,t}^{y,P}\right) + a\left(\beta_{n,t}^{\pi,F} - \beta_{t}^{\pi,P}\pi_{t}^{*}\right)\right\}... \\ &\qquad ... - f_{n} - \xi^{'}\Pi_{n,t} - k_{n,t} - \varphi^{F}, \\ V^{*}(\theta_{n,t} = 0|\theta_{n,t-1} = 0) &= 0, \\ &\therefore \\ V^{*}(1|0) > V^{*}(0|0) &\Leftrightarrow E_{n,t-1}\left\{\left(\beta_{n,t}^{y,F} - \beta_{n,t}^{y,P}\right) + a\left(\beta_{n,t}^{\pi,F} - \beta_{t}^{\pi,P}\pi_{t}^{*}\right)\right\}... \\ &\qquad ... > f_{n} + \xi^{'}\Pi_{n,t} + k_{n,t} + \varphi^{F}. \end{aligned}$$

– if $\theta_{n,t-1} = 1$:

$$\begin{aligned} V^{*}(\theta_{n,t} = 1|\theta_{n,t-1} = 1) &= E_{n,t-1}\left\{\left(\beta_{n,t}^{y,F} - \beta_{n,t}^{y,P}\right) + a\left(\beta_{n,t}^{\pi,F} - \beta_{t}^{\pi,P}\pi_{t}^{*}\right)\right\}... \\ &\qquad ... - f_{n} - \xi^{'}\Pi_{n,t} - k_{n,t}, \\ V^{*}(\theta_{n,t} = 0|\theta_{n,t-1} = 1) &= -\varphi^{P}, \\ &\therefore \\ V^{*}(1|1) > V^{*}(0|1) &\Leftrightarrow E_{n,t-1}\left\{\left(\beta_{n,t}^{y,F} - \beta_{n,t}^{y,P}\right) + a\left(\beta_{n,t}^{\pi,F} - \beta_{t}^{\pi,P}\pi_{t}^{*}\right)\right\}... \\ &\qquad ... + \varphi^{P} > f_{n} + \xi^{'}\Pi_{n,t} + k_{n,t} \end{aligned}$$

Therefore, the optimal policy is as follows:

$$\theta_{n,t} = \mathbb{I}_{\left[E_{n,t-1}\left(\beta_{n,t}^{y,F} + a\beta_{n,t}^{\pi,F}\right) - \varphi^{F}(1-\theta_{n,t-1}) > E_{n,t-1}\left(\beta_{n,t}^{y,P} + a\beta_{t}^{\pi,P}\pi_{t}^{*}\right) + K_{n,t} - \varphi^{P}\theta_{n,t-1}\right]},$$

where $\mathbb{I}_{[\cdot]}$ is the indicator function.

C
Likelihood Function

We must build up the likelihood function in order do perform the posterior distribution of the model, and then maximize it to obtain the estimated parameters. Writing it as product of conditional densities, we have

$$\begin{aligned}
\mathcal{P}\left(D^T|\alpha\right) &= \mathcal{P}\left(y_T, \hat{\pi}_T, \theta_T, \Pi_T | y_{T-1}, ..., \Pi_1, \alpha\right) ... \\
&\qquad\qquad ... \cdot \mathcal{P}\left(y_{T-1}, ..., \Pi_1|\alpha\right), \\
&= \mathcal{P}\left(D_T|D^{T-1}, \alpha\right) ... \\
&\qquad\qquad ... \cdot \mathcal{P}\left(y_{T-1}, \hat{\pi}_{T-1}, \theta_{T-1}, \Pi_{T-1}|y_{T-2}, ..., \Pi_1, \alpha\right) ... \\
&\qquad\qquad ... \cdot \mathcal{P}\left(y_{T-2}, ..., \Pi_1|\alpha\right), \\
&\vdots \\
&= \mathcal{P}\left(D_1|\alpha\right) \cdot \prod_{t=2}^{T} \mathcal{P}\left(D_t|D^{t-1}, \alpha\right).
\end{aligned}$$

The individual factors can also be written as product of conditional densities. Thus,

$$\begin{aligned}
\mathcal{P}\left(D_t|D^{t-1}, \alpha\right) &= \mathcal{P}\left(y_t|\hat{\pi}_t, \theta_t, \Pi_t, D^{t-1}, \alpha\right) \cdot \mathcal{P}\left(\hat{\pi}_t|\theta_t, \Pi_t, D^{t-1}, \alpha\right) ... \\
&\qquad ... \cdot \mathcal{P}\left(\theta_t|\Pi_t, D^{t-1}, \alpha\right) \cdot \mathcal{P}\left(\Pi_t|D^{t-1}, \alpha\right) \qquad \text{(C-1)}
\end{aligned}$$

Moreover, we assume that economic growth, inflation and variables in the floating ERR cost do not depend on the policymaker's beliefs, but only on actual ERR. Thus, equation (C-1) can be simplified into

$$\begin{aligned}
\mathcal{P}\left(D_t|D^{t-1}, \alpha\right) &= \mathcal{P}\left(y_t|\hat{\pi}_t, \theta_t, \Pi_t, D^{t-1}\right) \cdot \mathcal{P}\left(\hat{\pi}_t|\theta_t, \Pi_t, D^{t-1}\right) ... \\
&\qquad ... \cdot \mathcal{P}\left(\theta_t|\Pi_t, D^{t-1}, \alpha\right) \cdot \mathcal{P}\left(\Pi_t|D^{t-1}\right).
\end{aligned}$$

So,

$$\begin{aligned}\mathcal{P}\left(D^{T}|\alpha\right) &= \mathcal{C}\cdot\mathcal{P}\left(\theta_{1}|\Pi_{1},\alpha\right)\cdot\prod_{t=2}^{T}\mathcal{P}\left(\theta_{t}|\Pi_{t},D^{t-1},\alpha\right),\\ &= \mathcal{C}\cdot\prod_{n=1}^{N}\left[\mathcal{P}\left(\theta_{n,1}|\Pi_{n,1},\alpha\right)\cdot\prod_{t=2}^{T}\mathcal{P}\left(\theta_{n,t}|\Pi_{n,t},D^{t-1},\alpha\right)\right],\end{aligned}$$

where $\mathcal{C}$ is a constant term relative to α.

Using the optimal policy, we obtain

$$\text{Prob}\left(\theta_{n,t}=1|\Pi_{n,t},D^{t-1},\alpha\text{ with }\theta_{n,t-1}=0\right)=$$

$$\begin{aligned}&= \text{Prob}\left[E_{n,t-1}\left(\beta^{F}_{n,t}-\beta^{P}_{n,t}\right)+a\left(\beta^{\pi}_{n,t}-\beta^{\pi,P}_{t}\pi^{*}_{t}\right)-f_{n}-\xi'\Pi_{n,t}-k_{n,t}-\varphi^{F}>0\right],\\ &= \text{Prob}\left[E_{n,t-1}\left(\beta^{F}_{n,t}-\beta^{P}_{n,t}\right)+a\left(\beta^{\pi}_{n,t}-\beta^{\pi,P}_{t}\pi^{*}_{t}\right)-f_{n}-\xi'\Pi_{n,t}-\varphi^{F}>k_{n,t}\right],\\ &= \Phi\left(\frac{E_{n,t-1}\left(\beta^{F}_{n,t}-\beta^{P}_{n,t}\right)+a\left(\beta^{\pi}_{n,t}-\beta^{\pi,P}_{t}\pi^{*}_{t}\right)-f_{n}-\xi'\Pi_{n,t}-\varphi^{F}}{\varrho_{n}}\right),\end{aligned}$$

and

$$\text{Prob}\left(\theta_{n,t}=0|\Pi_{n,t},D^{t-1},\alpha\text{ with }\theta_{n,t-1}=0\right)=$$

$$= 1-\Phi\left(\frac{E_{n,t-1}\left(\beta^{F}_{n,t}-\beta^{P}_{n,t}\right)+a\left(\beta^{\pi}_{n,t}-\beta^{\pi,P}_{t}\pi^{*}_{t}\right)-f_{n}-\xi'\Pi_{n,t}-\varphi^{F}}{\varrho_{n}}\right).$$

In addition,

$$\text{Prob}\left(\theta_{n,t}=1|\Pi_{n,t},D^{t-1},\alpha\text{ with }\theta_{n,t-1}=1\right)=$$

$$\begin{aligned}&= \text{Prob}\left[E_{n,t-1}\left(\beta^{F}_{n,t}-\beta^{P}_{n,t}\right)+a\left(\beta^{\pi}_{n,t}-\beta^{\pi,P}_{t}\pi^{*}_{t}\right)-f_{n}-\xi'\Pi_{n,t}-k_{n,t}>-\varphi^{P}\right],\\ &= \text{Prob}\left[E_{n,t-1}\left(\beta^{F}_{n,t}-\beta^{P}_{n,t}\right)+a\left(\beta^{\pi}_{n,t}-\beta^{\pi,P}_{t}\pi^{*}_{t}\right)-f_{n}-\xi'\Pi_{n,t}+\varphi^{P}>k_{n,t}\right],\\ &= \Phi\left(\frac{E_{n,t-1}\left(\beta^{F}_{n,t}-\beta^{P}_{n,t}\right)+a\left(\beta^{\pi}_{n,t}-\beta^{\pi,P}_{t}\pi^{*}_{t}\right)-f_{n}-\xi'\Pi_{n,t}+\varphi^{P}}{\varrho_{n}}\right),\end{aligned}$$

and

$$\text{Prob}\left(\theta_{n,t}=0|\Pi_{n,t},D^{t-1},\alpha \text{ with } \theta_{n,t-1}=1\right)=$$

$$= \quad 1-\Phi\left(\frac{E_{n,t-1}\left(\beta^F_{n,t}-\beta^P_{n,t}\right)+a\left(\beta^{\pi}_{n,t}-\omega\pi^*_t\right)-f_n-\xi'\Pi_{n,t}+\varphi^P}{\varrho_n}\right).$$

Thus,

$$\mathcal{P}\left(\theta_{n,t}|\Pi_{n,t},D^{t-1},\alpha\right)=$$

$$\left[\Phi\left(\frac{\hat{\beta}^F_{n,t-1}-\hat{\beta}^P_{n,t-1}-f_n+a\left(\beta^{\pi,F}_{n,t}-\beta^{\pi,P}_t\pi^*_t\right)-\xi'\Pi_{n,t}-\varphi^F\left(1-\theta_{n,t-1}\right)+\varphi^P\theta_{n,t-1}}{\varrho_n}\right)\right]^{\theta_{n,t}}\cdots$$

$$\cdots\left[1-\Phi\left(\frac{\hat{\beta}^F_{n,t-1}-\hat{\beta}^P_{n,t-1}-f_n+a\left(\beta^{\pi,F}_{n,t}-\beta^{\pi,P}_t\pi^*_t\right)-\xi'\Pi_{n,t}-\varphi^F\left(1-\theta_{n,t-1}\right)+\varphi^P\theta_{n,t-1}}{\varrho_n}\right)\right]^{1-\theta_{n,t}},$$

where $\Phi(\cdot)$ is the standard Gaussian cumulative distribution.

D
Countries Estimates Distributions

Here, we report the empirical distribution of other estimates in the nestled model. As we can see, no extreme value is verified in order to fit the data.

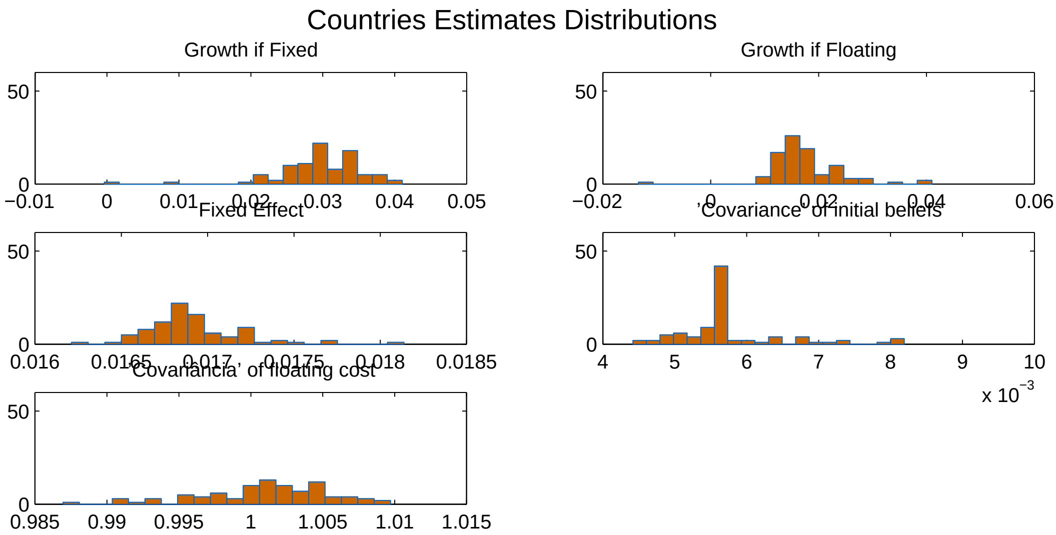

Figure D.1: Countries Estimates Distributions

E
No Learning Figures

Here, we report the results for the nested model with no learning feature. The model also fits well to the data. Moreover, by construction, the beliefs do not change over time, except for inflation if fixed. In this case, it is the “pass-through” from US inflation which remains constant.

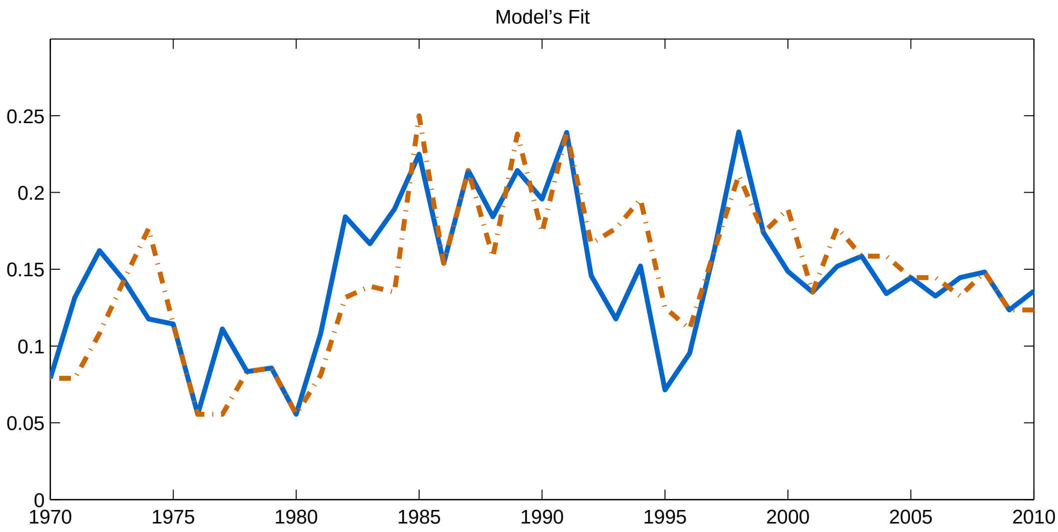

Figure E.1: No Learning Model's Fit

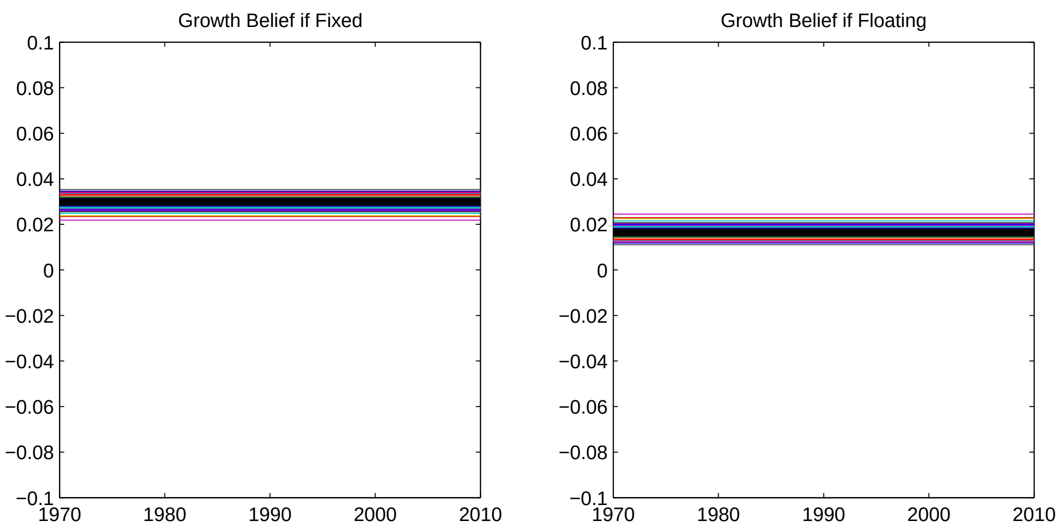

Figure E.2: No Learning Growth Beliefs

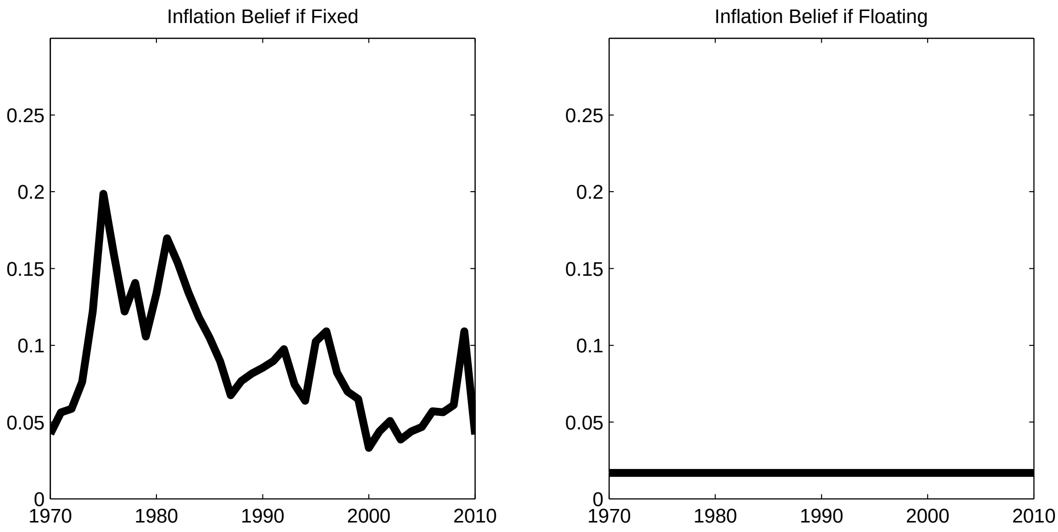

Figure E.3: No Learning Inflation Beliefs