DEPARTAMENTO DE ECONOMIA

PUC-RJ

TEXTO PARA DISCUSSÃO

No. 284

CONSUMPTION AND EQUILIBRIUM ASSET PRICING: AN EMPIRICAL ASSESSMENT

MARCO BONOMO
RENÉ GARCIA

JULHO 1992

Consumption and Equilibrium Asset Pricing: An Empirical Assessment[1]

by

Marco Bonomo

Pontifícia Universidade Católica do Rio de Janeiro

and

René Garcia

Département de sciences économiques
and C.R.D.E., Université de Montréal

[1] We would like to thank John Campbell and Pierre Perron for their useful advice and Gregory Chow, Jon Faust, Franque Grimard and Christoph Schmidt for useful comments and discussions

In the debate over the efficiency of markets in the financial literature, a number of empirical facts have been put forward as a challenge to the proponents of an equilibrium view of the world. The first and most famous concerns the so-called equity premium puzzle unveiled by Mehra and Prescott (1985): the difference between the return on equity and the return on a risk-free asset is too high historically to be explained in a complete-market Arrow-Debreu equilibrium framework. More recently, other ambivalent empirical facts have been detected: there seems to be negative autocorrelation in both real and excess long-horizon returns (Fama and French (1988a), Poterba and Summers(1988)); the dividend-price ratio seems to have some forecasting power for equity returns (Fama and French (1988b)). The ambivalence of this evidence comes from the fact that it can be made consistent with both an efficient market and an inefficient market explanation.

The main thrust of the efforts to provide an equilibrium theory consistent with the facts has been aimed at the equity premium puzzle. Researchers tried to reshape the various building blocks of the Lucas exchange economy model initially proposed by Mehra and Prescott (1985) in order to come up with pieces that fit the puzzle. The characterization of the utility function has received a lot of attention, along with the postulated nature of the economy. No such scrutiny has been applied to the representation of the endowment process which has potentially an important role to play in an exchange economy. Most studies calibrate the model parameters to fit some moments of the data based on a finite state Markov-chain approximation.

In this paper, we therefore pay careful attention to the modeling and estimation of the endowment process. Given the skewness and kurtosis present in the actual consumption and dividend growth series[1], we assume a heteroskedastic joint bivariate Markov endowment process and use maximum likelihood estimates of the parameters of this process to test the model over the period 1871-1985. In the assessment of the model, we take a wider scope than the equity premium and try to replicate both the first and second unconditional moments of the return series, the negative serial correlation present in real and excess returns and the forecasting power of the dividend-price ratio for multiperiod returns.

As a general assessment of the results, we can say that, for the real returns, the model captures to some extent the main features of the data for values of the coefficient of risk aversion below 10. The value of the discount factor does not play a major role: the model is assessed with a value of 0.97 but any reasonable value between 0.95 and 0.99 would have produced similar results. The main failure of the model comes from the risk-free rate: the equity premium is, as in Mehra and Prescott (1985), hopelessly too low because the mean of the risk free rate is too high; no evidence of negative autocorrelation is found in excess returns; the dividend-price ratio has no forecasting power at all for excess

[1] This is documented in Cechetti, Lam and Mark (1990a).

returns.

These failures seem to confirm that aggregate consumption does not characterize the endowment process in such an economy and we are therefore inclined to conclude as Mehra and Prescott (1985) and Weil (1989) that the assumptions of complete markets and of a representative agent are probably at fault. We thus ask in Section V the following questions: what would be the parameters of the assumed Markov endowment process that will rationalize the returns observed on the market for the risk-free rate and the equity returns? Would it resemble the consumption process of an individual stockholder?

This approach is in the spirit of Hansen and Jagannathan (1990) except that our inference is based on the particular parametric specification chosen in our model. We use the return equations implied by the model and the assumed specification for the dividend process to estimate by maximum likelihood the equity price-dividend ratios and risk-free asset prices from the actual returns and dividends. Assuming values for the remaining parameters (coefficient of risk aversion, discount factor and correlation between consumption and dividends innovations), we can infer the consumption parameters that will rationalize the observed returns. Using this experiment, one can choose an endowment process which matches the mean of the consumption growth of stockholders, as inferred from Mankiw and Zeldes (1989), but the corresponding standard deviation seems excessive.

Section I presents figures for the above mentioned stylized facts regarding asset returns for the period 1871-1985 and discusses the related theoretical issues. In Section II, a synthetic version of the asset pricing model is developed. In Section III, the parameters of the model are estimated by maximum likelihood. The model ability to replicate the stylized facts is assessed in Section IV. Section V infers the consumption parameters of an individual stockholder and assesses their plausibility. Section VI concludes and outlines future possible avenues of research.

II. Stylized Facts and Overview of the Theoretical Issues

Table 1 illustrates some empirical facts related to asset returns for the period 1871-1985.[2] Part A presents the first and second moments of the one-year Treasury Bill, the equity return and the equity premium. Over this period, the risk premium averaged 6.1% with a standard deviation of 18.6% and a covariance with the Treasury Bill rate of -0.0014. The Treasury Bill rate averaged 2.02% with a standard deviation of 6.43%.

In Part B, there is evidence of negative serial correlation in real returns at various lags. There is also evidence of negative autocorrelation in excess returns, although the regression coefficients start to be positive at lags 5 and higher. These point estimates suggest strong predictability. However, Poterba and Summers (1988) cannot reject, based on returns variance ratio

[2] See data sources in Appendix A.

tests, the hypothesis that stock prices follow a random walk[3] and Fama and French (1988b) and Kim, Nelson and Startz (1988) show that the mean reversion in prices implied by such negative autocorrelation in returns is mainly a feature of the 1926-46 period.

Finally, in Part C, the regressions of multiperiod returns (one to five years) on current dividend yield (dividend-price ratio) produce positive and significant coefficients, increasing with the horizon.[4] This is true for both real and excess returns.

Both the negative correlation in returns and the predictability of returns by the dividend-price ratio can be explained by conflicting theories regarding the behavior of asset prices. For the proponents of an inefficient market, high (low) dividend price ratios announce high (low) returns because stock prices are irrationally low (high) compared to their fundamental value. Mean reversion in prices is created by the intervention of arbitrageurs at some point to eliminate this swing away from the fundamental value.

The efficient market explanation (Fama and French (1988 a, b) argues that if equilibrium expected returns are highly correlated but mean reverting and if shocks to expected returns are independent of the shocks to rational forecasts of dividends, a

---

[3] Poterba and Summers perform a bias correction in estimating their variance ratios and obtain higher values than ours. We do not make this correction since all we need is a reference value for our simulated variance ratios.

[4] Fama and French (1988b) adjust the standard errors in the t-statistics for the sample autocorrelation of overlapping residuals by the method of Hansen and Hodrick. We do not make this adjustment because we want simply reference values for our simulated t-statistics.

shock to expected returns has no long term effect on expected prices and must therefore be compensated by an opposite movement in the current price. The mean reversion in prices is then implied by the mean reversion in expected returns.

In trying to reconcile these empirical facts with an equilibrium explanation, the strategy of research has been to construct a simple economy (usually a Lucas (1978) exchange economy with a representative agent having a power utility function), to specify an exogenous law of motion for the endowment process (usually a discrete state-space representation), and to choose the parameters of the model to "mimic roughly" some empirical characteristics of asset returns.

There are some noteworthy exceptions to this basic framework: Epstein and Zin (1989) and Weil (1989) explore the implications of abandoning the equality between the coefficient of relative risk aversion and the inverse of the intertemporal elasticity of substitution imposed by the usual choice of a time-separable Von Neumann-Morgenstern utility and a power function; Constantinides (1988) abandons the time separability of utility and builds a model of habit formation; Tauchen (1986) and Kocherlota (1988) abandon the original Lucas assumption of a singular joint distribution for consumption and dividends; Kandel and Stambaugh (1990) propose a leveraged economy with a risky bond where equity becomes a residual claim on output; Rouwenhorst (1989) constructs a production economy. Despite these departures from the original Lucas and Mehra and Prescott framework, the results are disappointing: typically, the coefficient of risk aversion necessary to reproduce the empirical facts is implausibly too large and in some cases the discount factor is higher than one.

One justification for this general failure is to argue that aggregate consumption is too smooth to rationalize the observed risk premia. A solution to the equity premium and risk-free puzzles is then sought in the heterogeneity of consumers: Weil (1990) proposes a model where agents face an individual undiversifiable consumption risk which disappears in the aggregate; Mankiw and Zeldes (1989) provide evidence based on micro data that the consumption of stockholders differ considerably from the consumption of non-stockholders and covaries much more with the equity returns than the latter. Another approach for testing intertemporal asset pricing models is to avoid reference to consumption. For example, Campbell (1990) derives testable restrictions that relate first and second moments of asset returns, while Hansen and Jagannathan (1990) infer non-parametric bounds for the mean and variance of the intertemporal marginal rate of substitution using return data.

In this paper, we will try to replicate the stylized facts reported in Table 1 using the standard exchange model except that we specify following Tauchen (1986) a joint bivariate process for consumption and dividends[5].

The reason for disentangling the consumption and dividend processes is first and foremost an empirical one: a look at figure

[5] Cechetti, Lam and Mark (1990b) also follow this path in trying to explain the equity premium.

1 will convince any reader that these series are very different and that a model that postulates that consumption should be equal to dividends is rejected on casual observation of the data. Therefore by not imposing the equality between consumption and dividends, we make the model more general than the original Lucas exchange model.

Another justification for the choice of a bivariate process is that a model based on either one of these series as characterizing the endowment process is unable to produce the kind of negative autocorrelation detected in the return data. This has been shown in a comment by Bonomo and Garcia (1990) to Cechetti, Lam and Mark (1990a). The latter claim to have found an equilibrium asset pricing model that generates negative serial correlation in returns of the magnitude found in the data, but the univariate model chosen for the endowment process (a two-state Markov model with a very low mean in one of the states) is not the best in the class of Markov models. Once the proper specification (with one mean and two variances) is selected, the mean reversion effect disappears.

Cechetti, Lam and Mark (1990a) valuable contribution was however to estimate the endowment process instead of calibrating it as researchers had done until then. They specify a homoskedastic Markov switching model for the endowment process and show that it performs at least as well as AR(1) or AR(2) models for consumption or GNP growth series and much better for the dividend growth series. In Bonomo and Garcia(1990), we show that a heteroskedastic Markov switching model characterizes even better the endowment series. Therefore we choose in this paper a joint heteroskedastic

Markov switching model for dividend and consumption growth series. We do not impose a number of states a priori but let the data decide between a 2-state and a 3-state specification.[6] We believe that this approach is the best compromise between an endowment model that fits the data and a model that gives a closed-form solution for the asset returns.

We also want to establish if the rejection of the model comes from specific characteristics of aggregate consumption. This is why we assess the model according to various measures with the hope to identify common patterns that will point to an explanation. In Section IVa, we compute the population moments implied by our model for the risk free rate, the equity return and the equity premium. In Section IVb, we generate by Monte-Carlo simulations the small sample and large sample distributions of the variance ratios and regression coefficients implied by the model for both real and excess returns in order to see if the actuals exhibited in Table 1 could have been produced by such a model. We repeat the Monte Carlo experiment in Section IVc, this time for various statistics associated with the regression of multiperiod returns on dividend-price ratios: the regression coefficients, their t-statistics and the R^2 of the regression.

---

[6] This is the maximum number of states we can specify to obtain reliable estimates given the available number of observations.

II. The Asset Pricing Model

Many identical infinitely-lived agents maximize their intertemporal utility and receive each period an endowment of a nonstorable good. Assuming additive time separability of the utility function and a constant discount factor β, the representative agent utility at time t, V_t, can be written as:

$$V_t = E_t \sum_{j=0}^{\infty} \beta^j U(C_{t+j}) \tag{1}$$

where E_t denotes expectation conditional on information available at time t and C_t the per capita consumption . We assume that U(.) is the power utility function with constant relative risk aversion γ:

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \tag{2}$$

Equilibrium requires that the prices of existing assets are such that the representative agent is satisfied to consume her expected endowment. In other words, the agent cannot increase her utility by changing the expected intertemporal allocation of her endowment through the purchase or sale of an asset or portfolio of assets.

Defining equity in this economy as an asset that gives as payoff a dividend D_t, its price (P^e) in equilibrium should satisfy the following first-order condition:

$$P_t^{e} U'(C_t) = \beta E_t U'(C_{t+1}) \, [P_{t+1}^{e} + D_{t+1}] \tag{3}$$

Using the power utility function defined in (2), this condition can be rewritten as:

$$P_t^{e} = \beta E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} [P_{t+1}^{e} + D_{t+1}] \tag{4}$$

In this version of the Lucas model proposed by Tauchen, the consumption endowment is not equal to dividends and includes payoffs from other assets unspecified in the model. This separation of consumption and dividends does not take away any generality from the model and has the advantage to make it more realistic given the observed different behavior for consumption and dividend growth.

The price-dividend ratio for the equity is then given by:

$$\frac{P_t^{e}}{D_t} = \sum_{j=1}^{\infty} \beta^j E_t \left(\frac{C_{t+j}}{C_t} \right)^{-\gamma} \frac{D_{t+j}}{D_t} \tag{5}$$

Similarly, in this economy, one can define a riskless asset which pays one unit of consumption with certainty the next period. In equilibrium, its price (P^f) will satisfy the following first-order condition:[7]

$$P_t^{f} U'(C_t) = \beta E_t U'(C_{t+1}) \tag{6}$$

Using (2), the price of the risk free asset can therefore be written as:

---

[7] Equations (3) and (4) are valid for unconstrained individual stockholders even if markets are incomplete.

$$P_t^f = \beta E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \tag{7}$$

We postulate that the logarithms of consumption and dividends follow a bivariate random walk where both the means and the variances change according to a Markov variable S_t which takes the values 0, 1, ...,K-1 (in the case of K states). The sequence $\{S_t\}$ of Markov variables evolves according to the following transition probability matrix Π:

$$\Pi = \begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0(k-1)} \\ p_{10} & p_{11} & \cdots & p_{1(k-1)} \\ \vdots & \vdots & \vdots & \vdots \\ p_{(k-1)0} & p_{(k-1)1} & \cdots & p_{(k-1)(k-1)} \end{bmatrix} \tag{8}$$

The bivariate consumption-dividends process can then be written as:

$$\begin{aligned} c_t - c_{t-1} &= \alpha_0^c + \alpha_1^c S_{1,t} + \cdots + \alpha_{k-1}^c S_{k-1,t} + (\omega_0^c + \omega_1^c S_{1,t} + \cdots + \omega_{k-1}^c S_{k-1,t}) \epsilon_t^c \\ d_t - d_{t-1} &= \alpha_0^d + \alpha_1^d S_{1,t} + \cdots + \alpha_{k-1}^d S_{k-1,t} + (\omega_0^d + \omega_1^d S_{1,t} + \cdots + \omega_{k-1}^d S_{k-1,t}) \epsilon_t^d \end{aligned} \tag{9}$$

where $S_{i,t}$ is a function of the state of the economy, S_t, taking value 1 whenever $S_t = i$ and 0 otherwise; c_t and d_t are respectively ln C_t and ln D_t; ϵ_t^c and ϵ_t^d are N(0,1) error terms with correlation ρ_{cd}. Then, in state i, the means and standard deviations of the growth rates of consumption and dividends will be given respectively by $(\alpha_0^c+\alpha_i^c, \omega_0^c+\omega_i^c)$ and $(\alpha_0^d+\alpha_i^d, \omega_0^d+\omega_i^d)$.

Given the joint process defined by (9) and the transition probability matrix Π, we can find closed form solutions for the asset prices and easily derive the formulas for returns.

Iterating each equation n times in the system (9), we obtain:

$$\left(\frac{C_{t+n}}{C_t}\right)^{-\gamma}=\exp\left(-\gamma\sum_{j=1}^{n}\alpha_0^c+\alpha_1^c S_{1,t+j}+\cdots+\alpha_{k-1}^c S_{k-1,t+j}+(\omega_0^c+\omega_1^c S_{1,t+j}+\cdots+\omega_{k-1}^c S_{k-1,t+j})\,\epsilon_{t+j}^c\right) \quad (10)$$

$$\left(\frac{D_{t+n}}{D_t}\right)=\exp\left(\sum_{j=1}^{n}\alpha_0^d+\alpha_1^d S_{1,t+j}+\cdots+\alpha_{k-1}^d S_{k-1,t+j}+(\omega_0^d+\omega_1^d S_{1,t+j}+\cdots+\omega_{k-1}^d S_{k-1,t+j})\,\epsilon_{t+j}^d\right) \quad (11)$$

Multiplying (10) by (11), taking the conditional expectation with respect to the information set at time t[8] and using the independence of the sequences $\{S_t\}$ and $\{\epsilon_t^c,\epsilon_t^d\}$, we obtain:

$$E_t\left(\frac{C_{t+n}^{-\gamma}D_{t+n}}{C_t^{-\gamma}D_t}\right)=E_t\exp(\mu_0 n+\mu_1 i_{1\,t,n}+\cdots+\mu_{k-1} i_{k-1\,t,n}) \quad (12)$$

where:

$$\begin{aligned}
\mu_0&=-\gamma\alpha_0^c+\alpha_0^d+\frac{1}{2}\,(\gamma^2\omega_0^{c\,2}+\omega_0^{d\,2}-2\gamma\omega_0^c\omega_0^d\rho_{cd})\\
\mu_j&=-\gamma\alpha_j^c+\alpha_j^d+\frac{1}{2}\,[\gamma^2\omega_1^{c\,2}+\omega_j^{d\,2}-2\gamma\,(\omega_j^d\omega_0^c+\omega_0^d\omega_j^c+\omega_j^d\omega_j^c)\,\rho_{cd}\\
&\qquad +2\gamma^2\omega_0^c\omega_j^c+2\omega_0^d\omega_j^d] \qquad j=1,\ldots,k-1\\
i_{j\,t,n}&=\sum_{h=1}^{n}S_{j,t+h} \qquad j=1,\ldots,k-1
\end{aligned} \quad (13)$$

The expectation term on the right hand side of (12) can be

[8] We assume that S_t belongs to the information set at time t.

written in matrix form :

$$E_t \exp(\mu_0 n+\mu_1 i_{1\,t,n}+\cdots+\mu_{k-1} i_{k-1\,t,n}) = I_{k,t} A^{n} 1 \tag{14}$$

where $I_{k,t}$ is a kx1 row vector with 1 in the column corresponding to the state at time t and zeros in the other columns, *1* is a kx1 column vector of ones and the matrix A is given by:

$$A=\Pi\,\mathbf{M} \quad with: \qquad \mathbf{M}=diag(e^{\mu_0}, e^{\mu_0+\mu_1}, \cdots, e^{\mu_0+\mu_{k-1}}) \tag{15}$$

So, expression (11) can be substituted in the equity price-dividend equation (5) to obtain the following formula:

$$P_t^{e}=D_t \rho(S_t) \tag{16}$$

where $\rho(S_t)$ is given by:

$$\rho(S_t)=I_{k,t}[(I-\beta A)^{-1}-I]\,1 \tag{17}$$

Similarly, the price for the risk-free asset will be given by:

$$\phi(S_t) = \beta I_{k,t} \Pi W \tag{18}$$

where:

$$\begin{aligned} &W=diag(w_0, w_0 w_1, \cdots, w_0 w_{k-1}) \quad with: \\ &w_0=\exp(-\gamma\alpha_0^c+\frac{1}{2}(\gamma^2\omega_0^{c\,2})) \\ &w_j=\exp(-\gamma\alpha_j^c+\frac{\gamma^2}{2}(2\omega_0^c\omega_j^c+\omega_j^{c\,2})) \quad j=1,\cdots,k-1 \end{aligned} \tag{19}$$

The return formulas can now be easily derived. For the safe asset, the return will be:

$$R_t^f = \frac{1}{\phi(S_t)} \tag{20}$$

For the equity, the one-period gross return will be:

$$R_t^e = \frac{P_{t+1}^e + D_{t+1}}{P_t^e} = \left(\frac{P_{t+1}^e + D_{t+1}}{D_{t+1}}\right)\left(\frac{D_t}{P_t^e}\right)\left(\frac{D_{t+1}}{D_t}\right) \tag{21}$$
$$R_t^e = \frac{\rho(S_{t+1})+1}{\rho(S_t)} \exp\left(\alpha_0^d + \cdots + \alpha_{k-1}^d S_{k-1,t+1} + (\omega_0^d + \cdots + \omega_{k-1}^d S_{k-1,t+1}) e_{t+1}^d\right)$$

These formulas will allow us in Section IV to compute the theoretical moments implied by the model, but also to generate the distributions of the measures of serial correlation (variance ratios and regression coefficients) and of the coefficients, R-square and t-statistics in the returns on dividend yield regressions. First, however, we need values for the mean(α), standard deviation (ω) and probability (p) parameters. In Section III, we estimate these values by maximum likelihood.

III. Maximum Likelihood Estimation of the Model Parameters

When Mehra and Prescott (1985) tried to reproduce the equity premium in an equilibrium framework, they assumed that consumption was equal to dividends. Given that assumption, they chose to limit the number of Markov states to two, to constrain the transition probabilities to be equal in both states and to set the ω

parameters to zero. They selected the remaining parameters so that the average growth rate of per capita consumption and the standard deviation and first-order serial correlation of the growth rate matched the sample values for U.S. consumption growth over the period 1889-1978. This calibration procedure has the advantage to be simple but imposes serious restrictions on the parameter values. To be sure that the model rejection does not come from a misspecification of the endowment process, one has to be careful about choosing a model that fits the data as closely as possible. To be tractable though, the model should allow to find closed-form solutions for the asset returns.

In our estimation, we do not impose any restrictions on the parameters and we let the data determine the number of states, except that in practice the number of available observations limits us to a maximum of three states (which requires already the estimation of 19 parameters). We therefore estimated the bivariate model with two states and three states. Results are reported in Table 2.

Apart from the mean parameter in the dividend equation, the parameters introduced for the new state seem to be significant, but the problem is that the specifications are non-nested. Under the null hypothesis of two states, both the identification and the rank conditions are violated and the standard testing procedures fail to apply as explained in Garcia and Perron (1990) where various testing procedures are used in this context. We report in Table 2 the Davies (1987) test[9] result, which is an upper bound for the p-value of a null of two states. Its value close to zero strongly supports the 3-state model.

Looking at the estimation results for the means and variances, we can label State 1 as the bad state, since consumption growth has a low mean (1.3%) and a high standard deviation (4.8%) and dividend growth a high standard deviation (18.3%). Note that the mean of the dividend growth rate is not significantly different from zero in all three states. In the second state, dividend growth is still fairly variable (standard deviation of 4.7%), but consumption growth has an intermediate mean (1.9%) and a low standard deviation (1.6%). Finally, in state 0, dividend growth has the smallest standard deviation (2%), while consumption growth has a high mean (3.6%) and an intermediate standard deviation (3.3%).

The probability parameters indicate that state 2 is the most persistent (p_{22}=0.957), followed by state 1 (p_{11}=0.854) and by state 0 (p_{00}=0.661). The unconditional probabilities π_0, π_1 and π_2[10] indicate how often in the limit the respective states are reached. State 2 is certainly the most visited state (0.815), while state 1 (the bad state) has a 0.129 probability to be reached. State 0 is

---

9 See Appendix A in Garcia and Perron (1990) for a description of the test.

10 The unconditional probabilities are defined by:

$$\pi_i = \frac{C_{ii}}{\sum_{j=0}^{k-1} C_{jj}}$$

where C_{ii} denotes the i^{th} cofactor of the matrix C= I - Π, with I a KxK identity matrix and Π as defined in (8) in the text.

the least likely to be visited (0.056).

The estimation algorithm gives as a sub-product the probabilities of being in the various states at each point in the sample, given the information available at that point. These so-called filter probabilities[11] are shown in Figure 2. State 2 is mainly reached between the late 50s and the end of the sample, while state 1 is mostly present before the 50s. State 0 occurs at numerous occasions, for lapses of a year or two.

IV. Assessing the Model According to Various Measures

Given the theoretical formulas derived in Section II for the safe asset and equity returns and the values estimated in Section III for the endowment parameters, we are now able to derive and compute the unconditional moments of returns implied by the model and to generate series of returns on which variance ratios, regression coefficients and other statistics can be calculated. Our objective is to give the model a thorough assessment. Most of the efforts in the literature have been geared towards finding a general equilibrium asset pricing model that will explain the equity premium puzzle. But the equity premium is only one statistic and as we have seen in Section I, there are other stylized facts that can serve as benchmarks for an assessment of the model. As mentioned before, Cechetti, Lam and Mark (1990a) did a similar exercise with measures of serial correlation in returns, but it can

[11] To know how these filter probabilities are calculated, see Hamilton (1989).

be argued that univariate tests have low power and, therefore, using them as a benchmark might not be very useful. A bivariate test involving returns and dividend yields should be more satisfying to assess the predictability of returns at various horizons. We therefore run regressions of multiperiod returns on dividend yields and add the corresponding statistics (regression coefficients, Student-t and R^2) to the previous criteria to get a full array of measures upon which we can base our assessment of the model.

The analysis proves very useful since all three series of measures point to the same deficiency of the model: its inherent inability to replicate the empirical facts involving the risk-free rate. The equity premium mean is too low, there is no negative autocorrelation in excess returns, and the dividend-price ratio has no forecasting power for excess returns. However, the model performs reasonably well in reproducing the mean and variance of the equity return, the negative autocorrelation in real returns, and the forecastability of real returns by the dividend yield. In other words, we reaffirm in other dimensions the phenomenon dubbed by Weil (1989) the risk-free puzzle since all measures involving the risk-free rate fail to pass the test. The following three subsections provide a detailed analysis of the results.

IVa. The Unconditional Moments

The formulas for the first and second unconditional moments of the equity return, the risk-free rate and the equity premium are derived in Appendix B. Table 3 presents the results for three values of γ, the coefficient of relative risk aversion (or the inverse of the elasticity of intertemporal substitution). We choose a range of 1.5 to 10 in order to stay within the bounds originally set by Mehra and Prescott (1985). We keep a fixed value of 0.97 for the discount factor β since this parameter does not affect the results in a significant way if it is maintained within the reasonable range of 0.95 to 0.99.

The results are not surprising. For low values of γ, the model is far away from the actual values on all scores. For a γ of 10, the mean of the equity premium is still desperately low (1.08%), but the results are better in terms of second moments. The standard deviation of the equity premium is close to 13%, compared to an actual of 18.6%, while the covariance between the risk-free rate and the equity return is almost equal to the actual (0.0027) and the covariance between the risk-free rate and the equity premium is about half the actual. The results for the mean of the risk-free rate illustrate in a dramatic way the risk-free puzzle: a γ of 10 produces a mean close to 22% (for an actual of 2%)!

These results show that even allowing the dividends to be different from aggregate consumption in an exchange economy does not go any length in resolving the equity premium puzzle. This was one doubt expressed by Weil (1989) in analyzing the robustness of his results.

IVb. Simulated Measures of Serial Correlation in Returns

Strong negative serial correlation in long-horizon returns can be induced by a slow mean reverting temporary component in asset prices, but as mentioned in Section I the economic rationale for this mean reversion in prices is not unique: both inefficient and efficient market explanations can be made consistent with this observation. One way to distinguish between the two theories is to propose an equilibrium model that imposes restrictions on the evolution of expected returns. In this section, we show that the model presented in Section II produces some negative autocorrelation in real returns, but fails to do so for excess returns, another piece of evidence in line with our results on the unconditional moments.

Tables 4 and 5 present the results of our Monte-Carlo experiment for real returns and excess returns respectively. Given a randomly drawn vector of N(0,1) errors $\epsilon_{t+1}{}^{d}$ and a randomly drawn vector of $S_{0,t}$, $S_{1,t}$, and $S_{2,t}$ according to the transition probabilities estimated in Section III, we generate series of equity and excess returns according to formulas (20) and (21) with the estimates obtained in Section III for the α and ω parameters. We replicate the procedure a 1,000 times and compute each time the variance ratios at lags 2 to 10 and the 1 to 10 multiperiod returns regression coefficients as defined at the top of tables 4 and 5. We therefore obtain the respective distributions of variance ratios and regression coefficients at various lags. For the length of the series, we choose 116 observations (the number of observations for the actual returns) to generate the small sample distributions and 1,160 observations for the large sample ones to account for small sample bias. We report the medians of the distributions for the variance ratios and the regression coefficients both for small samples (SS) and large samples (LS) as well as, in the case of the small sample distribution, the percentage (%) of the distribution below the actuals. These percentages are to be interpreted as p-values for the hypothesis that the actuals are produced by the model. The closer they are to the 50% line, the more support for the hypothesis. As in Section IVa, we have selected the values 1.5, 5 and 10 for γ and 0.97 for β.

For the real returns (Table 4), the best results among these 3 values of γ are obtained for $\gamma=5$. The actuals all lie close to the 40% line of the small sample variance ratio distributions, while there is still evidence of some negative autocorrelation in the large sample medians.[12] This evidence is confirmed by the regression coefficients, where for $\gamma=5$ the only actual regression coefficient below the 20% line is at lag 10.

However, for the excess returns (Table 5), even if the actuals of the variance ratios are all above the 30% line of the small sample distributions for $\gamma=1.5$, all evidence of negative autocorrelation disappears in large samples for all values of γ.

---

[12] It can be argued that the negative serial correlation exhibited by the actual returns is purely due to small sample bias and should disappear in large samples. Our purpose in this simulation exercise is to assess first how likely it is for the actuals to have been produced by such a model (this is the % of the distribution below the actuals) and second if the model can produce negative correlation in large samples, which is essential for the equilibrium explanation of mean reversion in asset prices.

Moreover, the actual values of the regression coefficients all lie close to the 20% or 80% lines of the small sample distributions for all values of γ.

We therefore conclude that the model can produce some negative autocorrelation in real returns, although not quite as much as in the actual data, but fails to produce any in excess returns, which seems to be another manifestation of the same risk-free rate problem experienced with the unconditional moments.

IVc. Simulated Statistics for the Regressions of Multiperiod Returns on Dividend Yields

Given the fact that univariate tests on returns have little power, Fama and French (1988b) proposed to regress one- to four-year returns on the dividend-price ratio. They found statistical evidence of a larger predictable component for long-horizon returns. The actuals in Table 6 confirm their results for the period 1871-1985[13] and show that the regression coefficients increase slightly less than in proportion with the horizon for both the real returns and the excess returns. As Fama and French (1988b) explain, this means, since multiperiod returns are cumulative sums of one-period returns, that the dividend-price ratio does not predict as much variation in the distant one-period expected returns, an indication of slow mean reversion in short term expected returns. Because of this slow mean reversion, short term

[13] They study the period 1926-1985 and various sub-periods.

expected returns are persistent, and the variance of multiperiod expected returns grows more than in proportion with the return horizon. However, the variance of the regression residuals grows much less with the return horizon, as shown in Table 1 (a standard deviation of 0.27 for two-year returns compared to 0.34 for four-year returns). The latter fact indicates that the residuals from the one-year regressions must on average be negatively autocorrelated. This explains why the forecasting power increases with the horizon.

We proceed as in Section IVb to generate the return series but we note that if we assume that the agent knows the state at time t, the model gives us only three values for the dividend-price ratio. To obtain a continuous variable for the price-dividend ratio, we therefore assume that the state is not directly observable[14] and allow the agent to make an optimal inference about the probabilities of states 0, 1, and 2 at time t given his information up to time t and of course the values of the parameters of the model. These inferred probabilities are precisely what we called in Section III the filter probabilities. We therefore obtain the price-dividend ratio by weighing the three values of $\rho(S_t)$ by these filter probabilities.

As shown in the upper part of Table 6, the model produces the same patterns as in the data for values of γ between 7 and 10 in the case of real returns, except that the values of the regression

---

[14] This assumption changes of course the model we used to generate the measures of serial correlation. The implications for serial correlation in returns of this new assumption regarding the information available to the agent should be investigated.

coefficient medians are too low: all the coefficients are positive, growing with the return horizon, as does the forecasting power. The actuals stand at about 40% of the distribution of the t and R^2 statistics. However, the model fails totally when it comes to excess returns. The coefficients are negative and totally insignificant, which translates into R^2 close to zero.

To summarize our assessment of the model, we can say that the model fails in all dimensions involving the risk-free rate. However, because we relaxed the restriction about the equality of consumption and dividends, we were able to reproduce, although not fully and with γ values between 5 and 10, the features associated with the real equity returns. In view of these results and of the numerous previous rejections of this model in the literature, we are inclined to conclude, as Mehra and Prescott (1985) and Weil (1989), that the assumptions of complete markets and of a representative agent seem to be the most damaging for replicating the stylized facts we started with.

The solution to the equity premium puzzle mentioned by Mehra and Prescott was to introduce heterogeneity between consumers. Mankiw (1986) and Weil (1990) propose models showing that if the agents bear some idiosyncratic undiversifiable risk, their consumption will be more risky, justifying a high equity premium. Also, in this case, the agents will value more an asset that pays a unit of consumption with certainty, which implies a lower return for the safe asset. But how risky (variable) should the consumption of an individual agent holding the stock and the risk-free asset be to justify the observed equity and safe asset returns? We will give a numerical answer to this question in the next section within the framework of our parametric model.

V. Inferring the Parameters of Individual Consumption from the Actual Returns

Our goal in this section is to assess the empirical plausibility of the model in terms of mean and variance of individual consumption growth inferred from asset market data. This is similar in spirit to Hansen and Jagannathan (1990), since given our choice of power utility function, the means and variances of the consumption growth can be translated into intertemporal marginal rates of substitution (IMRS) by raising them to the power γ and discounting them with β. Their goal, however, is more general and consists in deriving non-parametrically admissible regions for the means and variances of IMRS in order to test the validity of many classes of models. Of course, by not imposing a parametric structure, they only derive bounds for the IMRS and we trade-off this generality for getting estimates of the means and variances of the consumption process.

To infer the consumption parameters from actual returns, we start with the theoretical formulas given by the model for the returns. Taking the natural logarithm of the return formulas (20) and (21) for the risk-free asset and the equity respectively (assuming three states) and using the dividend process assumed in (8), we obtain the following system:

$$
\begin{aligned}
\ln R_t^e-(\ln D_{t+1}-\ln D_t) &= \ln[\rho(S_{t+1})+1]-\ln[\rho(S_t)]+v_t^e \\
\ln R_t^f &= -\ln \phi(S_t) \qquad\qquad\qquad +v_t^f \\
\ln D_t-\ln D_{t-1} &= \alpha_0^d+\alpha_1^d S_{1,t}+\alpha_2^d S_{2,t} \qquad +(\omega_0^d+\omega_1^d S_{1,t}+\omega_2^d S_{2,t})\epsilon_t^d
\end{aligned}
\tag{22}
$$

where normal mean-zero measurement error terms v_t^e and v_t^f, with variances σ^2_d and σ^2_f respectively, have been added to the equity return and risk-free return equations respectively. This system can be estimated with a trivariate version of the Markov switching model algorithm, as follows:

$$
\begin{aligned}
r_t^e &= a+bS_{1,t}+cS_{2,t}+\ln(\exp(a)+1)P(S_t,0)+\ln(\exp(a+b)+1)P(S_t,1) \\
&\quad +\ln(\exp(a+c)+1)P(S_t,2)+v_t^e \\
r_t^f &= d+eS_{1,t}+fS_{2,t}+v_t^f \\
d_t-d_{t-1} &= \alpha_0^d+\alpha_1^d S_{1,t}+\alpha_2^d S_{2,t}+(\omega_0^d+\omega_1^d S_{1,t}+\omega_2^d S_{2,t})\epsilon_t^d
\end{aligned}
\tag{23}
$$

We do not impose any restrictions on the covariance matrix of the errors, which leaves us with 23 parameters to estimate. The estimation results are reported in Table 7.

Note first that the estimates of the transition probability parameters are close to what we obtained with the estimation of the joint consumption-dividend process, except that state 2 (labelled state 1 before) is more persistent (0.949 instead of 0.854). The other transition probabilities are almost identical. This is a good feature since we are estimating the transition probability matrix of the same Markov variable. We also observe that the means of the dividend process are close to zero as in our estimation of the joint consumption-dividend process, but that the standard deviation parameter in state 1 (labelled state 0 before) is much higher. This is probably due to the fact that we do not allow the variances in the other two equations to change with the state as we did before when estimating the joint process with consumption.

The estimate of -2.669 for the coefficient a translates into a price-dividend ratio of 14.43, but the estimates of b and c are not significantly different from zero, so that the dividend-price ratio does not vary significantly between states.

For the risk-free asset, the prices in state 0, 1 and 2 are respectively 0.988, 1.14 and 0.644. In state 1, the agent is ready to pay more than the asset is worth and incur a negative real return.

Finally, as it should be, the correlation coefficient parameters are all small and not significantly different from zero except ρ^{df}, the correlation coefficient between ϵ_t^d and v_t^f.

Given these estimates, we can solve the set of six non-linear equations defined by (17) and (18) to recover the consumption parameters, although we need to assume some values for γ, β and ρ^{cd}.

Table 8 shows the implied parameters for the consumption process for various γ values, when the utility discount rate is

0.97 and the correlation between consumption and dividend innovations is 0.85.[15] We see that all the conditional mean and variance parameters, as well as the unconditional mean and standard deviation of the consumption growth, decrease in magnitude when γ increases. This result conforms to intuition since a higher standard deviation and a larger difference between state means are needed with a lower γ to rationalize the characteristics of equity and riskless asset returns, and especially a large equity premium. The more risk averse a consumer is, the less variability is required in her consumption (which covaries with the equity return) to explain her preference for the riskless asset over the equity.[16]

The reason for the decreasing relation between the average consumption growth and γ is also clear. The lower the elasticity of intertemporal substitution, the lower the consumption growth necessary to generate the marginal rate of substitution which rationalizes a given riskless return. Then, since in the particular utility function we use γ represents both the coefficient of relative risk aversion and the inverse of the elasticity of intertemporal substitution, we obtain a positive correlation between the mean and the standard deviation of consumption growth in the results of Table 8. Note that when γ is equal to 10, the mean of the consumption growth is of the same order of magnitude as

---

[15] This value has been chosen since we expect the correlation with the innovations in dividend growth to be higher for consumption growth of an individual stockholder than for the aggregate consumption growth, for which we find a value of 0.506.

[16] The same intuition can explain why the correlation between consumption and dividend innovations decreases, ceteris paribus, when γ increases.

the actual mean of aggregate consumption growth, but the standard deviation is much higher. This result illustrates why the model could not succeed in solving the equity premium or the risk-free puzzles: much more variability than present in the aggregate consumption series is needed to match the actual equity premium and the risk-free rate.

If we abandon the representative agent paradigm, we should ask ourselves to which agent or class of agents belong the consumption parameters we estimated. If markets are incomplete, individual consumption may differ in important ways from aggregate consumption. A large part of the population may be liquidity-constrained, which could explain why only a small fraction participates in the stock market. However, the absence of complete markets does not invalidate the equation which relates the intertemporal rate of substitution and asset returns at an individual level if the individual is unconstrained. Furthermore, as documented by Mankiw and Zeldes (1989), the consumption of stockholders behaves differently from the consumption of non-stockholders. We therefore feel tempted to interpret our derived consumption process as the typical individual consumption of stockholders. Is the inferred process reasonable given the postulated model?

Unfortunately, there is not much information about the consumption of stockholders and a thorough micro-level study would be required to answer the question properly. So, what follows is an unpretentious numerical exercise. If the relationship found in

Mankiw and Zeldes between the difference in the consumption growth of stockholders and non-stockholders and excess returns holds for the much longer period of our sample (a leap of faith)[17], the average food consumption growth of stockholders will exceed that of non-stockholders by something in the order of 0.5%.[18] If we assume that the same relationship holds for total consumption, the mean consumption growth implied by γ=10 in Table 8 will not be a bad estimate of the average consumption growth of stockholders. As far as the variance is concerned, one would expect it to be much higher for an individual stockholder than the variance found in aggregate consumption[19], but our inferred value of 0.31 for the standard deviation seems excessive. For the standard deviation of individual income growth, MaCurdy (1982) reports a value of 0.25. Clearly, the standard deviation of consumption for unconstrained consumers should be lower. Of course, we could decrease the standard deviation by increasing γ, but by doing so we would also lower the mean. For γ=15, the standard deviation is still high and the mean already too low[20]. This limitation in finding utility function parameters which match both moments is probably due to the double

---

[17] Mankiw and Zeldes estimate the following regression:

$$GC^{stockholders} - GC^{non\text{-}stockholders} = \alpha+\beta\,(r^m-r^f)$$

and find a β ranging from 0.054 to 0.09, the higher betas corresponding to the stockholders with the largest holdings. We use the 75%-25% percentages of non-stockholders and of stockholders reported in their study to obtain an order of magnitude for the difference in the consumption growth between the two groups.

[18] However, for the relatively short period examined by Mankiw and Zeldes (1989) the consumption growth of non-stockholders exceed that of stockholders and the mean excess return is negative.

[19] Even the variability of the aggregate stockholder consumption growth is likely to be higher than that of the aggregate non-stockholder, as the sample of food consumption in Mankiw and Zeldes (1989) suggests.

[20] When we change the value of ρ_{cd}, the consumption growth mean and standard deviation covary positively, and the previous problem reappears.

role played by γ. It is possible that in following Weil (1989) and using a Kreps-Porteus non-expected utility function, one can find reasonable values for the consumption growth mean and variance using non-extreme parameter values for the utility function.

It would also be desirable to find an individual consumption process that, when aggregated, gives figures for aggregate consumption close to the actual data. To this end, a Markov specification with different means and variances in the various states does not seem appropriate. A great part of the variability of the consumption growth process derived in Table 8 is due to the different state means. Since the states are implicitly assumed to be states for the whole economy (the same states which govern dividends and returns), this kind of variability in means cannot be reduced by aggregation. The same is not true for the variances: the ω parameters multiply the error term, where the idiosyncratic part could be factored away by aggregation.

VI. Conclusion

This paper started with some stylized facts regarding real and excess returns. Our conclusion is that an equilibrium asset pricing model based on a Lucas exchange economy - the particular version presented or an improved version based on existing refinements - can replicate roughly the features associated with real returns, but is totally incapable of replicating the excess returns characteristics. The equity premium puzzle and the risk-free puzzle are precisely a version of this result, but we confirmed this evidence using other statistics such as the negative autocorrelation of returns and the forecasting power of the dividend-price ratio for multiperiod returns.

Given this evidence, we are inclined to follow Mehra and Prescott's (1985) advice in searching for an incomplete market and heterogeneous consumers explanation. In Section IV, we illustrated how an intertemporal asset pricing model could still be used to guide our search in trying to characterize some individual consumption process that will rationalize the returns observed on the market. These benchmark figures can be compared to figures resulting from future micro-level studies aimed at identifying the unconstrained participants in the stock market if, for example, one is ready to invoke liquidity constraints as a source of friction in the economy. These micro-studies could also help the model building exercise and especially the choice of a good utility function. These are important issues we intend to address in future research.

REFERENCES

Bonomo, Marco, and Garcia, René, "Mean Reversion in Equilibrium Asset Prices: Comment", Financial Research Center Memorandum No. 120, Princeton University, September 1990.

Campbell, John Y., "Intertemporal Asset Pricing Without Consumption," mimeo., Princeton University, June 1990.

Cechetti, Stephen G., Lam, Pok-sang and Mark, Nelson C., (1990a) "Mean Reversion in Equilibrium Asset Prices," *American Economic Review,* June 1990, *80,* 398-418.

Cechetti, Stephen G., Lam, Pok-sang and Mark, Nelson C., (1990b) "The Equity Premium and the Risk Free Rate: Matching the Moments," mimeo., Ohio State University, August 1990.

Constantinides, George M., "Habit Formation: A Resolution of the Equity Premium Puzzle," mimeo., University of Chicago, October 1988.

Davies, R.B., "Hypothesis Testing when a Nuisance Parameter is Present Only Under the Alternative," *Biometrika,* 1987, *74,* 33-43.

Epstein, Lawrence and Zin, Stanley, "Substitution, Risk Aversion, and Temporal Behavior of Consumption and Asset Returns II: An Empirical Analysis," manuscript, University of Toronto, 1989.

Fama, Eugene F. and French, Kenneth, R., (1988a) "Permanent and Temporary Components of Stock Prices," *Journal of Political Economy,* April 1988, *96,* 246-73.

Fama, Eugene F. and French, Kenneth, R., (1988b) "Dividend Yields and Expected Stock Returns," *Journal of Financial Economics,* March 1988, 3-25.

Garcia, René and Perron, Pierre, "An Analysis of the Real Interest Rate under Regime Shifts," Econometric Research Program Research Memorandum No. 353, Princeton University, August 1990.

Hamilton, James D., "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica,* March 1989 ,*57, 357-384.*

Hansen, Lars P. and Jagannathan, Ravi, "Implications of Security Market Data for Models of Dynamic Economies," N.B.E.R. Technical

Working Paper No. 89, May 1990.

Kandel, Shmuel and Stambaugh, Robert F., "Modeling Expected Stock Returns for Long and Short Horizons," mimeo., University of Pennsylvania, December 1988.

Kocherlakota, Narayana R., "In Defence of the Time and State Separable Utility-Based Asset Pricing Model," mimeo., Northwestern University, October 1988.

Kim, Myung Jig, Nelson, Charles R. and Startz, Richard, "Mean Reversion in Stock Prices? A Reappraisal of the Empirical Evidence," mimeo., National Bureau of Economic Research, December 1988.

Lucas, Robert E. Jr., "Asset Prices in an Exchange Economy," *Econometrica,* November 1978, 1429-1445.

Mankiw, N. Gregory, "The Equity Premium and the Concentration of Aggregate Shocks," *Journal of Financial Economics,* 1986, *17,* 211-219.

Mankiw, N. Gregory and Zeldes, Stephen P., "The Consumption of Stockholders and Non-Stockholders," mimeo., Harvard University and University of Pennsylvania, May 1989.

Mehra, Ranjnish and Prescott, Edward C., "The Equity Premium," *Journal of Monetary Economics,* 1985, 145-161.

Poterba, James M. and Summers, Lawrence H., "Mean Reversion in Stock Prices," *Journal of Financial Economics,* March 1988, 27-59.

Rouwenhorst, Geert K., "Asset Returns and Business Cycles: a General Equilibrium Approach," mimeo., University of Rochester, October 1989.

Tauchen, George, "The Statistical Properties of Generalized Method-of-Moments Estimators of Structural Parameters Obtained From Financial Market Data," *Journal of Business & Economic Statistics,* October 1986, 397-425.

Turner, Christopher M., Startz, Richard and Nelson, Charles T., "A Markov Model of Heteroskedasticity, Risk, and Learning in the Stock Market," National Bureau of Economic Research, Inc., January 1989.

Weil, Philippe, "Equilibrium Asset Prices with Undiversifiable Labor Income Risk," mimeo., Harvard University, June 1990.

Weil, Philippe, "The Equity Premium Puzzle and The Risk-Free Rate Puzzle," *Journal of Monetary Economics*, December 1989, *24*, 401-421.

Appendix A

Data Sources

- Nominal dividends and stock prices: Campbell and Shiller (1987) data set.
- Price index: CPI 1871-1926 : Wilson and Jones (1987)
  1930-1985 : Ibbotson and Sinquefield (1988)
- Nominal Interest Rate: constructed from four different sources as in Cechetti, Lam and Mark(1990a, Data Appendix)
- Consumption: 1889-1928 : Kendrick Consumption series (Balke and Gordon, 1986)
  1929-1985 : NIPD Accounts
- Population: 1869-1938 : Historical Statistics of the United States (Series A7 for 1869-1928, Series A6 1929-1938)
  1938-1985 : Economic Report of the President (1989), Table B-31.

References:

Balke, Nathan S. and Gordon, Robert J., "Appendix B. Historical Data," in R. J. Gordon, ed. The American Business Cycle, Chicago: University of Chicago Press for NBER, 1986.

Campbell, John Y. and Shiller, Robert J., "Cointegration and Tests of Present Value Models", Journal of Political Economy, October 1987, 95, 1062-88.

Ibbotson, Roger G. and Sinquefield, Rex A., Stocks, Bonds, Bills and Inflation 1988 Yearbook, Chicago: Ibbotson Associates, 1988.

Wilson, Jack W. and Jones, Charles, "A Comparison of Annual Common Stock Returns: 1871-1925 with 1926-1985", Journal of Business, April 1987, 60, 239-58.

Table 1

A- First and Second Moments of Returns		
	Mean (%)	Standard Deviation (%)
Treasury Bill Rate (R_f)	2.02	6.43
Equity Return (R_q)	8.13	18.93
Equity Premium (R_p)	6.11	18.57
Covariance (R_f , R_q)	0.0027	
Covariance (R_f , R_p)	-0.0014	

B- Measures of Serial Correlation				
	Variance Ratios $VR = Var(R_{t,t+k})/kVar(R_t)$		Regression Coefficients $R_{t,t+k} = a + bR_{t-k,t} + \epsilon_t$	
k	Real Returns	Excess Returns	Real Returns	Excess Returns
1	1.0000	1.0000	0.0188	0.0871
2	1.0256	1.0957	-0.1539	-0.1142
3	0.9044	1.0012	-0.1432	-0.1668
4	0.8785	0.9849	-0.1376	-0.2128
5	0.8572	0.9361	-0.1601	-0.1680
6	0.7941	0.8509	-0.0957	0.0082
7	0.7625	0.7906	-0.1037	0.1133
8	0.7781	0.7795	-0.1982	0.1025
9	0.7646	0.7710	-0.2792	0.0841
10	0.7395	0.7828	-0.3829	0.0606

C- Return Forecastability based on Dividend Yield Regression: $R_{t,t+k} = \alpha + \beta(D_t/P_t) + u_{t,t+k}$								
	Real Returns				Excess Returns			
k	Coef.	t	R^2	σ	Coef.	t	R^2	σ
1	2.87	1.91	.031	0.19	3.46	2.35	.047	0.18
2	5.31	2.48	.053	0.27	6.45	3.01	.076	0.27
3	7.05	2.89	.071	0.30	8.29	3.36	.092	0.31
4	10.10	3.72	.113	0.34	11.16	3.99	.127	0.35
5	13.59	4.68	.169	0.36	14.51	4.92	.183	0.36

Table 2

Estimation Results for the Consumption-Dividend Joint Markov Models (1889-1985)				
	Two-State Model		Three-State Model	
	Coefficient Estimate	Standard Error	Coefficient Estimate	Standard Error
α^c_0	0.0238	0.0039	0.0355	0.0075
α^c_1	-0.0091	0.0085	-0.0228	0.0112
α^c_2	---	---	-0.0162	0.0083
ω^c_0	0.0237	0.0029	0.0330	0.0054
ω^c_1	0.0242	0.0057	0.0154	0.0075
ω^c_2	---	---	-0.0167	0.0057
α^d_0	-0.0028	0.0067	-0.0040	0.0069
α^d_1	0.0019	0.0267	0.0053	0.0278
α^d_2	---	---	-0.0011	0.0115
ω^d_0	0.0415	0.0047	0.0204	0.0059
ω^d_1	0.1337	0.0195	0.1629	0.0196
ω^d_2	---	---	0.0267	0.0085
p_{01}	---	---	0.3395	0.1290
p_{02}	---	---	0.00	0.00
p_{11}	0.905	0.072	0.8539	0.0700
p_{12}	---	---	0.0202	0.0201
p_{21}	---	---	0.00	0.00
p_{22}	0.895	0.057	0.9568	0.0359
ρ_{cd}	0.4119	0.087	0.506	0.077
L	455.17		468.47	

Davies Test for Two States vs Three States: 0.00169
(Upper bound of p-value for a null hypothesis of two states)

Table 3

Population Moments for the risk free return and the equity premium implied by the model				
	Actual	$\gamma=1.5$ $\beta=0.97$	$\gamma=5$ $\beta=0.97$	$\gamma=10$ $\beta=0.97$
Mean Equity Return	8.13%	6.29%	13.48%	23.04%
Standard Deviation Equity Return	18.93%	8.89%	9.80%	13.66%
Mean Risk free Asset	2.02%	6.14%	12.98%	21.96%
Standard Deviation Risk free Asset	6.43%	0.82%	1.83%	5.44%
Mean Risk Premium	6.11%	0.15%	0.50%	1.08%
Standard Deviation Risk Premium	18.57%	8.8%	9.7%	12.98%
Covariance Risk-free Equity Return	0.0027	0.00002	0.0002	0.0024
Covariance Risk-free Risk Premium	-0.0014	-0.000005	-0.00009	-0.0006

Table 4

Simulated Measures of Serial Correlation

Real Returns

Median of Distribution of Variance Ratios of Real Returns for Model calibrated to the Joint Consumption-Dividend Three-State Markov Model

k	Actual	γ=1.5 SS	γ=1.5 %	γ=1.5 LS	γ=5 SS	γ=5 %	γ=5 LS	γ=10 SS	γ=10 %	γ=10 LS
2	1.0256	0.9894	0.61	0.9966	0.9687	0.69	0.9758	0.9667	0.71	0.9736
3	0.9044	0.9744	0.34	0.9925	0.9435	0.39	0.9595	0.9476	0.40	0.9558
4	0.8785	0.9576	0.33	0.9916	0.9199	0.40	0.9478	0.9313	0.39	0.9492
5	0.8572	0.9433	0.34	0.9899	0.9009	0.42	0.9387	0.9212	0.39	0.9471
6	0.7941	0.9292	0.29	0.9868	0.8833	0.35	0.9312	0.9113	0.32	0.9491
7	0.7625	0.9142	0.28	0.9836	0.8647	0.34	0.9286	0.9054	0.29	0.9553
8	0.7781	0.8946	0.35	0.9814	0.8435	0.39	0.9242	0.8975	0.34	0.9604
9	0.7646	0.8784	0.35	0.9785	0.8312	0.40	0.9202	0.8822	0.34	0.9682
10	0.7395	0.8665	0.34	0.9745	0.8173	0.39	0.9154	0.8735	0.32	0.9743

Median of Distribution of Regression Coefficients of Real Returns for Model calibrated to the Joint Consumption-Dividend Three-State Markov Model

k	Actual	γ=1.5 SS	γ=1.5 %	γ=1.5 LS	γ=5 SS	γ=5 %	γ=5 LS	γ=10 SS	γ=10 %	γ=10 LS
1	.0188	-.0123	.59	-.0031	-.0326	.68	-.0241	-.0330	.69	-.0267
2	-.1539	-.0316	.16	-.0040	-.0497	.21	-.0310	-.0351	.18	-.0210
3	-.1432	-.0458	.26	-.0087	-.0604	.31	-.0306	-.0292	.23	-.0324
4	-.1376	-.0639	.34	-.0095	-.0682	.36	-.0277	-.0300	.28	.0153
5	-.1601	-.0767	.31	-.0102	-.0862	.33	-.0243	-.0304	.24	.0303
6	-.0957	-.0925	.49	-.0101	-.1040	.52	-.0240	-.0242	.37	.0423
7	-.1037	-.1038	.50	-.0121	-.1207	.54	-.0232	-.0261	.38	.0526
8	-.1982	-.1124	.37	-.0117	-.1364	.40	-.0236	-.0346	.27	.0605
9	-.2792	-.1243	.26	-.0114	-.1465	.30	-.0251	-.0528	.19	.0693
10	-.3829	-.1391	.17	-.0130	-.1547	.18	-.0265	-.0713	.11	.0749

Table 5

Simulated Measures of Serial Correlation

Excess Returns

Median of Distribution of Variance Ratios of Excess Returns for Model calibrated to the Joint Consumption-Dividend Three-State Markov Model

k	Actual	γ=1.5 SS	γ=1.5 %	γ=1.5 LS	γ=5 SS	γ=5 %	γ=5 LS	γ=10 SS	γ=10 %	γ=10 LS
2	1.0957	0.9975	0.81	1.0019	1.0028	0.77	1.0114	1.0144	0.77	1.0270
3	1.0012	0.9822	0.55	1.0027	0.9955	0.51	1.0141	1.0229	0.44	1.0459
4	0.9849	0.9667	0.53	1.0027	0.9826	0.50	1.0188	1.0319	0.41	1.0636
5	0.9361	0.9473	0.48	1.0037	0.9653	0.44	1.0220	1.0290	0.34	1.0745
6	0.8509	0.9396	0.38	1.0005	0.9531	0.33	1.0242	1.0217	0.24	1.0847
7	0.7906	0.9268	0.32	1.0015	0.9417	0.28	1.0282	1.0229	0.19	1.0934
8	0.7795	0.9149	0.33	0.9996	0.9354	0.29	1.0319	1.0129	0.21	1.0991
9	0.7710	0.9053	0.35	0.9973	0.9181	0.31	1.0344	1.0083	0.22	1.1054
10	0.7828	0.8903	0.39	0.9979	0.9109	0.36	1.0344	0.9966	0.26	1.1104

Median of Distribution of Regression Coefficients of Excess Returns for Model calibrated to the Joint Consumption-Dividend Three-State Markov Model

k	Actual	γ=1.5 SS	γ=1.5 %	γ=1.5 LS	γ=5 SS	γ=5 %	γ=5 LS	γ=10 SS	γ=10 %	γ=10 LS
1	.0871	-.0034	.80	.0015	.0000	.75	.0114	.0127	.75	.0270
2	-.1142	-.0224	.25	.0004	-.0129	.23	.0121	.0144	.15	.0358
3	-.1668	-.0341	.20	.0000	-.0376	.18	.0105	.0040	.12	.0363
4	-.2128	-.0513	.17	.0000	-.0434	.17	.0093	-.0080	.11	.0361
5	-.1680	-.0582	.31	-.0033	-.0577	.30	.0105	-.0273	.20	.0357
6	.0082	-.0767	.66	-.0062	-.0784	.65	.0091	-.0357	.58	.0363
7	.1133	-.0886	.81	-.0085	-.0844	.80	.0103	-.0503	.76	.0359
8	.1025	-.1139	.81	-.0093	-.1010	.79	.0120	-.0627	.74	.0348
9	.0841	-.1263	.79	-.0112	-.1061	.77	.0095	-.0702	.74	.0356
10	.0606	-.1440	.78	-.0150	-.1158	.76	.0091	-.0847	.72	.0320

Table 6

Simulated statistics on the Forecastability of Future Returns by Current Dividend Yields
Regression: $R_{t,t+k}=\alpha+\beta(D_t/P_t)+u_{t,t+k}$

Median of Distribution of various regression statistics
REAL RETURNS

	Actual			γ=1.5 β=0.97			γ=7 β=0.97			γ=10 β=0.97		
k	Coef.	t	R^2	Coef.	t	R^2	Coef.	t	R^2	Coef.	t	R^2
1	2.87	1.91	0.03	0.85	0.21	.004	1.40	1.71	0.03	1.39	2.83	0.07
2	5.31	2.48	0.05	1.69	0.30	.006	2.53	2.18	0.04	2.61	3.80	0.11
3	7.05	2.89	0.07	2.05	0.30	.009	3.52	2.40	0.05	3.67	4.52	0.16
4	10.10	3.72	0.11	2.47	0.33	.012	4.33	2.63	0.06	4.62	4.96	0.18
5	13.59	4.68	0.17	3.19	0.34	.015	5.05	2.76	0.07	5.40	5.32	0.21
	Actual			Percentage of distribution below the actual values								
1	2.87	1.91	0.03	0.72	0.95	0.94	0.91	0.60	0.60	0.94	0.16	0.16
2	5.31	2.48	0.05	0.71	0.95	0.94	0.92	0.60	0.60	0.96	0.15	0.15
3	7.05	2.89	0.07	0.69	0.94	0.92	0.92	0.61	0.61	0.96	0.18	0.18
4	10.10	3.72	0.11	0.72	0.96	0.94	0.96	0.70	0.70	0.98	0.26	0.26
5	13.59	4.68	0.17	0.74	0.97	0.96	0.98	0.79	0.79	0.99	0.39	0.39

Median of Distribution of various regression statistics
EXCESS RETURNS

	Actual			γ=1.5 β=0.97			γ=7 β=0.97			γ=10 β=0.97		
k	Coef.	t	R^2	Coef.	t	R^2	Coef.	t	R^2	Coef.	t	R^2
1	3.46	2.35	0.05	-.12	-.03	.003	-.36	-.42	.005	-.21	-.44	.005
2	6.45	3.01	0.08	-.34	-.06	.006	-.61	-.51	.009	-.39	-.55	.008
3	8.29	3.36	0.09	-.22	-.03	.009	-.88	-.58	.011	-.51	-.58	.011
4	11.16	3.99	0.13	-.21	-.03	.011	-.95	-.60	.014	-.58	-.60	.014
5	14.51	4.92	0.18	-.55	-.05	.015	-1.01	-.57	.017	-.66	-.57	.018
	Actual			Percentage of distribution below the actual values								
1	3.46	2.35	0.05	0.85	0.98	0.97	.997	0.99	0.97	.998	0.99	0.96
2	6.45	3.01	0.08	0.84	0.98	0.96	.996	0.99	0.95	--	0.99	0.96
3	8.29	3.36	0.09	0.79	0.98	0.96	.996	0.99	0.94	--	0.99	0.95
4	11.16	3.99	0.13	0.79	0.98	0.96	.996	0.99	0.96	--	0.99	0.95
5	14.51	4.92	0.18	0.80	0.99	0.98	.997	0.99	0.97	--	0.99	0.97

Table 7

Estimation of the Trivariate Markov System with Actual Returns and Dividend Growth

Parameter	Coefficient Estimate	Standard Error
a	-2.669	(0.221)
b	-0.058	(0.169)
c	-0.005	(0.210)
d	0.012	(0.007)
e	-0.146	(0.016)
f	0.032	(0.009)
α_0^d	-0.005	(0.008)
α_1^d	-0.035	(0.07)
α_2^d	0.007	(0.02)
ω_0^d	-0.048	(0.008)
ω_1^d	0.245	(0.046)
ω_2^d	0.202	(0.017)
σ^e	0.133	(0.009)
σ^f	0.043	(0.003)
ρ^{ef}	-0.009	(0.095)
ρ^{de}	-0.118	(0.09)
ρ^{df}	-0.235	(0.09)
p_{01}	0.056	(0.00)
p_{02}	0.000	(0.00)
p_{11}	0.546	(0.161)
p_{12}	0.353	(0.159)
p_{21}	0.032	(0.023)
p_{22}	0.949	(0.027)
L	641.83	

Note: To identify the estimated parameters, refer to the system of equations (23).

Table 8

Implied Parameters for the
Individual Consumption Process

γ	2	5	10	15
α_0	0.2149	0.0860	0.0430	0.0287
α_1	0.2506	0.1003	0.0501	0.0334
α_2	-0.1911	-0.0763	-0.0382	-0.0255
ω_0	-0.4630	-0.1852	-0.0926	-0.0617
ω_1	1.246	0.4985	0.2492	0.1662
ω_2	0.5667	0.2267	0.1133	0.0756
mean[1]	0.1922	0.05206	0.0230	0.0147
s.d.[2]	1.109	0.4859	0.3121	0.2473

Notes: .For the results above we set β=0.97 and ρ_{cd}=0.85
1.mean of the consumption growth
2.standard deviation of the consumption growth

Figure 1

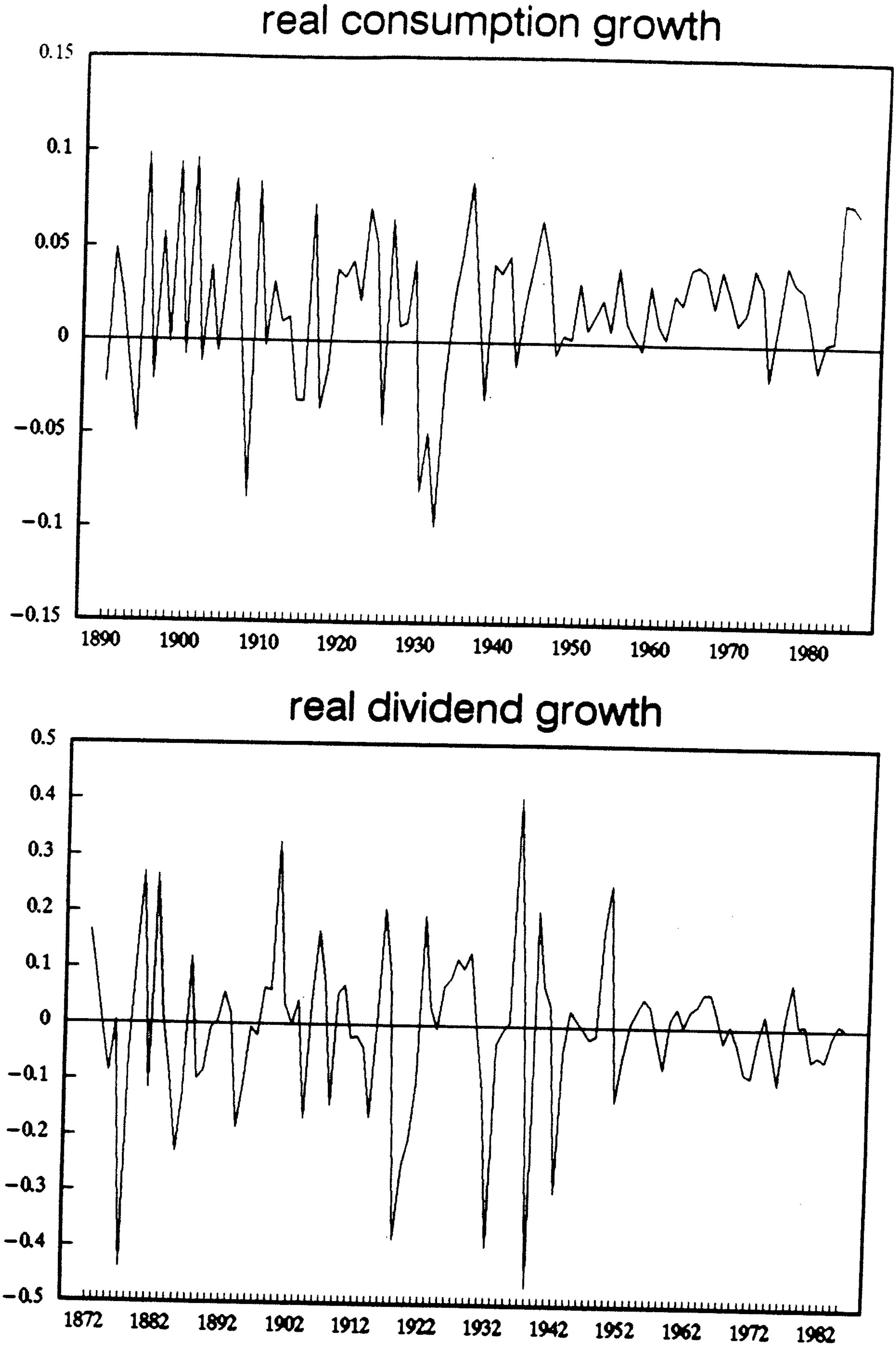
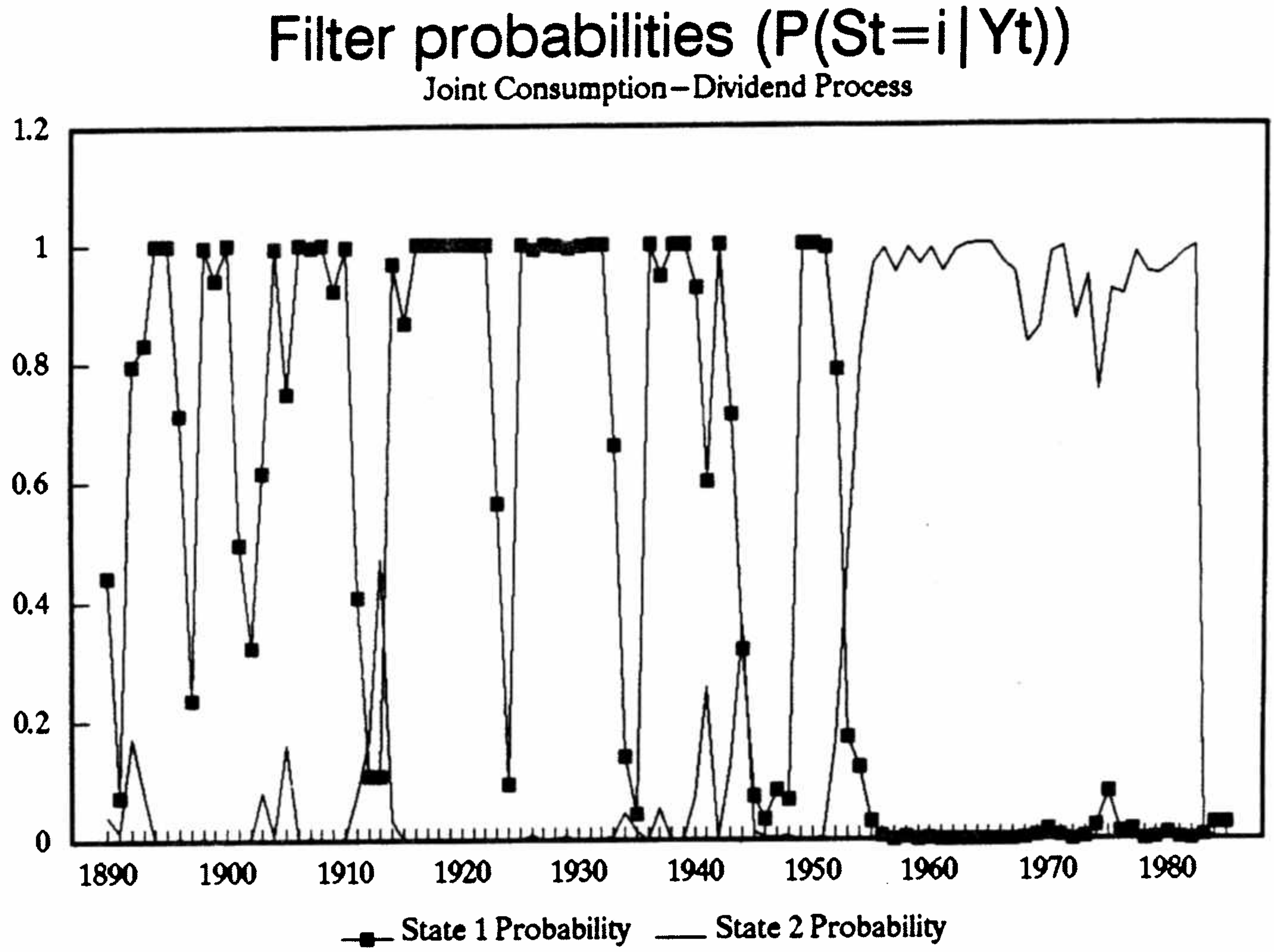

real consumption growth
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
1890
1900
1910
1920
1930
1940
1950
1960
1970
1980
real dividend growth
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
1872
1882
1892
1902
1912
1922
1932
1942
1952
1962
1972
1982

Figure 2


Filter probabilities (P(St=i|Yt))
Joint Consumption–Dividend Process
1.2
1
0.8
0.6
0.4
0.2
0
1890
1900
1910
1920
1930
1940
1950
1960
1970
1980
State 1 Probability
State 2 Probability

Textos para Discussão

266. Franco, G.H.B., "Dolarização: Mecanismos, Mágicas e Fundamentos".

267. Garcia, M., "The Formation of Inflation Expectations in Brazil: A Study of the Fisher Effect in a Signal Extraction Framework".

268. Fritsch, W. & G.H.B. Franco, "Trade Policy Issues in Brazil in the 1990s".

269. Garcia, M., "The Formation of Inflation Expectations in Brazil: A Study of the Futures Market for the Price Level".

270. Bonomo, M.A. & Garcia, R. "Can a well fitted equilibrium asset pricing model produce mean reversion?"

271. Amadeo, E.J. "Adjustment, stabilization and investment performance: Chile, Mexico and Bolivia"

272. Amadeo, E.J. "Causes for persistent unemployment and fluctuations in monetary economics"

273. Amadeo, E.J. & Camargo, J.M. "Liberalização comercial, distribuição e emprego"

274. Amadeo, E.J. & Landau, E. " Indexação e dispersão de preços relativos: análise do caso brasileiro (1975-1991)"

276. Amadeo, E.J.; Camargo, J.M.; Marques, A.E.S. & Gomes, C. " Fiscal crisis and assymetries in educational system in Brazil"

278. Bonelli, R.; Franco, G.H.B. & Fritsch, W. "Macroeconomic instability and trade liberalization in Brazil: Lessons from 1980s to the 1990s"

279. Abreu, M.P. "Trade policies in a heavily indebted economy: Brazil, 1979-1990"

280. Abreu, M.P. "O Brasil e o GATT: 1947-1991"

281. Bonomo, M. & Garcia, R. "Indexation, staggering and disinflation"

282. Werneck, R.L.F. "Fiscal federalism and stabilization policy in Brazil"

283. Carneiro, D.D. & Werneck, R.L.F. "Public savings and private investment: Requirements for growth resumption in the Brazilian economy"