TEXTO PARA DISCUSSÃO

Nº 116

Notes on growth, distribution and capacity utilization*

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January 1986

* Forthcoming in *Contributions to Political Economy*, an annual supplement of the *Cambridge Journal of Economics*.
**I am grateful to Murray Milgate, Lance Taylor and Amitava Dutt for their helpful comments although they are not responsible for my errors and misinterpretations. The author is assistant professor at the Pontifical Catholic University, Rio de Janeiro (PUC-Rio).

There are two (almost) indisputable propositions in one sector growth theory: firstly, that there is an inverse relation between the wage rate and the rate of profit, and secondly, its dual, that there is an inverse relation between the rate of growth and consumption per capita. Such relations arise from an assumption either of *fixed* degree of capacity utilization or of fixed level of aggregate output: more resources being allocated to the production of capital goods (leading to a higher rate of growth) requires that a smaller share of output per worker employed be devoted to consumption; if investment is financed by profits, it also requires a smaller real wage.

Keynesian and Marxian type models share these propositions differing only over the direction of causality between growth and distribution. The typical Keynesian exercise would begin with a change in investment demand which, assuming that output is fixed and capitalists have access to credit while workers do not, implies a reduction of consumption per head and real wage. The adjustment to the expenditure shock operates through changes in the price level given the money wage rate. In the typical Marxian model, where distribution is determined exogenously, it is the growth rate that accommodates. An increase in the real wage due to, say, a temporary exhaustion of the 'reserve army', reduces the profit rate and, thus, the rate of accumulation – compared with the Keynesian scheme causality between growth and distribution is reverted.

Although the notion that capacity may not (or, rather, does not) adjust to demand in the short period is widely agreed upon, the same is not true in the long period. The latter, as Marshall taught us, is characterized by capacity adjusting to demand; there is no place for the concept of capacity *utilization* in the long period: utilization is, *ex-hypothesis*, 'planned', 'optimal' or 'full'. Here we shall consider the possibility of utilization (rather than the price level) being the adjustment variable between steady states and, most importantly, of utilization being different from the planned or normal degree in the long period. Our purpose is to call the attention of those who, accepting the notion that utilization may play the role of adjustment variable – such as Vianello (1985) and Ciccone (1985) –, do not realize the implications for the long period configuration. In particular, it shall be argued that once capacity utilization is allowed to vary, the Classical notion that there is a "unique, inverse relation which must obtain in the long period between the real wage and the rate of profits" (Garegnani, 1979, p. 77) may not necessarily hold. Furthermore, it shall be shown that both the Keynesian and Marxian propositions will not hold either. In their place an alternative proposition according to which a higher wage rate is associated with a higher rate of growth, rate of profit and degree of capacity utilization will be developed. This proposition can be associated with the names of Kalecki and Steindl.

The plan of the paper is as follows. In section 1 a simple model is built. Sections 2 and 3 present the typical Keynesian and Marxian propositions. In section 4 changes in utilization are introduced and the alternative proposition is discussed. section 5 concludes the paper by showing that if utilization is allowed to vary, the Keynesian and Marxian propositions do not hold. An appendix extends the model to the case where both capitalists and workers save – since in the text we will assume without loss of generality) that workers only consume and capitalists only save.

1. *A Simple Model*

In this section, we will assume that there is only one commodity being produced in a closed economy and that it can be used for both consumption and investment purposes. The output: labor ratio (a) is assumed to be given and not to change through time. Firms operate having a long period capacity utilization target (k). Finally, we shall assume that only two consumption groups exist; 'workers' whose consumption expenditure is a function of the economy's wage bill and 'capitalists' whose propensity to consume depends *inter alia* on the firms' decision to distribute profits. The two general equations – common to all three models – are a modified form of the Cambridge equation and a profit rate identity.

1.1 *The Profit Rate Identity*

We start with the following price equation according to which labor and capital costs plus gross profits on the ownership of capital exhaust the price of a unit of output:

$$p = \frac{w}{a} + p\frac{r}{u}$$

where w is the wage rate, r is the gross rate of profit, $a \equiv X/N$ where X is the level of aggregate output and N the level of employment and $u \equiv X/K$ is the degree of capacity utilization.

By definition, the share of wages in income (α) and the real wage (W) are given by:

$$\alpha \equiv 1 - \frac{r}{u} \text{ and}$$

$$W \equiv a\alpha = a\left[1 - \frac{r}{u}\right]$$

which, together with the price equation define the rate of profit identity:

$$r \equiv (1 - \alpha)u \tag{1}$$

When the actual degree of capacity utilization (u) is equal to the planned degree (k), equation (1) describes the capacity *distribution* frontier depicted in figure 1.

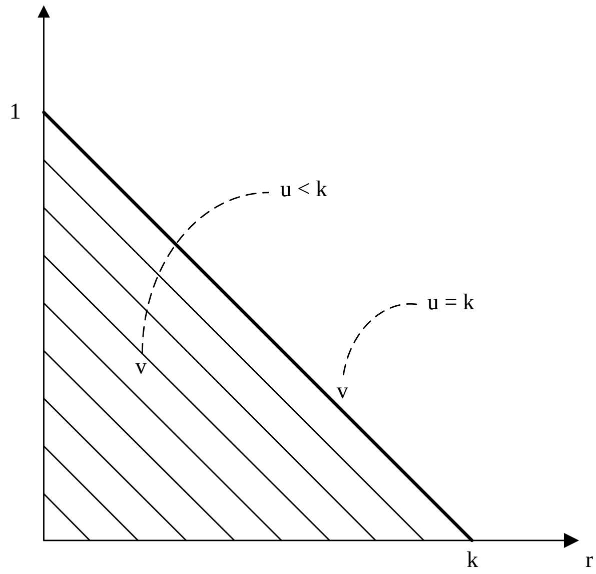

Figure 1

For $u < k$, the frontier becomes the upper bound for the distribution set represented by the shaded area on the graph. The reader will appreciate that the inverse correlation between the wage and profit rates only holds true for the special case, where $u = k$.

1.2 *A Modified Cambridge Equation*

The origin of the second general equation is the following aggregate expenditure function:

$$pX = C_{\omega}WN + C_k(pX - WN) + pI \tag{2}$$

where C_{ω} and C_k are, respectively, workers' and capitalists' propensities to consume. Dividing equation (2) through by pK we get:

$$g^s = \lambda u \tag{3}$$

where $\lambda \equiv [1 - \alpha(C_{\omega} - C_k) - C_k]$ and g^s is the growth rate expressed as a function of consumption propensities, the distribution of income and the degree of utilization. Assuming $C_{\omega} = 1$, equations (1) and (3) yield the conventional Cambridge equation:

$$g^s = (1 - C_k)r$$

For sake of simplicity, we shall develop all models assuming not only that $C_\omega = 1$, but also that $C_k = 0$, that is, that all profits are saved[1] implying the following modified Cambridge equation:

$$g^s = r \tag{4}$$

The general model has two equations – (1) and (4) – and 4 unknowns – g, r, u and α.

1.3 *The Capacity Production Frontier*

In order to assess the second proposition mentioned in the introduction to this paper – concerning the relation between consumption per head and the rate of growth – we shall derive an additional equation[2]. We define $C \equiv (\omega/p)\, N$ as aggregate real consumption and $c \equiv \omega/p \equiv W$ as real consumption per unit of labor employed. Rearranging equation (2) we may obtain:

$$c = a\left[1 - \frac{g}{u}\right] \tag{5}$$

which, for $u = k$, gives rise to the capacity *production* frontier depicted in figure 2.

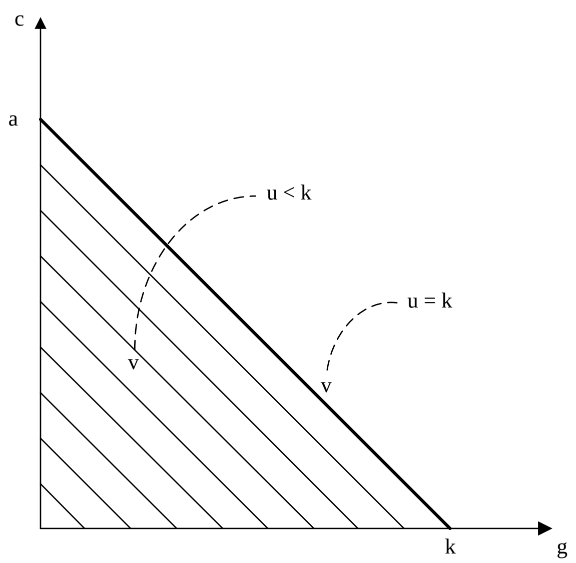

Figure 2

[1] An appendix to this article extends the exercises to the case where both, capitalists and workers, consume and save; the reader will then realize that none of the results presented in the text are affected by the simplifying assumptions.
[2] The equation to be derived is not independent of equations (1) and (4) and, therefore, plays no role in closing the general model.

Again, the production set is represented by the shaded area and bounded above by equation (5) for $u = k$; the proposition of inverse relation between c and g depends on the giveness of the degree of capacity utilization.

2. *A Keynesian Analysis*

As we have already mentioned, the Keynesian and Marxian type models share the propositions of inverse correlation in distribution and production. Obviously enough, they also share the underlying assumption responsible for those propositions to hold true, that is to say, the assumption that actual capacity utilization is equal to the planned degree:

$$u = k \tag{6}$$

Equation (6) provides one of the two equations required to close the general model. The specifically Keynesian equation is an investment demand function of the form[3]:

$$g' = g(r), \;\; g' > 0$$

where r can be interpreted as a proxy for the expected rate of profit. Put in linear form, such equation can be written as

$$g' = d + hr, \;\; h > 0 \tag{7}$$

where h measures the responsiveness of investment to changes in the rate of profit and d the "psychology of the business community" – as Marglin (1934 , p. 80) refers to it – or simply 'animal spirits'. Equations (1), (4), (6) and (7) close the model giving rise to the following equilibrium configuration:

$$g^* = r^* = \frac{d}{1-h} \tag{8}$$

$$\alpha^* = 1 - \frac{d}{k(1-h)} \tag{9}$$

$$W^* = c^* = a\alpha^* \tag{10}$$

[3] See Robinson (1962, pp. 36-8) and Marglin (1984, pp. 79-81) for a discussion of the investment demand function. Such closure is specifically Robinsonian; Kaldor's (1956) closure is $g = n$ (where n is the rate of population growth), hardly Keynesian in spirit.

The stability condition for this model[4] is given by $h - 1 < 0$, which simply means that the investment function must be less responsive to changes in the rate of profit than the saving function. The equilibrium configuration is depicted in figure 3.

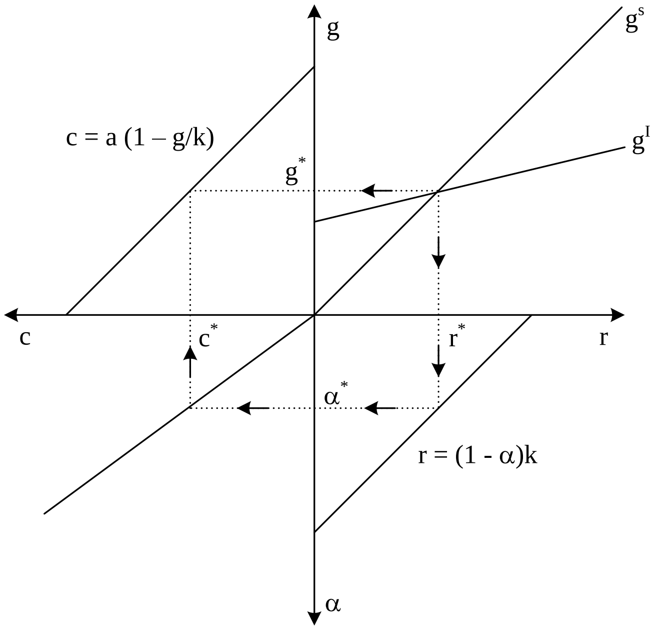

Figure 3

Causality in this Keynesian-type model runs from investment and saving to the equilibrium rates of growth and profit; once the latter is determined, given the degree of utilization, the share of wages and consumption per capita are determined.

The Keynesian shock *par excellence* is a change in animal spirits (Δd). For $\Delta d > 0$, the investment function would be upwardly displaced. Looking at the equilibrium configuration equations and taking the stability condition into account, it can be easily concluded that while the rates of growth and profit will increase, the real wage and share of wages will fall. For a given degree of capacity utilization – and, thus, aggregate output – an increase in investment must be off set by a reduction in consumption per unit of labor employed. The mechanism through which this occurs is a rise in the price level given the money wage rate[5].

The reduction in the purchasing power of workers or 'forced saving' is the mechanism through

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[4] The rate of growth is supposed to increase over time whenever, for a given profit rate, the investment function exceeds the saving function. The movement of g over time can be described by the following dynamic equation: $dg/dt = \theta[d + (h-1)r],\ \theta' > 0$. Stability requires $d\theta/dr < 0$ or $h - 1 < 0$.

[5] See Pasinetti (1974, pp. 101 and 105) and Harglin (1984, pp. 88-95) for a discussion of the Neo-Keynesian adjustment mechanism. As Harglin there notices, an implicit assumption for such mechanism to take place is that money wages are given and only capitalists have access to the credit system.

which the system adjusts to any exogenous change in aggregate expenditure. The central proposition derived from the above exercise is that a reduction of the real wage rate (or share of wages for that matter) is a *necessary condition* for the economy to achieve a higher rate of growth.

3. *A Marxian Analysis*

What differentiates the Marxian-type models from those of a Keynesian variety is the substitution of an exogenous real wage rate for the investment function[6]. The 'subsistence wage' (W^s) is *historically* determined not only according to Marx writings but, indeed, all Classical economists as well. The share o-f wages in income (α) depends on the subsistence wage and the methods of production – here represented by the output/labor ratio – which are, in turn, determined by both the development of the forces of production and the social relations of production to use the Marxian terminology.

For a given subsistence wage and output/labor ratio, the share of wages is given by:

$$\alpha = \alpha^s = W^s/a \tag{11}$$

Equations (1), (4), (6) and (11) close the model which equilibrium configuration is given by the following equations and depicted in figure 4:

$$g^* = r^* = (1 - \alpha^s)k \tag{12}$$

$$c^* = W^s \tag{13}$$

Causality here runs from distribution and the saving coefficients towards the rate of accumulation. The exogenous shock *par excellence* being a change in the subsistence wage or share of wages, say, $\Delta\alpha^s > 0$, which effect on equilibrium is given by:

$$\frac{\partial g^*}{\partial \alpha^s} = -k < 0$$

$$\frac{\partial c^*}{\partial \alpha^s} = a > 0$$

The notion of 'profit squeeze' seems to reasonably synthetize this result: as wages rise, since

[6] Although Marx refers to the 'possibility of crisis' when discussing Ricardo's theory of the rate of profit (Marx, Theories of Surplus Value, part II, ch. 17), he did not formulate any systematic theory of demand as a whole, and neither a theory of investment demand.

capacity utilization is fixed, profits are squeezed leading to a reduction on the rate of accumulation. The proposition reads, therefore, as follows: a reduction in the real wage is a *sufficient condition* for a higher rate of growth.

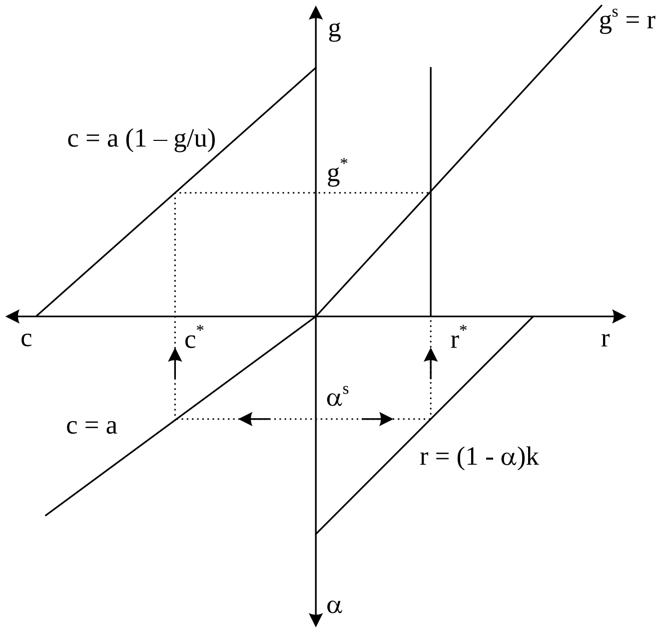


Figure 4

4. *An Analysis of Capacity Utilization*

The model to be examined in this section shares with the Marxian model the exogeneity of distribution and with the Keynesian model, the investment demand function. What differentiates it from both is the endogeneity of capacity utilization. First, let us assume an oligopolized market structure in which firms determine prices by fixing a mark-up (π) over prime costs:

$$p = (1 + \pi)(\omega/a)$$

Which implies the following share of wages and real wage equations:

$$\alpha = \frac{1}{1+\pi} \tag{14}$$
$$W = \frac{a}{1+\pi}$$

The second equation is an investment function with the rate of profits and capacity utilization as arguments:

$$g' = g(r, u)$$
$$g_r > 0$$
$$g_w > 0$$

The sensitiveness of investment to changes in capacity utilization is widely supported by empirical evidence[7]. Very little, if anything, however, has been written in recent contributions on the theoretical content of introducing capacity utilization in the investment demand function. At least to our knowledge, Steindl (1952, ch. 10) has been alone in spelling out its theoretical foundation, In an oligopolized industry – so Steindl's argument runs – firms can engage in agreements to protect their profits (say, during recessions) by rising the profit margin (π). If this would be the case, a reduction in capacity utilization due to deficient aggregate expenditure could be offset by a rise in the share of profits in income ($1 - \alpha$) leaving the rate of profit roughly unaltered.

Steindl's argument is that if the rate of profit becomes a variable firms can control through movements in the profit margin, capacity utilization (rather than the rate of profit) should be seen as the most important index of changes in aggregate demand and, therefore, an essential determinant of investment demand. This being the rationale to endogeneize capacity utilization in growth models designed to study mature industrialized economies.

A question that may arise at this point is why maintain the rate of profit as an argument for of the investment function. Indeed, in face of the above argument, it becomes theoretically meaningless[8]. We shall, therefore, assume a modified investment function in which the only argument is the degree of capacity utilization[9]. We will further assume that firms take the planned degree of utilization (k) as a target in their investment decision; for $u > k$, they have more incentive to invest and otherwise for $u < k$. The investment function in linear form thus becomes:

$$g' = v + z(u - k) \tag{15}$$

Equations (l), (4), (14) and (15) yield the following equilibrium configuration, also depicted in figure 5[10]:

[7] See, e.g., Kuh (1963) and Cowling (1982, pp. 46-7) for surveys of the econometric work on the role of capacity utilization oh investment demand.

[8] Except if one is willing to separate the effects of expected demand (through the expected rate of profit) and current demand (through capacity utilization).

[9] See Taylor (1983, ch. 2) and Dutt (1984) for models where both utilization and the rate of profit affect investment demand.

[10] The stability condition in this model is given by $z - (1 - \alpha) < 0$ which means that the investment function must be less responsive to changes in capacity utilization than the saving function

$$u^* = \frac{v - zk}{\sigma} \tag{16}$$

$$g^* v + z\left[\frac{v - zk}{\sigma} - k\right] \tag{17}$$

$$c^* = a\left[\frac{1}{1+\pi}\right] \tag{18}$$

where $\sigma \equiv \frac{\pi}{1+\pi} - z$

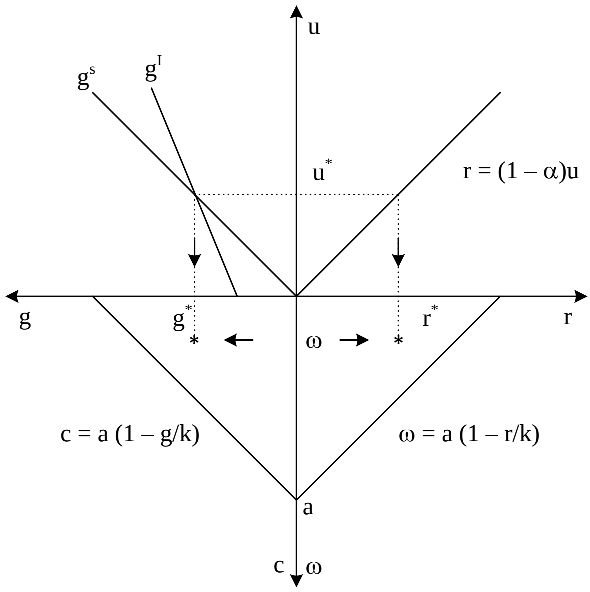


Figure 5

Causality in this model runs from distribution, on the one hand, and investment and saving, on the other, to the rates of growth, profit and capacity utilization. As the figure clearly shows, the equilibrium configuration is such that there is still 'space' for both, growth and consumption (and wage and profit rates), to increase[11].

4.1 *The Marxian and Keynesian Propositions Revisited*

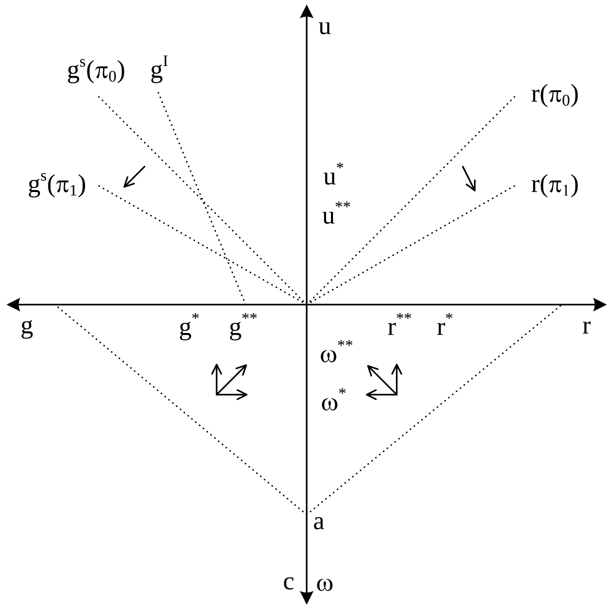
We can now re-examine the Keynesian and Marxian propositions in a model with endogenous capacity utilization. The effect of a change in the share of wages is given by the following equations and depicted in figure 6:

$$\frac{\partial u^*}{\partial \alpha} = \frac{v - zk}{\sigma^2} > 0$$

[11] Rowthorn (1982) differentiates the Neo-Keynesian from the Kaleckian approaches by associating the former to the assumption that $u = k$ and the latter to the assuaption that $u < k$.

$$\frac{\partial g^*}{\partial \alpha} = \frac{\partial r^*}{\partial \alpha} = \frac{v(v - zk)}{\sigma^2} > 0$$

$$\frac{\partial c^*}{\partial \alpha} = a > 0$$



Effect of $\Delta\pi > 0$

Figure 6

The above exercise leads to the following 'stagnationist' proposition[12]: a reduction of the real wage (or share of wages for that matter) is a *sufficient condition* for a lower rate of growth to be attained. This proposition, the reader will appreciate, simply reverts the Marxian 'profit squeeze' proposition.

One can also study the effect of a change in animal spirits, say, $\Delta v > 0$. The effects on the rates of capacity utilization, growth and profit are unambiguously positive and none on the real wage or share of wages. This result contradicts the Keynesian 'force saving' proposition; it says that a reduction in real wages is *not* a necessary condition for higher growth rates.

The introduction of capacity utilization as an endogenous variable has two important implications. First, it implies that the inverse relations between the rate of profit and the real wage, on the one hand, and the rate of growth and consumption per head, on the other, do not necessarily obtain. According to the model, to higher wage rates there corresponds higher degrees of capacity utilization and, thus, higher rates of profit up to the point where the economy reaches a situation of full utilization of capacity – say $u = k$. Refer to figure 7 where $u_0 < u_1 < u_2 < k$.

[12] The stagnationist result was first presented in a one-commodity growth model by Taylor (1983) following an early draft of Dutt (1984).

The inverse relation between the wage rate and the rate of profit does not hold as long as capacity is not fully employed. The second implication concerns the Keynesian and Marxian propositions: neither the 'protit squeeze' nor the 'forced saving' adjustment mechanisms hold if utilization is assumed to adjust. An increase in the real wage, instead of leading to lower rates of profit and growth, is associated with a greater degree of capacity utilization and, thus, to higher rates of profit and growth. An autonomous change in the rate of growth rather than accommodated by a reduction in the real wage will be accompanied by higher rates of profit and growth whereas the wage will remain unaltered.

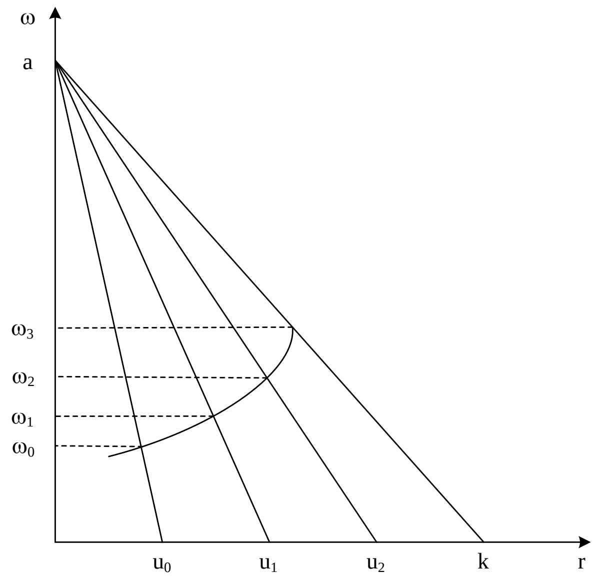


Figure 7

Appendix: The General Cambridge Equation

In this appendix we simply repeat the exercises developed in the text under the assumptions that $0 < C_w < 1$ and $0 < C_k < 1$.

The two general equations common to the three models are:

$$r = (1 - \alpha)u \ \text{ and} \tag{1}$$

$$g^s = \lambda u, \text{where } \lambda \equiv 1 - \alpha(C_w - C_k) - C_k \tag{3}$$

(a) *A Keynesian Model*

The Keynesian model adopts the following two equations:

$$u = k \; and \tag{6}$$

$$g' = d + hr \tag{7}$$

The equilibrium configuration is given by

$$r^* = \frac{d - (1 - C_k)k}{\xi}$$

$$g^* = d + \frac{h[d - (1 - C_w)k]}{\xi}$$

$$\alpha^* = 1 - \frac{d - (1 - C_w)k}{\xi^k}$$

where

$$\xi \equiv C_w - C_k - h$$

The stability condition is given by

$$h < C_w - C_k$$

The effect of a change in animal spirits is given by:

$$\frac{\partial r^*}{\partial d} = \frac{1}{\xi} > 0$$

$$\frac{\partial g^*}{\partial d} = 1 + \frac{h}{\xi} > 0$$

and

$$\frac{\partial \alpha^*}{\partial d} = -\frac{1}{\xi^k} < 0$$

(b) *A Marxian model*

The Marxian model adds

$$u = k \tag{6}$$

and

$$\alpha = \alpha^s = \frac{W^s}{a} \tag{11}$$

The equilibrium configuration is given by:

$$g^* = \lambda k$$

and

$$r^* = (1 - \alpha^s)k$$

The effect of an increase in α^s is given by:

$$\frac{\partial r^*}{\partial \alpha^s} = -k < 0$$

$$\frac{\partial g^*}{\partial \alpha^s} = -(C_w - C_k) \geq 0 <==> C_w \geq C_k$$

(c) *A Model with Endogenous Capacity Utilization*

The two equations that close the model dealing with capacity utilization are:

$$\alpha = \frac{1}{1 + \pi} \quad (14)$$

and

$$g' = v + zu \quad (15)$$

The equilibrium configuration in this model is:

$$u^* = \frac{v}{\gamma}$$

$$g^* = \frac{v\lambda}{\gamma}$$

and

$$r^* = \frac{(1 - \alpha)v}{\gamma}$$

where

$$\gamma \equiv \lambda - z$$

The stability condition is:

$$z < \lambda$$

The effect of a change in distribution is:

$$\frac{\partial u^*}{\partial \alpha} = \frac{v(C_w - C_k)}{\gamma^2} > 0$$

$$\frac{\partial g^*}{\partial \alpha} = \frac{vz(C_w - C_k)}{\gamma^2} > 0$$

$$\frac{\partial r^*}{\partial \alpha} = \frac{v[z - (1 - C_w)}{\gamma^2} \geq 0$$

And the effect of a change in animal spirits:

$$\frac{\partial u^*}{\partial v} = \frac{1}{\gamma} > 0$$

$$\frac{\partial g^*}{\partial v} = \frac{\lambda}{\gamma} > 0$$

$$\frac{\partial r^*}{\partial v} = \frac{1 - \alpha}{\gamma} > 0$$

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