INDEXATION, STAGGERING AND DISINFLATION[1]

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Indexation, either formal or informal, is widespread in high inflation economies. It is also a current view among policymakers that indexation makes disinflation more difficult. This paper provides theoretical foundations for this view by introducing indexation in a model of staggered price setting, where prices, when not adjusted by past inflation, are set optimally. We show that it is more difficult to disinflate in the model with indexation than in a standard staggering model where prices are fixed for a period twice as long[2]. As a consequence, the paper offers an explanation of why disinflation may be more difficult in high inflation economies.

There are several ways to incorporate indexation in a pricing rule. We chose one that seems realistic to us. Individual prices are adjusted by the accumulated (from the last adjustment) inflation at some points in the interval between optimal price adjustments[3].

Although the optimality of a pricing rule which includes both fixed price and indexation was not demonstrated up to now, it is possible to give a rationale for the emergence of such kind of rule when inflation is high[4]. When a firm has its nominal

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[2] By combining staggering with some other features it is also possible to have costs of disinflation that are higher than the ones which follow from the standard fixed price staggering model; e.g., Bonomo and Garcia (1992), which assumes limited rationality by a fraction of the agents, and Ball (1992), which relaxes the assumption of full credibility.

[3] As a contrast, Gray (1976,1978) models indexation as a continuous adjustment, where a nominal price (wage) is changed in any finite interval of time.

[4] Gray (1978) derives the optimal degree of indexation and the contract length in an economy subject to both real and

price fixed, it incurs a loss for not changing its price while the optimal price moves. This loss has to be weighted against the cost of ajusting the price. With high inflation, individual price adjustments have to occur very often to prevent the current price from drifting away from the optimal price. However, optimal price decisions or price negotiations may be too costly to implement so frequently. In contrast, price adjustments by past inflation use free and widely available information[5]. Thus, indexation by past inflation may emerge, either as a rule of thumb or as an institutional arrangement, to update prices at some points in the time interval between optimal price adjustments.[6]

To capture those features in the simplest setup, we assume that individual price adjustment alternates between optimal price setting and indexation by past inflation. Each firm, when deciding its optimal price, takes into account that its price will be automatically adjusted by past inflation at the midpoint of the time interval between optimal price adjustments. There is also uniform staggering of the optimal price setting. Thus, it is not surprising that the dynamics generated by this model is complex. Linear disinflation causes a boom, followed by a recession. For relatively fast disinflations the relative size

monetary shocks. However, because menu costs are absent in her model, prices are free to vary.

[5] A similar argument can be found in Blanchard (1979). He derives a rule which indexes the nominal wage to the price level when it is costly to apply the most efficient rule. In that context it is in general optimal to index by a fraction of the price level. In our case, since we assume high inflation and prices are fixed between adjustment, this fraction tends to be very close to one.

[6] The validity of this argument depends on the existence of both menu costs and costs of collecting information.

and length of the recession tends to be large, when compared to the boom.

Indexation does help to explain the costs of disinflation. In a model without indexation, disinflation can be achieved very quickly without a recession (Ball (1990)). In a model with preset time-varying individual prices (Phelps (1978), Taylor (1983)), disinflation is more difficult than when individual prices are fixed between optimal adjustments[7]. With indexation, disinflation is still more difficult: disinflation accomplished in any reasonable amount of time is accompanied by an unavoidable recession. Indexation also delays considerably disinflation with constant output. The disinflation path which keeps output at its natural rate takes at least three times longer to converge when there is indexation.

Our work also provides an explanation of why disinflation may be more difficult in high inflation economies. The standard staggering model implies the contrary. In that framework, the shorter the length of the period a price is fixed, the easier it is to disinflate. Since a higher inflation implies that the periods without adjustments are shorter, it would be easier to disinflate when inflation is higher. However, this argument assumes that a higher inflation induces more frequent optimal adjustments, with no other kind of adjustment between them. As we argued above, because very frequent optimal adjustments may be too costly, it is likely that high inflation introduces adjustments that are not optimal - like adjustments by the price index - between two optimal adjustments. We show that the

[7] See Ball (1990), for a comparison.

introduction of an adjustment by past inflation at the middle of the period makes disinflation more difficult, despite price adjustments being twice more frequent.

Previous work on the consequences of indexation for disinflation use different assumptions[8]. Friedman (1974) argues that indexation would make disinflation less costly (implicitly assuming perfect indexation). Fischer (1986) constructs a model of imperfect indexation which supports the same qualitative results, but indexation is made with respect to changes in expectation about future price level rather than to the changes in the price level. Simonsen (1983) finds that lagged indexation makes disinflation harder. However, indexation is modeled as part of a price adjustment which is undertaken every period in a discrete time setting. Hence, the comparison is made with respect to a standard (Lucas type) macro model without price rigidity. In our continuous time model indexation updates the price using the the accumulated inflation until the adjustment date. So, indexation is not lagged, but infrequent. Moreover, individual price changes are staggered and each price receives an optimal adjustment periodically. The optimal adjustment takes into account that each price is going to be adjusted by the accumulated inflation in the middle of the period. The structure of our model allows us to compare our results with the ones obtained under staggering and price rigidity.

The rest of the paper contains four sections. Section II

[8] There is a part of the literature which is concerned with the effects of monetary policy on the price level, e.g. Fischer (1977) and Gray (1976). Since those results about levels do not carry over to inflation rates, we concentrate on the literature which deals explicitly with inflation rates.

presents the model and derives the behavior of the economy under steady inflation. Section III derives the behavior of the economy for an arbitrary disinflation, computes the ouput path for linear and delayed "cold-turkey" disinflations, and calculates the money path which would yield a disinflation while maintaining output constant at the natural rate. Section IV interprets the results, and Section V concludes.

II. The Model[9]

There is a continuum of firms, indexed in the interval [0,1]. The firms are identical, except that they adjust their price at different times. The optimal relative price for a firm is increasing in aggregate output (all variables are in log):

$$p_i^{*}-p=vy \qquad (1)$$

where p_i^{*} is the firm's optimal nominal price, y is the aggregate output and p is the general price index given by:

$$p=\int_0^1 p_i di \qquad (2)$$

The intuition for equation (1) is that an increase in aggregate demand, by shifting out the demand curve the firm faces, causes an increase in the profit-maximizing real price. The exact functional form of Equation (1) can be derived from isoelastic demand and cost functions. We will see that the

[9] The model is similar to Ball (1990), the difference being the introduction of indexation.

magnitude of v affects our results in an important way. The parameter v is small if the marginal cost is just slightly increasing and/or if the demand is very elastic.[10]

We assume aggregate demand to be determined by the quantity of real money:

$$y=m-p \tag{3}$$

where m is the nominal amount of money[11].

Combining (3) and (1), we obtain:

$$p^{*}=vm+(1-v)p \tag{4}$$

where we dropped the subscript i from p_i^*, since the optimal price is common to all firms.

According to equation (4) the optimal price is a convex combination of the quantity of money and the price level, and the parameter of this combination (v) is the same parameter which represents the degree of real rigidity of the optimal price in equation (1)[12]. Since (1-v) expresses the dependence of the optimal price of a firm i on the prices of others firms, we say that the smaller is v the higher the degree of strategic

[10] When the demand equation for firm i's product is given by $y_i=y-\epsilon(p_i-p)$, $\epsilon>1$ and its log cost function is θy_i+K, $\theta>1$, for a convenient choice of K we obtain equation (1) with $v=(\theta-1)/(1+\epsilon(\theta-1))$.

[11] More realistically, money demand depends on the expected inflation rate through the nominal interest rate. To take this into consideration, m can be interpreted as nominal income. So equation (3) becomes an identity relating nominal and real GNP. In this context, the monetary policy can be interpreted as providing the quantity of money compatible with an announced target for the nominal income.

[12] This equation can be derived directly from an utility function involving money and a production function as in Blanchard and Kiyotaki (1987). A simplified version is presented in chapter 8 of Blanchard and Fischer (1989).

complementarity of prices.

We assume that a firm adjusts its price twice in an interval of length one. The first time is at the beginning of the period, when the price is set optimally. The second time is at the midpoint of the time interval, where the price is adjusted according to the inflation accumulated since the last adjustment (half period before). Optimal adjustments by different firms are staggered uniformly over time.

These assumptions can be motivated by the results of previous work on microfoundations of price rules. Small costs of price adjustments lead firms to adjust infrequently. Costs of gathering information lead firms to adjust at a constant interval of time, as shown in Caballero (1989). If past inflation is an information freely available, and if the inflation index is released very often, it may be optimal to adjust based on past inflation at points where it is not worth to pay the cost of gathering the precise information necessary to set the price optimally. Finally, Ball and Cecchetti (1988), and Caballero (1989) present explanations for staggered timing based on imperfect information and firm-specific shocks.

We assume that the instantaneous loss incurred by a firm when its price differ from the optimal price is proportional to the square of the price deviation from the optimal price. This follows from a second-order Taylor approximation to the profit loss. A firm chooses a price $x(t)$ to minimize the loss over $[t,t+1)$, knowing that its price will be adjusted at $t+1/2$ by $p(t+1/2)-p(t)$:

$$Z(t) = \int_0^{1/2} E_t[(x(t)-p^*(t+s))^2]\,ds + \int_{1/2}^{1} E_t[(x(t)+p(t+1/2)-p(t)-p^*(t+s))^2]\,ds \tag{5}$$

Minimizing (5) yields

$$x(t)=\int_0^1 E_t p^*(t+s)\,ds-\frac{1}{2}E_t[p(t+1/2)-p(t)] \tag{6}$$

A firm's price for the first half of the period is the average of its expected optimal price over the entire period minus half of the expected inflation for this half period. The firm's price for the second half-period is equal to the price for the first half-period adjusted by the inflation of the first half-period. When there is a steady inflation, with money and average price growing at the same rate, the prices in both the first and second half-periods are averages of the optimal prices in the respective half-periods (see figure 1).

The assumptions above imply that the price is given by:

$$p(t)=\int_0^1 x(t-s)\,ds+\int_0^{1/2} p(t-s)-p(t-1/2-s)\,ds \tag{7}$$

Now we derive the behavior of the economy under a steady inflation that is expected to last forever. Suppose that,

$$m(t)=t \tag{8}$$

which implies that money stock grows at a constant rate equal to one. It is easy to see that the following paths for the endogenous variables are consistent with equations (3),(4),(6) and (7):

$$
\begin{aligned}
p(t) &= t \\
p^{*}(t) &= t \\
y(t) &= 0 \\
x(t) &= t+1/4
\end{aligned}
\tag{9}
$$

Taken together, these equations represent the behavior of the economy under steady inflation. The price level grows at the same rate as the money stock and output is constant at the natural rate.

III. Disinflation

In this section, we derive the behavior of prices and output for an arbitrary disinflation path for the money stock, evaluate the results for two kinds of disinflation paths - linear and "cold-turkey" with a delay - and calculate the money path consistent with a "zero-output" disinflation.

Assume that the economy is in the steady inflation regime described in the last section, which is believed to last forever, and that, at time zero, a new path for the money supply is announced, believed, and carried out without further surprises. For t<0, the price set by a firm, x(t), and the price level, p(t) are given by (9). For t>0, x(t) is given by (6), which because of perfect foresight becomes:

$$x(t)=\int_{0}^{1} p^{*}(t+s)\,ds-\frac{1}{2}\left(p(t+1/2)-p(t)\right) \tag{10}$$

In Appendix A we show that substituting those expressions in the price equation (7) yields:

$$p(t)=\int_0^t sp^*(s)\,ds+\int_t^1 tp^*(s)\,ds+\int_1^{t+1}(1+t-s)p^*(s)\,ds$$
$$-\ \frac{1}{2}\int_0^t p(t-s+\frac{1}{2})\ -\ p(t-s)\,ds \qquad \textbf{(11)}$$
$$+\int_0^{1/2} p(t-s)-p(t-s-1/2)\,ds-\frac{t^2}{2}+\frac{3t}{4}-\frac{1}{4}\ , \qquad 0\le t<1$$

$$p(t)\ =\ \int_0^1 (1-s)p^*(t-s)\,ds\ +\ \int_0^1 (1-s)p^*(t+s)\,ds$$
$$-\ \frac{1}{2}\Bigg[\int_0^{1/2} p(t-s+\frac{1}{2})\ -\ p(t-s)\,ds \qquad \textbf{(12)}$$
$$-\ \int_{1/2}^1 [p(t-s+\frac{1}{2})\ -\ p(t-s)]\,ds\Bigg], \qquad t\ge 1$$

The formula for p(t) changes at t=1, when the last firms which set prices before t=0 have a chance to set prices again.

Since $p^* = vm(t)+(1-v)p(t)$, (11,12) define p(t) implicitly. We use numerical methods to find the solution for p(t) and substitute p(t) in the quantity equation for money to get y(t).

1. Linear Disinflation

Here we assume that the path for the growth rate of the money supply announced at t=0 is:

$$\frac{dm(t)}{dt}\ =\ 1-\frac{t}{k}\ , \qquad 0\le t<k \qquad \textbf{(13)}$$
$$=\ 0\ , \qquad t\ge k$$

Since m(0) = 0, this implies that the money stock has the following path:

$$m(t) = t - \frac{t^2}{2k}, \quad 0 \le t < k$$
$$= \frac{k}{2}, \quad t \ge k \tag{14}$$

Money growth decays linearly until reaching zero at time k. Figure 2-A shows the path for m(t) when k=1. The money path is concave with the money stock stabilizing at t=1. Since the interval of time between two optimal price adjustments for an individual firm is of length one (a normalization), we can, by choosing k, set the speed of the disinflation relative to the length of the individual price rules. For example, it is reasonable to assume that the length of individual price rules is not greater than one year. This implies, that when we set k=1, we are talking about a disinflation which takes less than one year; this is a quick disinflation by real economy standards.

Figure 2-B,C,D shows the path of output resulting from linear disinflations which take 1, 3 and 5 periods. The pattern is the same in all cases. Initially, there is a boom, followed by a recession. Further on, there is a very small boom, which lasts until the output converges to its natural rate. This second boom will be neglected in our analysis. As we can see also in Table 1, the intensity and duration of the booms and recessions are larger, the higher the strategic complementarity in prices (that is, the smaller is v). As a consequence, the lower is v, the slower is the convergence of the output to the natural rate, as indicated by the sum of the lengths of the boom and the recession, in the last column of Table 1. The intensity of the recession decreases with k. Nonetheless, whatever k, a recession always happens immediately after the period when the money stock

is stabilized (k). As Table 1 illustrates, there is not a monotonic relationship between the intensity of the boom and k. The intensity of the boom grows when k increases from 1 to 3, but decreases when k goes from 3 to 5.

Although a recession always happens when stabilization is undertaken in a reasonable amount of time, the average output during stabilization becomes positive for some k smaller than 3. In the third entry of Table 1 we calculate the output average during the periods when output is away from its natural rate. We note that there is not a monotonic relationship between the output average and the stabilization horizon. Output average changes from a 1.7% recession for k=1, to a 0.847% boom when k=3, but decreases slightly when k increases further to 5.

2. "Cold-turkey" with a delay

Here we assume that the announced path for the money growth at t=0 is to keep growing at the same rate until t=k, when growth is halted. Formally:

$$\frac{dm(t)}{dt} = 1\,, \qquad t<k$$
$$\frac{dm(t)}{dt} = 0\,, \qquad t\geq k \tag{15}$$

which implies the following path for m:

$$m(t) = t\,, \qquad t<k$$
$$m(t) = k\,, \qquad t\geq k \tag{16}$$

Figure 3-A depicts the path of m(t), for k=1.

The effect on output is graphed in Figure 3B,C,D for disinflations which take place 0.5, 1 and 3 periods after the announcement. In this kind of stabilization, when k is not smaller than 1, the value of k does not have a significant effect on output. The output stays constant at the natural level until immediately before k. Then, it increases sharply, reaching a peak at t=k. It decreases rapidly thereafter, with the boom turning into a recession. When the recession ends, the output converges to the natural rate through a very small boom. As Table II shows, the boom and recession maximums and durations are not substantially affected by k, when k is not smaller than 1. Although a recession is always generated, there is not a substantial negative effect on average output. For k=0.5 there is a very mild recession (average output is 0.24% below the natural rate when we assume v=0.5). Even for a short one-period disinflation, the average output is 1.7% above the natural rate, when averaged over periods in which the output is different from zero.

3. <u>Disinflation with constant output</u>

Figure 4 shows the disinflation trajectory (of the money supply) which keeps output constant at its natural rate. Money growth decreases sharply until t=1, jumps up at this point, decreasing in a smoother way after that. Observe that disinflation is attained approximately after 4.5 periods[13]. That is, disinflation with constant output takes 4.5 times the length of a contract. As a contrast, if indexation is absent,

[13] The path of money growth converges asymptotically through dying oscillations around zero. After 4.5 periods, money growth is always very close to zero.

disinflation with constant output takes only 1.5 period (see Ball(1990))[14].

4. Different initial inflation rates

Different initial inflation rates do not affect our results qualitatively. That is, if the central bank disinflates in k periods, the level of initial inflation does not affect the time intervals where a recession and or a boom occurs. The initial level of inflation influences only the intensity of the output deviations from the natural rate. The higher is the initial level of inflation, the higher is the intensity of the recessions and booms generated. Appendix B demonstrates this point.

IV. Interpretation

Ball (1990) examines disinflation with price rigidity and staggering, but without indexation. He obtains a boom for linear and "cold-turkey" disinflations in one period. In his model, there are two effects taking place during disinflation. One is the effect of the surprise announcement of disinflation, at t=0. The surprise generates an overhang of prices set too high (according to the previous expectations), which tends to cause a recession. The other effect can be isolated assuming perfect foresight. First, observe that the trajectory of money during

[14]Note that this is not necessarily the fastest disinflation without recession, since a boom is not allowed either.

disinflations is concave[15], and prices are set according to an average of future optimal prices. Individual prices, set to be fixed during a period, tend to be below the trajectory of money, generating a boom. If disinflation is very fast, the surprise effect is predominant, producing a recession. However, even for reasonably short horizons, the concavity effect is prevalent. Any linear disinflation which takes longer than 0.68 period or any "cold-turkey" disinflation which is announced more than 0.37 period in advance causes a boom and not a recession.

By introducing indexation, we generate a third effect. This effect can be seen through equation (12), which rules the price level behavior under perfect foresight[16]. The first two terms are also present in the model without indexation. They are responsible for Ball's concavity effect. The remaining part, between square brackets, arises only because of indexation. It can be interpreted as the acceleration of the "average" inflation between period t+1/2 and t, where the "average" inflation in period t+1/2 is measured as the average price in [t,t+1/2] minus the average price in [t-1/2,t]. In a steady inflation regime, when inflation is constant, the term between square brackets is zero. So, the expression for the price level is the same as in the model without indexation. A decrease in the inflation, by making the square bracket term negative, makes the price level in the indexation model greater than in the model without indexation. This "indexation effect" tends to cause a recession,

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[15] This is true for linear and "cold-turkey" disinflations. Only awkward trajectories of disinflation are not concave.

[16] There is also an "indexation-surprise" effect. However, this does not seem to be quantitatively important.

making disinflation more difficult in our model than in Ball's. In our disinflation simulations we saw that there is always a boom and a recession. This can be explained by the different timing of the effects[17].

To gain intuition on why the "indexation effect" depends on the inflation acceleration, first recall that the optimal price set by a firm at time t decreases with the expected inflation rate between t and t+1/2 (equation 6). A price set at time t is fixed in this level until t+1/2. At period t, there are also prices of other firms which set their prices at time u, where u is between t-1/2 and t, taking into account expected inflation between u and u+1/2. So, the average price of those firms in t depends negatively on future inflation. However, some individual prices at t are the sum of the prices which were set at some s, with s between t-1 and t-1/2, with the inflation rate between s and s+1/2. So, past inflation affects the average price of those other firms positively. The aggregation of the average price of those two subsets of firms causes the "indexation effect" on the aggregate price level to depend on the difference between future and past inflation.

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[17] For example, in a one-year linear disinflation, the concavity effect reaches its maximum at t=1. If we assume, for simplicity, that the price level is always equal to the money stock, we would conclude that the indexation effect reaches its peak at t=1.25.

V. Final Remarks

We have shown that combining indexation and staggering helps to explain the costs of disinflation. In a model without indexation, it is possible to disinflate in less than one period and produce a boom and without a recession. Ball (1990) argues that with time-varying prices it is more difficult to disinflate[18]. In this case, disinflation can be attained in one period without any recession (and without any boom either). In explaining the costs of disinflation, the model with indexation does even a better job than the model with time-varying prices. It has also the additional advantage of being much more realistic, especially for high inflation economies.

The results are not entirely satisfying. The model generates an initial boom which is not observable in disinflations induced by monetary policy[19]. It might be argued that when the initial inflation is high, the considerable decline of the inflation rate would cause a reduction in the money velocity. This would add a recessive effect to the model that could possibly offset the initial boom. In this context, the disinflation paths for the money supply analized above should be reinterpreted as disinflation paths for the nominal GNP, with the money supply being altered to take into consideration the changes in velocity (see also footnote 11)[20]. However, for moderate initial

[18] In the model with time-varying prices, each firm chooses at time t a path of prices from t to t+1. The time when each firm chooses a path of prices is uniformly staggered.

[19] A boom followed by a recession is a pattern of exchange-rate-based disinflations.

[20] Nominal GNP targeting was followed by the Budesbank from

inflations, the monetization effect is not likely to big, and initial booms are not observable either.

Thus, given our comparative results, we are able to explain why indexation makes disinflation harder, but our absolute results do not allow us to claim that our model fully account for the costs involved in a disinflation.

REFERENCES

.Ball, L. (1990), "Credible Disinflation with Staggered Price Setting," NBER Working Paper 3555.

.Ball, L. (1992), "Disinflation with Imperfect Credibility," NBER Working Paper 3983.

.Ball, L. and S. Cecchetti (1988), "Imperfect Information and Staggered Price Setting," American Economic Review 78: 999-1018; M-R Ch 11.

.Blanchard, O. (1979), "Wage Indexing Rules and the Behavior of the Economy," Journal of Political Economy 87: 798-815.

.Blanchard, O. (1983), "Price Asyncronization and Price Level Inertia," in R. Dornbusch and M. H. Simonsen (orgs.), Inflation, Debt and Indexation, MIT Press: 3-24.

.Blanchard, O. (1986), "The Wage-Price Spiral," Quarterly Journal of Economics 101: 543-565.

.Blanchard, O. and S. Fischer (1989), Lectures on Macroeconomics, MIT Press.

.Blanchard, O. and N. Kiyotaki (1987), "Monopolistic Competition and the Effects of Aggregate Demand," American Economic Review 77: 647-686.

.Bonomo, M. and Garcia (1992), "Limited Rationality and Disinflation," in preparation.

.Caballero, R. (1989), "Time Dependent Rules, Aggregate Stickiness and Information Externalities," Columbia University Working Paper #428.

1974 to 1980 (see Fischer (1988)).

.Fischer, S. (1977), "Long-term Contracts, Rational Expectations, and the Optimal Money Supply Rule," Journal of Political Economy 85: 191-205; M-R Ch 7.

.Fischer, S. (1986), "Contracts, Credibility and Disinflation," in: Indexing, Inflation and Economic Policy, MIT Press.

.Fischer, S. (1988), "Monetary Policy and Performance in U.S., Japan and Europe," in: Y. Suzuki and M. Olcabe (eds.) Toward a World of Economic Stability: Optimal Monetary Framework and Policy, University of Tokyo Press.

.Friedman, M. (1974), "Monetary Correction," in: H. Giersch et al. (eds.), Essays on Inflation and Indexation, American Enterprise Institute for Public Policy Research, Washington, D.C.

.Gray, J. (1976), "Wage Indexation: a Macroeconomic Approach," Journal of Monetary Economics 2: 221-235.

.Gray, J. (1978), "On Indexation and Contract Length," Journal of Political Economy 86: 1-18.

.Phelps, E. (1978), "Disinflation without Recession: Adaptive Guideposts and Monetary Policy," Welt. Archiv. 114: 783-809.

.Simonsen, M. (1983), "Indexation: Current Theory and the Brazilian Experience," in: R. Dornbusch and M. Simonsen (eds.) Inflation, Debt and Indexation, MIT Press.

.Taylor, J. (1979), "Staggered Wage Setting in a Macro Model," American Economic Review 69: 108-113.

.Taylor, J. (1980), "Aggregate Dynamics and Staggered Contracts," Journal of Political Economy 88: 1-23.

.Taylor, J. (1983), "Union Wage Settlements During a Disinflation." American Economic Review 73: 981-993.

APPENDIX A

Here we derive equations (11) and (12), which define implicitly the path of the price level for an arbitrary disinflation.

We first derive the expression (11), for t between 0 and 1. Rearranging expression (7):

$$p(t)=\int_0^1 x(t-s)\,ds+\int_0^{1/2} p(t-s)-p(t-1/2-s)\,ds$$
$$=\int_0^t x(t-s)\,ds+\int_t^1 x(t-s)\,ds+\int_0^{1/2} p(t-s)-p(t-1/2-s)\,ds$$

x(t) is given by (9), for $t<0$, and by (10), for $t>0$. Substituting in the last expression above, we get:

$$p(t)=\int_0^t\int_0^1 p^*(t-s+r)\,dr\,ds-\frac{1}{2}\int_0^t p(t+1/2-s)-p(t-s)\,ds$$
$$+\int_t^1 (t-s+1/4)\,ds+\int_0^{1/2} p(t-s)-p(t-1/2-s)\,ds$$

The third integral is $3/4t-t^2/2-1/4$. The double integral can be represented as a sum of simple integrals:

$$\int_0^t\int_0^1 p^*(t-s+r)\,dr\,ds=\int_0^1 sp^*(s)\,ds+\int_t^1 tp^*(s)\,ds+\int_t^{t+1} (1+t-s)p^*(s)\,ds$$

Replacing the double integral and the third integral by the expressions above, we arrive at equation (11):

$$p(t) = \int_0^t s p_i^*(s)\,ds + \int_t^1 t p_i^*(s)\,ds + \int_1^{t+1} (1+t-s) p_i^*(s)\,ds$$
$$- \frac{1}{2}\int_0^t [p(t-s+\frac{1}{2}) - p(t-s)]\,ds$$
$$+ \int_0^{1/2} p(t-s) - p(t-s-1/2)\,ds - \frac{t^2}{2} + \frac{3t}{4} - \frac{1}{4}$$

To get the expression (12) for p(t), when t>1, first, we substitute expression (10) for x(t) into (7):

$$p(t) = \int_0^1\int_0^1 p^*(t-s+r)\,dr\,ds - \frac{1}{2}\int_0^1 p(t-s+1/2) - p(t-s)\,ds$$
$$+ \int_0^{1/2} p(t-s) - p(t-s-1/2)\,ds$$

The double integral of the expression above can be represented as a sum of simple integrals:

$$\int_0^1\int_0^1 p^*(t-s+r)\,dr\,ds = \int_0^1 (1-s) p^*(t-s)\,ds + \int_0^1 (1-s) p^*(t+s)\,ds$$

Replacing the double integral by the expression above we arrive at:

$$p(t) = \int_0^1 (1-s) p_i^*(t-s)\,ds + \int_0^1 (1-s) p_i^*(t+s)\,ds$$
$$- \frac{1}{2}\int_0^1 [p(t-s+\frac{1}{2}) - p(t-s)]\,ds$$
$$+ \int_0^{1/2} [p(t-s) - p(t-s-1/2)]\,ds$$

Rearranging yields equation (12).

APPENDIX B

Here we derive the effect of different initial inflation rates on the level of output during a linear disinflation. The same kind of argument applies to "cold turkey" disinflations. Suppose the economy is initially in a steady inflation regime with inflation b. For convenience, when the initial level of inflation is b, we denote the money supply, the price level and the output at time t by m(b,t), p(b,t) and y(b,t), respectively. We write the money supply path in the steady inflation regime as:

$$m(b,t)=bt$$

The equilibrium aggregate price, output and individual price set at t are:

$$p(b,t)=bt$$
$$y(b,t)=0$$
$$x(b,t)=bt+\frac{b}{4}$$

If a k-period linear disinflation is announced at time 0 and carried over without further surprises, the path for money supply rate of growth will be the following:

$$\frac{dm(b,t)}{dt}=b-\frac{bt}{k}, \qquad 0\le t<k$$
$$=0, \qquad t\ge k$$

with the corresponding path for the money supply:

$$m(b,t)=bt-\frac{bt^2}{2k}, \qquad 0\le t<k$$
$$=\frac{bk}{2}, \qquad t\ge k$$

Since equations (6) and (7) do not depend on the level of initial inflation, we have

$$x(b,t)=\int_0^1 E_t p^*(b,t+s)\,ds-\frac{1}{2}E_t[p(b,t+1/2)-p(b,t)] \qquad (6')$$

$$p(b,t)=\int_0^1 x(b,t-s)\,ds+\int_0^{1/2} p(b,t-s)-p(b,t-1/2-s)\,ds \qquad (7')$$

Substituting (6') into (7') and using the fact that whenever an agent sets a price before t, he does it according to the steady inflation equation above, and that when an agent sets a price at a time later than t, he has perfect foresight, we get equations analagous to (10) and (11):

$$\begin{aligned} p(b,t)=&\int_0^t sp^*(b,s)\,ds+\int_t^1 tp^*(b,s)\,ds \\ &+\int_1^{t+1}(1+t-s)p^*(b,s)\,ds \\ &-\frac{1}{2}\int_0^t p(b,t-s+\frac{1}{2}) - p(b,t-s)\,ds \\ &+\int_0^{1/2} p(b,t-s)-p(b,t-s-1/2)\,ds \\ &-\frac{bt^2}{2}+\frac{3bt}{4}-\frac{b}{4}, \qquad 0\le t<1 \end{aligned} \qquad (11')$$

$$p(b,t)=\int_0^1 (1-s)p^*(b,t-s)\,ds+\int_0^1 (1-s)p^*(b,t+s)\,ds$$
$$-\frac{1}{2}\left[\int_0^{1/2} p(b,t-s+\frac{1}{2})-p(b,t-s)\,ds\right. \qquad \textbf{(12')}$$
$$\left.-\int_{1/2}^1 [p(b,t-s+\frac{1}{2})-p(b,t-s)]\,ds\right], \qquad t\geq 1$$

Equations (10) and (11) define the price path p(1,t). It is easy to see that if p(1,t) satisfy (10),(11), p(b,t)=bp(1,t) satisfy (10'),(11'). Since m(b,t)=bm(1,t), it follows that y(b,t)=by(1,t). So, when we find a recession (boom) for an initial inflation of 1, we would also find a recession (boom) for any other initial inflation level. The magnitude of the effects is stronger the higher is the initial inflation level.

FIGURE 1

INDIVIDUAL AND OPTIMAL PRICE

PATH IN A STEADY INFLATION REGIME

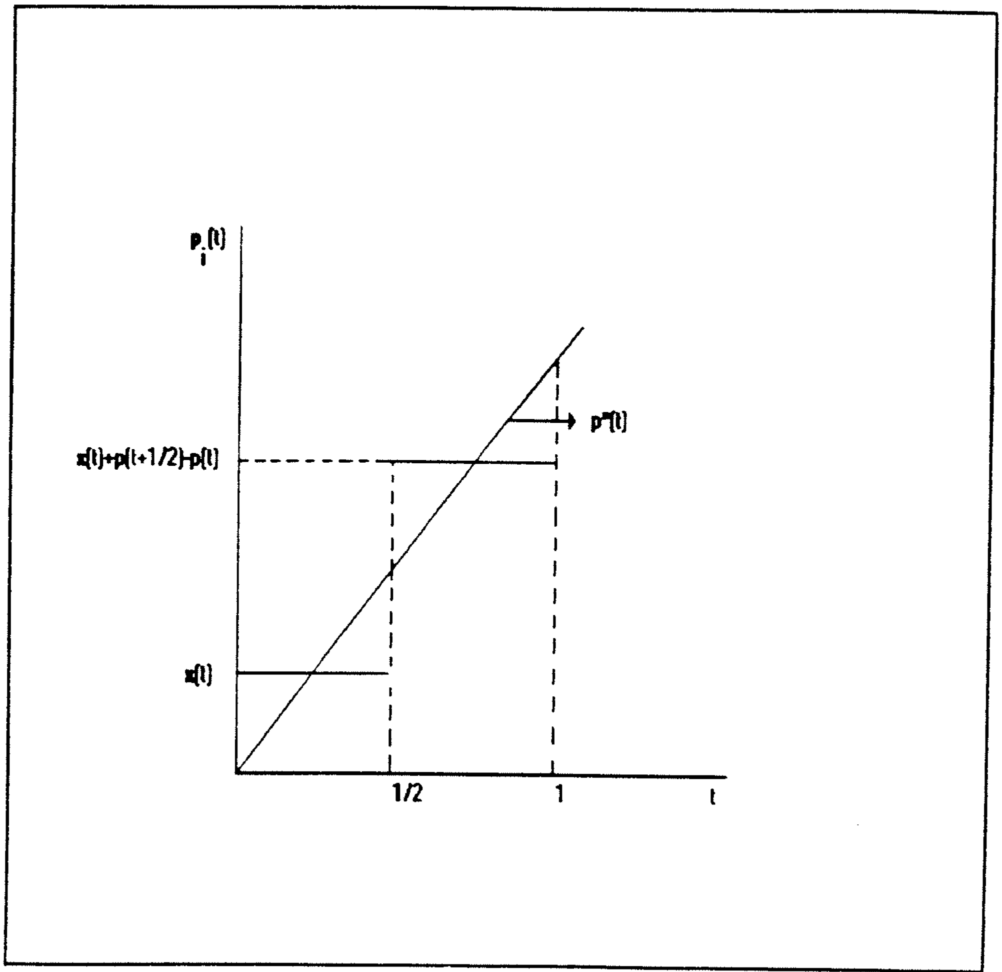

FIGURE 1

FIGURE 2

LINEAR DISINFLATION

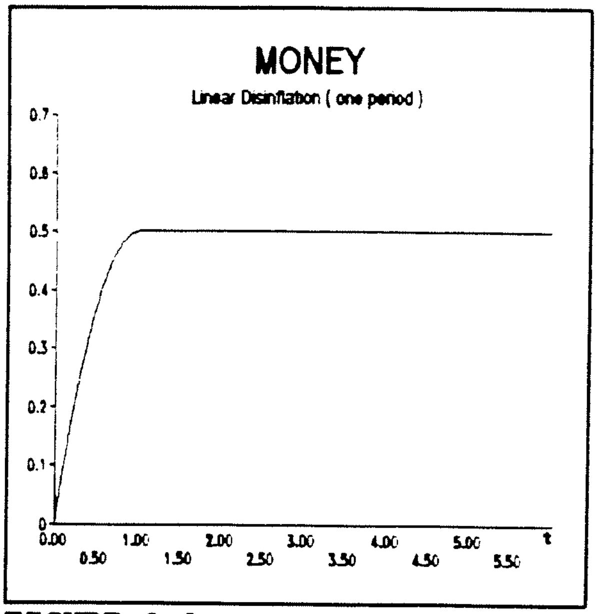

FIGURE 2-A

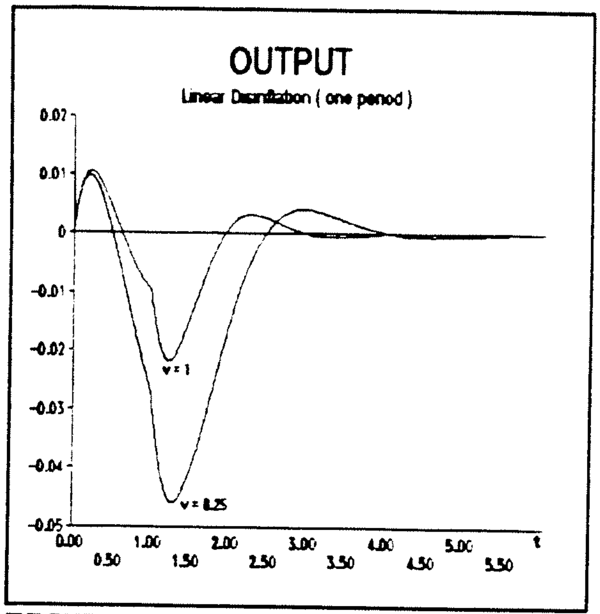

FIGURE 2-B

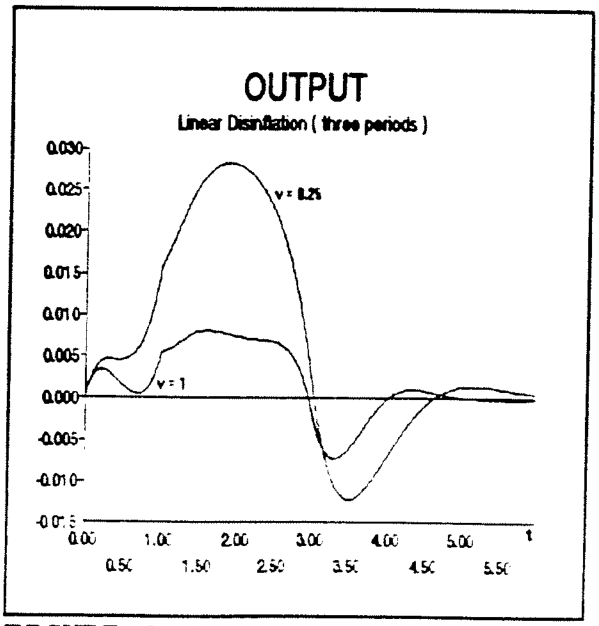

FIGURE 2-C

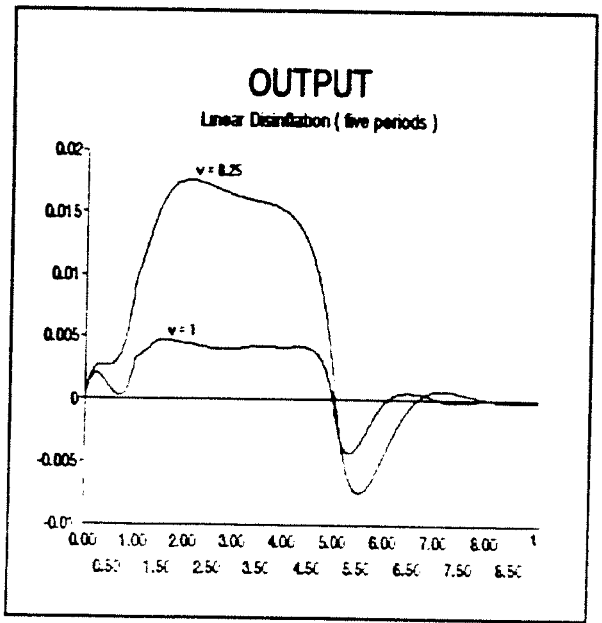

FIGURE 2-D

FIGURE 3

"COLD-TURKEY" DISINFLATION

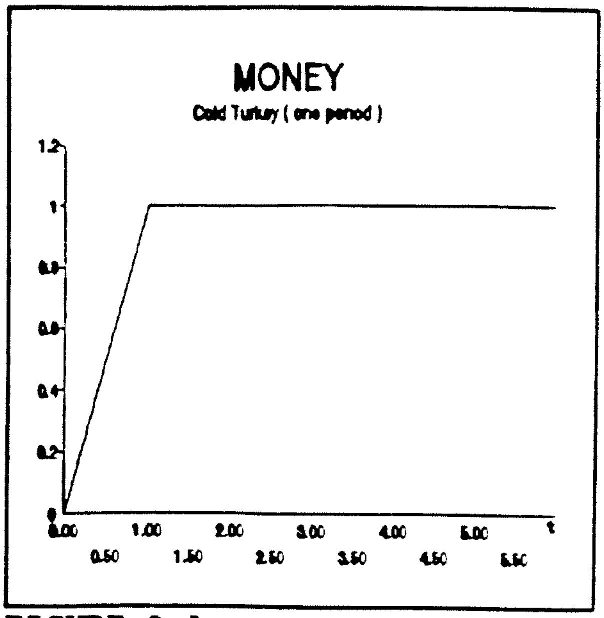

FIGURE 3-A

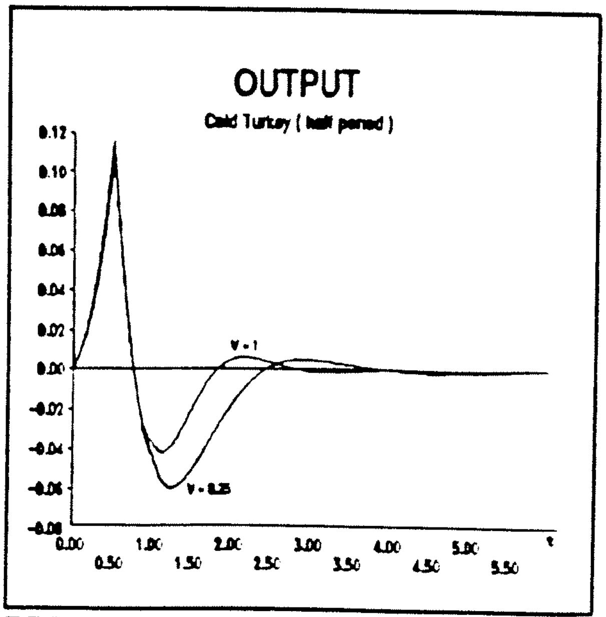

FIGURE 3-B

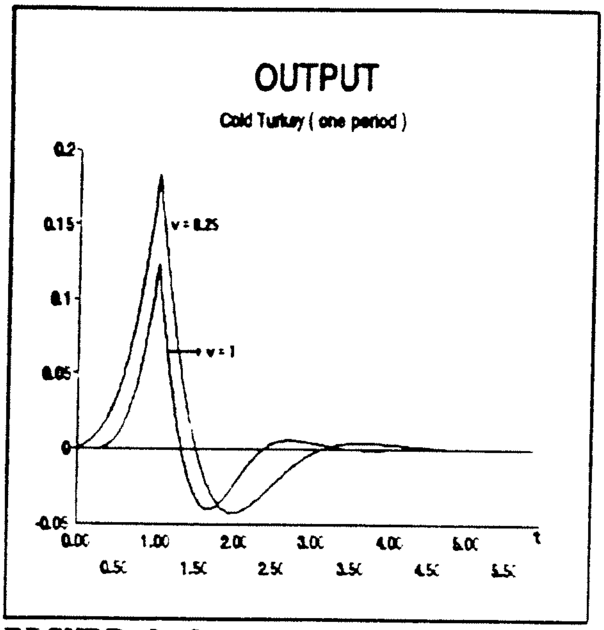

FIGURE 3-C

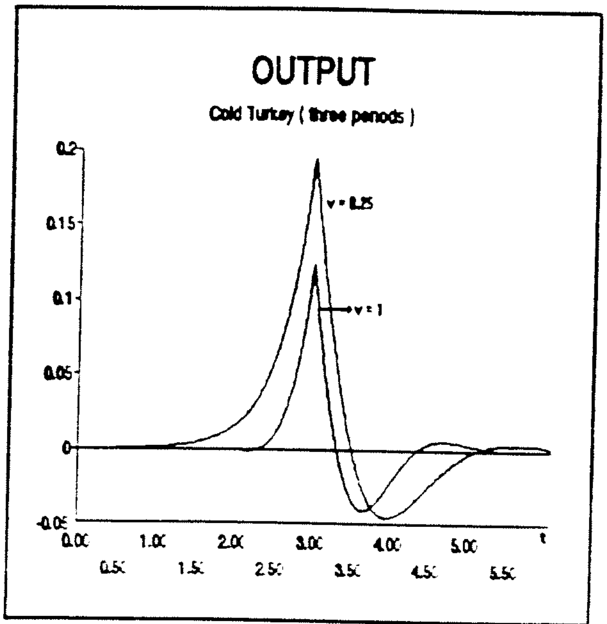

FIGURE 3-D

FIGURE 4

DISINFLATION WITH y(t)=0

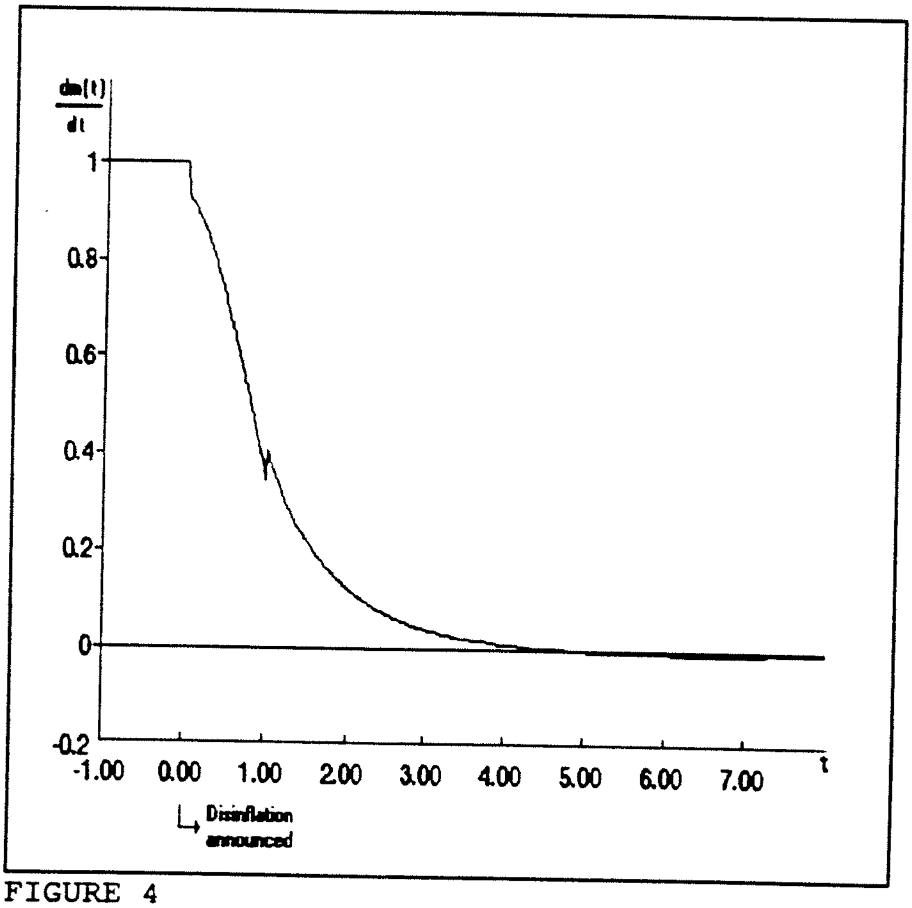

FIGURE 4

LINEAR DISINFLATION

TABLE I

K	V	Output Average (1)	Boom Maximum	Recession Maximum	Boom Duration	Recession Duration	End of Recession
1	0.25	-0.01712	0.00980	-0.04600	0.50	1.97	2.47
	1.00	-0.00524	0.01053	-0.02178	0.63	1.32	1.95
3	0.25	0.00847	0.02819	-0.01220	3.04	1.58	4.62
	1.00	0.00261	0.00802	-0.00722	2.96	1.05	4.01
5	0.25	0.00828	0.01756	-0.00748	5.02	1.61	6.63
	1.00	0.00242	0.00472	-0.00425	4.96	1.08	6.04

"COLD-TURKEY" DISINFLATION

TABLE II

K	V	Output Average (1)	Boom Maximum	Recession Maximum	Boom Duration	Recession Duration	Length of Recession plus Boom
0.5	0.25	-0.00242	0.11619	-0.05946	0.68	1.66	2.34
	1.00	0.00163	0.10993	-0.04149	0.66	1.07	1.73
1	0.25	0.01727	0.18321	-0.04260	1.45	1.65	3.10
	1.00	0.00891	0.12313	-0.04010	0.96	1.06	2.02
3	0.25	0.01889	0.19428	-0.04469	2.52	1.64	4.16
	1.00	0.00884	0.12302	-0.04005	0.95	1.06	2.01

(1) Output is integrated through the periods of boom and the recession and devided by the total length of time