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OPTIMAL TWO-SIDED AND SUBOPTIMAL ONE-SIDED STATE-DEPENDENT PRICING RULES[1]

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1. INTRODUCTION

Some recent literature was dedicated to the study of the macroeconomic implications of individuals adopting one-sided Ss rules (e.g., Blinder (1981), Caplin (1985), Caplin and Spulber (1987), Caballero and Engel (1991,1993)).

The growing interest in one-sided Ss rules reflects in part the attention shift from time-dependent to state-dependent policies. State-dependent policies have well-known microeconomic foundations[2] and their macroeconomic implications had been little explored until a few years ago. The focus of the state-dependent literature on one-sided Ss rules can be justified on the grounds of the latter being a reasonable description of reality. This is specially true in the context of pricing policy. Other reasons for the emphasis are the tractability and the appealing results obtained from this simple rule. For the latter, observe that when the frictionless optimal level of the controllable variable is monotonic, one sided Ss rules have a very important aggregation property: if the distribution of the individual deviations of the controlled variable from the frictionless optimal level is uniform, the average deviation is always constant (see, for example Caballero and Engel (1992) for that and other properties). In the pricing context this means that money is neutral, as it was first noted by Caplin and Spulber (1987)[3].

Given the prominence of the one-sided Ss pricing policies in the literature, it is time to devote some effort to the evaluation of their plausibility. This paper intends to reduce this

[2]Sheshinski and Weiss (1977,1983), Caplin and Sheshinski (1987) and Bénabou (1988) derive one-sided Ss pricing rules as optimal policies in different settings.

[3]Caballero and Engel (1993) showed that the condition imposed on the initial distribution need not be very stringent. More than that, if the idiosyncratic shocks are nonstationary, an arbitrary initial distribution converges to a distribution that satisfies these weaker conditions for neutrality.

gap.

The issue in question concerns the conditions for the optimality of the one-sided Ss rule or, alternatively, how far the optimal policies stand from one-sided Ss rules. The optimality of the one-sided rule requires a strict hypothesis for the stochastic process of the frictionless optimal level of the controllable variable: that its level is monotonic with respect to time. When the controllable variable is the price charged by an agent, this means that the frictionless optimal level for an individual price never decreases. If the frictionless optimal level of the control variable follows a process that has a trend, but it is not monotonic with respect to time, the optimal policy is a two-sided rule. If the drift is large, when compared to the variance, it is possible that one of the bounds will be very little active. When average inflation is very high, when compared to the variance, the upper bound of the band is likely to be large. More importantly, the probability that the deviation process increases by a given amount in a certain interval of time will also be small. So, the probability that the deviation process reaches the upper barrier, in a given interval of time, tends to be very small. Thus, one may argue that, in practice, it works as if the policy were one-sided. One may also argue that, in this case, the loss incurred in adopting a simpler suboptimal one-sided Ss rule is very small, and consequently the agents are likely to adopt such rules in that context. This could be justified by near rationality or, alternatively, by the existence of a small extra cost involved in using a more complex rule.

Our objective is to assess the validity of those arguments in the light of plausible parameter values for the frictionless optimal-price process.

Our simulations of both the optimal two-sided and the suboptimal one-sided pricing policies, with parameters for the frictionless optimal price process based on real economy data, show that these policies are close to one another. Furthermore, the additional cost of adopting a suboptimal one-sided rule is small. So, the adoption of the simpler suboptimal one-sided rule is even plausible. Due to their closeness, optimal two-sided and suboptimal one-sided rules have similar macroeconomic consequences. However, it is important to notice that optimal and suboptimal one-sided rules result from different specifications for the stochastic processes of the frictionless optimal price and, for that reason they entail different macroeconomic effects. Suboptimal one-sided rules do not produce the same kind of neutrality results generated by optimal one-sided rules. Even when a suboptimal one-sided rule is close to optimal, there might be frequent small negative shocks that have contractionary effects on the output. Thus, suboptimal one-sided rules not only are realistic microeconomic rules but also produce realistic macroeconomic effects, generating substantial price rigidity asymmetry, as found in the data[4].

This work is related to Tsiddon (1993), but its purpose is different. There a suboptimal one-sided rule is calculated only because it has a closed-form solution when there is no time discounting. It is assumed that it is close enough to the optimal two-sided rule to yield a good analytical approximation to it. Here, because we allow for time discounting, the one-sided rule does not have this analytical convenience. Rather than assuming the validity of the approximation, our goal is to assess its pertinence, and our motivation for doing that is to assess how plausible one-sided suboptimal rules are.

We proceed as follows. Section 2 characterizes the solution for the optimal policy, which is an asymmetric two-sided rule, assuming that the frictionless optimal value for the control variable follows a Brownian motion with drift. It solves also for the best one-sided

[4]Caballero and Engel (1992) study the effect of the drift and the variance of aggregate shocks on the asymmetry of shock effects when individual firms adopt symmetric two-sided rules. They also estimate those effects from a panel which includes 37 countries. Their estimates confirm the existence of substantial asymmetry between the effects of positive and negative aggregate shocks.

rule. Section 3 derives the expected time until the upper (lower) bound is reached for the first time. Section 4 makes a numerical assessment of the closeness between the optimal two-sided and the best one-sided rules. It calculates the expected time until the upper bound is attained and evaluates the additional cost incurred when the suboptimal one-sided Ss rule is adopted, instead of the optimal one. It first analyses how sensitive the results are to changes in the parameter values. With fixed and previously calibrated parameter values for the menu cost and the discount rate, the comparison is made based on mean and variance parameters obtained from international data. Section 5 speculates about the possible macroeconomic implications of the results obtained. The last section concludes.

2. OPTIMAL TWO-SIDED AND SUBOPTIMAL ONE-SIDED RULES

In this section we derive the optimal pricing policy, which is an asymmetric two-sided rule and the best one-sided policy.

The following are the basic assumptions related to the individual agent's decision problem. The optimal price of the firm follows a geometric Brownian motion. So the (unconstrained) optimal value for the logarithm of the price charged by the firm, $\mathbf{p}^*$, will follow a Brownian motion, that is:

$$dp_t^* = m\,dt + s\,dw_t \tag{1}$$

where $\{w_s\}$ is a Wiener process. However, there is a lump-sum adjustment cost, **k**, which is paid every time the price is changed, and there is a quadratic flow cost for being away from the optimal price. We assume that a deviation of the (log of the) control variable **p** from the unconstrained optimal level **p*** brings an instantaneous flow cost $h(p_t - p_t^*)^2 dt$. Time is discounted at the continuous rate, ρ, which is constant through time.

This is what is called a problem of impulse control, because the adjustment cost function makes unfrequent jumps of the control variable optimal instead of continuous small adjustments. An impulse control problem, similar to this, was first solved by Harrison, Selke and Taylor (1983). Following a different approach, Tsiddon (1993) characterizes the optimal policy for the undiscounted problem. Making use of a simpler framework, Dixit (1991b) presents the solution for classes of cost functions, which include the ones used here. We follow his approach.

Formally, the cost function can be written as:

$$C(x) = \min E\left[\int_0^\infty h x_t^2 e^{-\rho t} dt + \sum_i k e^{-\rho t_i} \middle| x_0 = x\right] \tag{2}$$

where

$$x_t = p_t - p_t^*$$

That is, the problem can be stated in terms of controlling the difference between the log of the original control variable and the log of its frictionless optimal value. It is clear that if no control is exercised, **x** will follow a Brownian motion with drift η=**-m** and variance σ=**s**. The problem consists in finding the optimal value for three numbers, **a**<**c**<**b**, such that if either **a** or **b** is reached, control is exercised and **x** is reset to **c**.[5]

If **a**<**x**<**b**, no jump takes place in a small interval of time dt. Then, the cost function at time zero can be written as the flow cost at the next infinitesimal interval of time, plus the expectation of the cost function at the end of this interval:

[5] That the optimal resetting from both the upper and lower barrier is made to the same place is a feature of the lump-sum adjustment costs.

$$C(x) = hx^2dt + e^{-\rho dt}E[C(x+dx_t)\,|x_t = x] \tag{4}$$

Since **a<x<b**, **x** is following a Brownian motion at the next infinitesimal time. So, we can apply Ito's Lemma to $dC(x_t)$ and take expectations conditioned on the knowledge of x_t to arrive at an expression for the expectation term in the equation above. Substituting it into (4) and then rearranging it, we obtain the following differential equation for C:

$$\frac{1}{2}\sigma^2 C''(x) + \eta C'(x) - \rho C(x) + hx^2 = 0 \tag{5}$$

which implies that C has the following general form:

$$C(x) = Ae^{\alpha x} + Be^{\beta x} + h\frac{x^2}{\rho} + 2\eta h\frac{x}{\rho^2} + h\frac{\sigma^2}{\rho^2} + 2h\frac{\eta^2}{\rho^3} \tag{6}$$

where:

$$\begin{aligned} \alpha &= -\frac{\eta}{\sigma^2} - \sqrt{\left(\frac{\eta}{\sigma^2}\right)^2 + 2\frac{\rho}{\sigma^2}} \\ \beta &= -\frac{\eta}{\sigma^2} + \sqrt{\left(\frac{\eta}{\sigma^2}\right)^2 + 2\frac{\rho}{\sigma^2}} \end{aligned} \tag{7}$$

The first two terms in equation (6) are the solutions for the homogeneous equation. The remaining ones constitute a particular solution, namely the expected discounted cost of the uncontrolled process[6]. The cost functions of the uncontrolled process and of any process controlled by barriers follow the same differential equation (5)[7]. The control adds other restrictions that determine the values of the constants A and B in the cost function (6).

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[6]This can be found by integrating the expression of the expected present value of the cost of the uncontrolled process.

[7]This is a feature of the differential approach where the probability of reaching a barrier at the next infinitesimal time, from a point inside the band, is also infinitesimal.

A. The two-sided optimal rule

The Value Matching Conditions (VMC) state that the cost at **a** (**b**) should be equal to the cost at **c** plus what is paid for moving from **a** (**b**) to **c**, that is **k**[8]. So, V(a)= V(c)+k and V(b)=V(c)+k . Using (6) we have the following equations:

$$A\left(e^{\alpha a}-e^{\alpha c}\right)+B\left(e^{\beta a}-e^{\beta c}\right)+h\left(\frac{1}{\rho}\left(a^{2}-c^{2}\right)+2\frac{\eta}{\rho^{2}}(a-c)\right)=k \tag{8}$$

$$A\left(e^{\alpha b}-e^{\alpha c}\right)+B\left(e^{\beta b}-e^{\beta c}\right)+h\left(\frac{1}{\rho}\left(b^{2}-c^{2}\right)+2\frac{\eta}{\rho^{2}}(b-c)\right)=k \tag{9}$$

The Smooth Pasting Conditions (SPC) tell us that the derivative of the value function at the points **a**,**c** and **b** should be equal to the derivative of the adjustment cost[9]. So, V'(a)=V'(c)=V'(b)=0 . Together with the VMC (8) and (9), they allow us to find the optimal values for **a**,**c** and **b**. Using (6) we get the following SPC:

$$\alpha Ae^{\alpha a}+\beta Be^{\beta a}+h\left(\frac{2}{\rho}a+2\frac{\eta}{\rho^{2}}\right)=0 \tag{10}$$

$$\alpha Ae^{\alpha b}+\beta Be^{\beta b}+h\left(\frac{2}{\rho}b+2\frac{\eta}{\rho^{2}}\right)=0 \tag{11}$$

$$\alpha Ae^{\alpha c}+\beta Be^{\beta c}+h\left(\frac{2}{\rho}c+2\frac{\eta}{\rho^{2}}\right)=0 \tag{12}$$

The VMC and SPC equations , (8,9,10,11,12), constitute a non-linear system of five

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[8]The VMC introduce mathematically the control into the solution. For chosen values **a**,**c**,**b** the VMC allow us to determine A and B, in order to find the cost function generated by this policy. The VMC are just consistency requirements for the expected present cost of a given policy.

[9]The SPC are optimality conditions for policy parameters. For a simple derivation of the SPC, see Dixit (1991b).

equations and five unknowns, which can only be solved numerically[10].

B. The one-sided suboptimal rule

Now we impose the form of the policy to be a one-sided rule, and determine the best policy of its form.

First observe that the cost function for the one-sided rule, for the same reasons given above for the two-sided rule, should satisfy the differential equation given by (5), and so, has a general solution given by (6). α and β are the roots of the characteristic equation of (5) and have opposite signs: $\alpha < 0$, $\beta > 0$. The form of our suboptimal one-sided rule will depend on the sign of the drift. In order to keep the most useful barrier, we will drop the upper barrier, if the drift of the uncontrolled process is negative, and will drop the lower one, if it is positive.

We will assume the drift is negative. The other case is similar. So, we will not have any upper barrier. The process is allowed to take any arbitrary large value. Starting from a very large value, the probability of hitting the lower barrier within a reasonable amount of time is very small, and, as a consequence, the cost function should be close to the cost function of the uncontrolled process. When $\mathbf{x}$ is very large, the first term of (6) is close to zero (α is negative), but the second also becomes very large, unless $\mathbf{B}$ is zero. For the cost function to be approximated by the four last terms in (6) (the cost function of the uncontrolled process), when $\mathbf{x}$ becomes very large, it is necessary that $\mathbf{B}=\mathbf{0}$. So, our general equation for the cost function, when the control rule is a one-sided resetting policy is:

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[10]When the uncontrolled process has no drift, the problem becomes much simpler with $\mathbf{a}=-\mathbf{b}$ and $\mathbf{c}=0$. Dixit (1991a) finds an approximated analytical solution for this case.

$$Ae^{\alpha x}+h\frac{x^2}{\rho}+2h\frac{\eta}{\rho^2}x+h\frac{\sigma^2}{\rho^2}+2h\frac{\eta^2}{\rho^3}=0 \tag{13}$$

Equation (13) together with the VMC linking **b** and **c** determine the cost function for an arbitrarily chosen one-sided policy (b,c). Using (13) the VMC becomes:

$$A\left(e^{\alpha a}-e^{\alpha c}\right)+h\left(\frac{1}{\rho}\left(a^2-c^2\right)+2\frac{\eta}{\rho^2}\left(a-c\right)\right)=k \tag{14}$$

Again, the SPC give optimality conditions for the choice of the parameters, **a** and **c**. The SPC are:

$$\alpha Ae^{\alpha a}+h\left(\frac{2}{\rho}a+2\frac{\eta}{\rho^2}\right)=0 \tag{15}$$

$$\alpha Ae^{\alpha c}+h\left(\frac{2}{\rho}c+2\frac{\eta}{\rho^2}\right)=0 \tag{16}$$

Equations (14), (15) and (16) determine the cost function and the suboptimal policy parameters a and c. As before, it is still not possible to find explicit solutions, forcing us to use numerical techniques.

The increase in cost of adopting the suboptimal one-sided rule, as a fraction of the cost when the optimal two-sided rule is adopted (**r**), can now be easily calculated. Let the expected cost starting from **x** using the optimal rule, $\mathbf{C_2(x)}$, be given by the cost function (6) when the constants **A** and **B** are calculated solving the equations (8) to (12). Let the expected cost starting from **x** when the suboptimal one-sided rule is used, $\mathbf{C_1(x)}$, be given by (13) when **A** is calculated solving the equations (14) to (16). Then, the relative increase in cost starting at **x** is given by:

$$r(x)=\frac{C_1(x)}{C_2(x)}-1 \tag{17}$$

3. THE EXPECTED TIME OF HITTING A SPECIFIC BARRIER FOR THE FIRST TIME

In order to assess how far the two-sided policy is from a one-sided one, it would be useful to have an idea of the time spent before an specific barrier (the one that is not hit very often) is hit, starting from a position x. Notice that calculating the distribution of the random time until hitting a specific barrier for the first time, is much more involving than finding the distribution of the time until hitting <u>any</u> barrier for the first time. The latter depends only on the probability law of the Brownian motion, whereas the former has to take into account that a possible resetting in the other barrier may occur, before the specific barrier is hit. Furthermore, the distribution that would interest us does not have a closed form solution. Nevertheless, following a relatively simple approach, we can find an explicit formula for the expected time until reaching a specific barrier starting from x. So, we chose to do it instead.

The approach we use is similar to the one employed to calculate the value function corresponding to a specific policy (a,c,b).[11]Let $\mathbf{T_b}$ be the time the controlled process **X** hits the barrier **b** for the first time. We define $\theta_b(\mathbf{x}) = \mathbf{E^x T_b}$, that is, $\theta_b(\mathbf{x})$ is the expected time, starting at **x**, until the process **X** hits the barrier **b** for the first time. So, θ_b satisfies the following Bellman equation:

$$\theta_b(x) = dt + E[\theta_b(x+dx_t) \mid x_t = x] \tag{18}$$

Applying Ito's Lemma to $d\theta(x_t)$ and taking expectations conditioned on the knowledge of x_t, one can arrive at an expression for the expectation term in the equation above. After substituting it into (18) we obtain the following differential equation for θ:

[11]This is an extension of Karlin and Taylor (1981,pp192-3).

$$\frac{1}{2}\sigma^2\theta''(x)+\eta\theta'(x)+1=0 \tag{19}$$

The general solution to this differential equation is:

$$\theta_b(x)=Ae^{-\frac{2\eta}{\sigma^2}x}-\frac{x}{\eta}+B \tag{20}$$

Now, consistency conditions, analogous to VMC, allow us to find the constants A and B corresponding to the policy parameters (a,c,b).[12]

The conditions are:

$$\begin{aligned}\theta_b(b)&=0\\ \theta_b(a)&=\theta_b(c)\end{aligned} \tag{21}$$

Equations (21) have obvious interpretations. The expected time until hitting **b** for the first time starting from **b** is 0. The expected time of hitting **b** from **a** has to be equal to the expected time of hitting **b** from **c**, since when the process is in **a**, it is instantaneously reset to **c**.

Using conditions (21) to determine the constants A and B in equation (20), we arrive at the following formula for θ_b:

$$\theta_b(x)=-\frac{c-a}{\eta\left(e^{-2\frac{\eta}{\sigma^2}a}-e^{2\frac{\eta}{\sigma^2}b}\right)}e^{-2\frac{\eta}{\sigma^2}x}-\frac{x}{\eta}+\frac{b}{\eta}+\frac{(c-a)\,e^{-2\frac{\eta}{\sigma^2}b}}{\eta\left(e^{-2\frac{\eta}{\sigma^2}a}-e^{-2\frac{\eta}{\sigma^2}c}\right)} \tag{22}$$

The formula of θ_a can be easily found by symmetry:

$$\theta_a(x)=-\frac{c-b}{\eta\left(e^{-2\frac{\eta}{\sigma^2}b}-e^{2\frac{\eta}{\sigma^2}a}\right)}e^{-2\frac{\eta}{\sigma^2}x}-\frac{x}{\eta}+\frac{a}{\eta}+\frac{(c-b)\,e^{-2\frac{\eta}{\sigma^2}a}}{\eta\left(e^{-2\frac{\eta}{\sigma^2}b}-e^{-2\frac{\eta}{\sigma^2}c}\right)} \tag{23}$$

[12]Observe that in equation (19) the policy parameters do not appear explicitly.

4. NUMERICAL ANALYSIS

In order to do numerical exercises with parameters based on real data, it is necessary to have an equation which relates the optimal individual price with the aggregate and idiosyncratic shocks. We assume that the (log of) the optimal individual price, p_i^*, is given by[13]:

$$p_i^* = y + e_i \tag{24}$$

where y is the (log) of nominal aggregate demand and e_i is an idiosyncratic component. In the absence of control, a change in the nominal aggregate demand will have an effect on the difference between the actual price and the optimal frictionless price of the same magnitude and opposite direction. We assume that the (log of the) nominal aggregate demand follows a Brownian motion, and equate the drift and diffusion parameters of the process with the symmetric of the mean and the standard deviation of the changes in the (log of the) nominal aggregate demand observations, respectively. As for the idiosyncratic component, we assume it follows a Brownian motion without drift, independent of the stochastic process followed by the nominal aggregate demand. So,

$$\begin{aligned} y &= m dt + \sigma_y dw_1 \\ e_i &= \sigma_e dw_{ei} \\ p_i^* &= m dt + s dw_i \end{aligned} \tag{25}$$

where $s = \sigma_y + \sigma_e$.

This section assesses how far optimal two-sided Ss rules are from suboptimal one-sided S-s rules. We use two notions for that. The first is related to the consequences of adopting

[13]This formulation implies that a 1% shock in nominal agregate demand has a 1% impact in the frictionless optimal price. Since we fix the reference period for the flows in one year, this is not without loss of generality.

a suboptimal pricing rule instead of the optimal one. Does an asymmetric two-sided rule in practice look like a one-sided Ss rule? If the upper bound is very rarely achieved, the two-sided Ss rule in practice looks like a one-sided one and has similar macroeconomic implications[14]. The expected time until reaching the upper bound provides us with useful information for this assessment. We chose to evaluate the expected time of reaching the upper bound at 0, when the actual price is equal to the optimal one, $\theta_b(\mathbf{0})$[15]. The expected time until reaching the most often reached bound - the lower bond **a**- is also calculated to give us a notion of how often a price adjustment occurs in this economy.

The second is related to the likelihood that an economic unit adopt the suboptimal one-sided rule instead of the optimal two-sided one. We evaluate how costly it is to adopt the suboptimal one-sided rule instead of the optimal two-sided one. For this purpose, we evaluate **r(0)**[16], which gives the increase in cost of adopting the suboptimal one-sided rule as a fraction of the cost when the optimal two-sided rule is adopted, evaluated at $\mathbf{x=0}$.

We proceed in two steps. We start by performing some simulations in order to get some qualitative assessment on how changes in parameter values affect the comparison. This helps us build intuition that will be useful in the interpretation of the results based on numbers for real economies, rendered in the second step.

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[14]As explained in section 5 below, it has similar macroeconomic implications to the ones of a one-sided rules when those rules are used in the same environment, where those rules are not optimal.

[15]In Bonomo (1992) the expected time until reaching each barrier starting from **c** - the point to which the difference between actual and optimal prices returns after an adjustment - is also reported.

[16]The function r(.) can be defined over the intersection of the regions delimited by the upper and lower bounds of the two rules. In all simulations we made, 0 fell inside both regions. However, it is possible for 0 to fall outside the control region of a suboptimal one-sided rule. A suboptimal rule may call for resetting a (low) actual price to a value lower than the optimal one to compensate for the absence of the upper barrier.

A. Evaluating the parameter effects

In Table 1 we vary one parameter at a time to appraise the influence of that parameter on the comparison[17]. The first column gives results for the base values we chose for the parameters: η=-0.1, σ=0.1, ρ=2.5% and k=0.01 (since **k** and **h** enter the solution only through k/h, we normalize **h** to one)[18,19]. Every time the actual price becomes 14% lower (**a**=-14%) or 20% (**b**=20%) higher than the optimal price, it is reset to a value 5% (**c**=5%) higher than the optimal one. Observe that the price is not reset to the value of the optimal price itself, because the optimal price has a tendency to increase. So, the anticipation of this tendency, and the knowledge that the price should remain fixed for a while because of the menu costs, lead the agent to reset the price to a level a little bit higher than the frictionless optimal one. Since the magnitudes of the upper and lower edge of the band are not so different, and there is a sizeable downwards drift, the lower edge is reached much more often than the upper edge. This is reflected in a value for the expected time until reaching the upper extreme which is much higher than the one for the expected time until reaching the lower extreme. Starting at the resetting price (c), the expected time until reaching the lower barrier for the first time is 1.82 years, whereas the same computation for the upper barrier gives 30.42 years. When we use the best one-sided policy instead of the two-sided policy, **A**, **a** and

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[17]We keep the discount rate constant at ρ=0.025 since it does not affect substantially the optimal or the suboptimal policies. In Bonomo (1992), it is reported the result of an increase in ρ to 0.10 while keeping the other parameters constant.

[18]The value of **k** is chosen to give reasonable predictions for the frequency of price adjustments when the model is calibrated to the U.S. The value chosen for ρ is standard in the literature, and alternative reasonable values produce very similar results.

[19]In Bonomo (1992), the results for the parameters **A** and **B** of the cost function are also shown. Recall that **A** and **B** give the reduction in cost achieved by the use of resetting from below and above, respectively. We get relatively large values for **A** and low values for **B**, as a consequence of the lower barrier being very active and the upper one being little active.

c are very similar to what we had before. The absence of the upper barrier makes it safer to reset to a price a little bit lower than before, so c is slightly smaller now. The percentual increase in cost caused by the use of the suboptimal one-sided rule is calculated in 6.6%, when we use the base values for the parameters.

In the second column we increase **k**, the ratio between the menu cost and the flow cost, from 0.01 to 0.05. The increase in **k** affects dramatically the results turning the optimal policy much closer to the suboptimal one. The band becomes much wider, increasing the expected time until reaching the lower and the upper barrier. The effect on the expected time until reaching the upper barrier is striking. It increases from 31.16 to 216.06 years. So, the upper bound becomes somewhat superfluous and the adoption of the one-sided rule becomes almost costless ($\lambda(0)$ is 0.9%).

It is intuitive that when the absolute value of the drift increases, ceteris paribus, the loss involved in adopting the suboptimal one-sided rule, instead of the optimal two-sided one, decreases. Also, since the stochastic component is symmetric, when the variance increases, ceteris paribus, the loss involved in adopting the one-sided rule increases. So, we pursue the more obscure question of what happens when both the variance and the drift vary in the same direction. In the third column we double both the drift and the standard deviation. A higher variance substantially reduces the expected time until reaching a barrier for the first time. So, the size of the band widens as a response, but the expected time continues to be smaller than before. We see that the effect of the increase in the variance dominates the effect of the higher drift since the additional cost of imposing a one-sided rule increased from 6.6% to 24.8%. In the fourth column we double the drift and variance. Now the effect of the drift is prevalent - r(0) is reduced from 6.6% to 3.1% - although the resulting effect is of smaller magnitude than the one we had before. Since the additional cost of adopting a suboptimal one-sided pricing rule is not a function alone of m/s^2, it becomes interesting to investigate what is the shape of the relation between m and s^2 which keeps r constant.

Figure 1 depicts the relation between m and s^2 for r equal to 0.01, 0.05 e 0.10. The uppermost curve is the one with the smallest r since for the same variance an increase in the drift makes the optimal policy closer to the best one-sided rule. We see that the relation is not a straight line: the increase in m, necessary to compensate a given increase in s^2 in order to keep r constant, is decreasing in s^2. As a consequence, an increase in s^2 requires a less than proportional increase in m to maintain r constant. In Figure 2, we explore the influence of k in the shape of the iso-r. We see that the lower is k, the higher the concavity of the curve. Figure 3 illustrates that if we substitute s for s^2 in the ordinate, the curve becomes convex. So, in general, to keep r constant when m is increased, it is necessary to increase the variance more than proportionally, but by less than the addition that would cause a proportional increase in the standard deviation.

B. Comparison of the rules based on real economies

In Table 2, we present results with parameter values based on real economy data. We chose one low inflation economy (U.S.), one high inflation economy (Colombia) and a set of 43 countries (Inter)[20]. We base our values on Ball, Mankiw and Romer (1988). We take the values for the average increase in the log of the nominal aggregate demand as the drift. To arrive at our values for the standard deviation, an allowance for the standard deviation of the idiosyncratic shocks is added to the standard deviation of the increase in the log of

[20] In Bonomo(1992), Brazil was chosen as the high inflation country. However, the Brazilian inflation process has different means in different periods, which causes an upward bias in the diffusion parameter estimate when one tries to fit a geometric Brownian motion for prices.

nominal aggregate demand. For the U.S. we follow Ball, Mankiw and Romer's (1988) assumption of a 3% standard deviation of the idiosyncratic shocks. For the set of countries (a sample of 43 countries in Ball, Mankiw and Romer (1988)), we use 0.037 and for Colombia, 0.042. The implicit assumption is that there is not a substantial correlation between the standard deviation of the idiosyncratic shocks and inflation. Observe that, when we use $k=0.01$, the model produces reasonable predictions for the U.S. economy. The expected time until an upwards price adjustment is made, starting from **c** (the value at which the price is reset), is a good approximation for the time between adjustments, since a downwards adjustment does not happen often. So, according to the model, the elapsed time between adjustments should be a little bit more than two years (since $\theta_a(c)=2.25$), which is consistent with the microeconomic evidence[21]. However, the expected time until a downwards revision is achieved, seems to be very big, 154 years. As a contrast, using Colombia's data, the number obtained for the expected time until adjustment is lower, 77.22 years, despite the larger inflation. This may suggest that the one-sided Ss rule is a better approximation of the optimal two-sided rule for the parameters based on the US data than for the parameters based on the Colombian data. However the additional cost of adopting an one-sided Ss rule is smaller for the Colombian numbers (0.4% as compared to 0.8% for the US values). The effect of the higher drift for the Colombian inflation is not totally offset by its higher standard deviation. For the international set, the loss of adopting a one-sided rule is substantially higher because the standard deviation of the international average is higher than the Colombian one and the drift is lower.

We can notice two features from those results which deserve attention. The first is that the additional cost of imposing a suboptimal one-sided rule is relatively small in all cases.

[21]For microeconomic evidence on price adjustments, see Cechetti (1986).

However, the results depend on unobservable parameter values as **k** and ρ. The value of **k** should be set to provide realistic price adjustment frequencies. A lower **k** would simultaneously decrease the frequency of price adjustments and increase the additional cost of adopting a suboptimal one-sided rule. It seems difficult at the level of generality of the analysis to decide what is a good value for **k**. The indeterminacy of ρ is not as problematic since any value in the acceptable range from 1% to 10% will not give substantially different results.

The second feature is that the effect of the variance is dominant on the results. We saw that if an increase in σ requires a more than proportional increase in μ, in order to keep λ constant (Figure 3). However, in real economies an increase in the inflation trend is, in general, associated with a close to proportional increase in standard deviation[22]. So, according to our model, it is not assured that one-sided Ss rules are closer to optimal in high inflation economies.

From the discussion in this section, we conclude that it is possible that in real world situations one-sided Ss pricing rules are good approximations of the optimal asymmetric two-sided ones. It is even possible that the one-sided rules are used in practice, since the cost involved in adopting the simpler and suboptimal one-sided rule, rather than the optimal two-sided one is relatively small. However, evaluations based solely on the ratio between the mean and the variance parameters of the stochastic process followed by the optimal price are unsafe.

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[22]An informal evidence is figure 3 of Ball, Mankiw and Romer (1989).

5. MACROECONOMIC IMPLICATIONS

The effect of an aggregate shock depends on the pricing rule (assumed to be the same for all units), on the cross-section distribution of the price deviations inside the inaction band, and, as noted by Caballero and Engel (1991b), on the idiosyncratic shocks which affect simultaneously each unit. Since the cross-section distribution depends on the history of aggregate shocks, we use the ergodic distribution (see the Appendix for the derivation), that is an average of the possible cross-section distributions, in our considerations. In what follows we neglect the simultaneous effect of idiosyncratic shocks, since it has no qualitative importance in the comparison of suboptimal one-sided rules with optimal two-sided rules.[23]

The case where the rule is one-sided and the cross-section distribution is uniform constitutes an useful benchmark for the analysis. Within these circumstances, while a positive shock in the money supply is neutral, since it preserves the same distribution, a negative shock has maximum effect because there is never a price reduction. What is interesting about this benchmark case, is the extreme asymmetry of the effects: average price is totally rigid downwards and totally flexible upwards. It is important to remark that because this rule is optimal only when there are no negative shocks, its effect was never considered in the one-sided rule literature. Since we treat one-sided rules explicitly as suboptimal rules, it makes sense to consider the effect of negative shocks.

When the rule is two-sided, the effects of both positive and negative shocks depend on the parameters of the rule and on the cross-section distribution. The former fix the size

[23]For a formal analysis of the macroeconomic implications of two-sided rules in presence of both aggregate and idiosyncratic shocks, see Caballero and Engel (1991b). The analysis is simplified because of the particular assumptions made for the individual rules and the distribution of the price deviations inside an industry.

of the adjustment while the latter determines the fraction of units changing prices. A symmetric two-sided rule is optimal when the stochastic process followed by the frictionless optimal price is driftless. In this case the ergodic distribution of the individual price deviations is obviously symmetric. When there is a positive drift in the frictionless optimal price process, the distribution of the price deviation becomes asymmetric, tilted downwards (see Figure 4). The fraction of units close to the upper bound decreases, so negative monetary shocks trigger less adjustments and the effect of a monetary contraction is increased. On the other hand, the fraction of units close to the lower bound increases, so the effect of positive monetary shocks decreases.

When the rule is one-sided, but the driving stochastic process has shocks in both directions, as it is the case with the Brownian motion, the ergodic distribution of the individual price (in log) deviations has positive decreasing density for values higher than the resetting point, **c**. For values smaller than **c** the density is increasing from **a** to **c**. The higher is the drift, the lower is the probability of having an individual price deviation greater than **c** and the flatter is the slope of the density between **a** and **c** (see Figure 5). When the drift becomes very large (with a fixed variance), the ergodic distribution of the price deviations approaches the uniform distribution between **a** and **c**. Like the two-sided case, the effect of a monetary expansion is larger when the cross-section distribution of price deviations is the ergodic distribution of individual price deviations corresponding to a process with a smaller drift. When the drift is small, since the density of the ergodic distribution increases with a steeper slope from **a** to **c**, a positive monetary shock induces a smaller number of units to adjust. The effect of a monetary contraction is independent of the cross-section distribution: since there is never a downwards adjustment when the units are following one-sided rules, all reductions in nominal money supply are real[24].

So, when the drift is positive, both one-sided and two-sided rules provide asymmetric responses to positive and negative monetary shocks. Average price is stickier downwards than upwards. The asymmetry is always bigger when the rule is one-sided, when average price is totally rigid downwards. However, when the drift increases (for a given variance), the difference between the effects of one-sided and two-sided rules are reduced and both rules and ergodic distributions converge to our benchmark case. So, the one-sided and two-sided rules have similar effects, when the adoption of a suboptimal one-sided rule is more plausible.

The simulations in section IV suggest that the one-sided rules are similar to the two-sided rules for parameter values based on real economies. Not surprisingly, the corresponding ergodic distributions are also close (see figures 6 and 7). The rules are close because it is not very likely, at the individual level, that the upper bound of the two-sided rule will be reached often. This corresponds to a cross-section distribution where the fraction of units close to the upper bound is small, and as a consequence, the effect of a negative monetary shock should be large. Thus, our numerical exercises lead us to conclude that an analysis based on a suboptimal one-sided rule would not give results that are substantially different from those derived from an optimal two-sided rule. In both cases, there is a substantial asymmetry between the effects of positive and negative monetary shocks.

[24]When the presence of idiosyncratic shocks is taken into account, the effect of a monetary contraction on the output is always negative, but the magnitude depends on the cross-section distribution. For example, if the drift is relatively large, there will be an important fraction of units close to the lower bound. Thus the idiosyncratic shocks will trigger some price increases, making the effect of the money contraction stronger.

6. CONCLUSIONS

One-sided Ss pricing rules are rarely optimal. This paper shows that they are often very close to the optimal rule. Since the additional cost of adopting suboptimal one-sided rules is small, it is possible that they are used in practice. The macroeconomic implications of the optimal two-sided rules are also similar to the ones of the suboptimal one-sided rules for the cases analyzed. However, the implications of suboptimal one-sided rules are different from the optimal ones. Negative shocks are possible and have large effects, reproducing the substantial asymmetry between positive and negative shocks found in the data.

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APPENDIX

Ergodic Distributions for Two-sided and One-sided rules

Two-sided Rules

The derivation of the ergodic distributions for optimal two-sided rules is shown in Bertola and Caballero (1990). The density function of the ergodic distribution for the two-sided rule has the following form (see Bertola and Caballero (1990)):

$$f(z)=\begin{cases} Me^{\gamma z}+N; & a\le z\le c \\ Pe^{\gamma z}+Q; & c\le z\le b \\ 0; & otherwise \end{cases} \qquad \textbf{(A1)}$$

with $\gamma=-2m/s^2$.

Because the density should die continuously, it should be zero at the extremes, that is, $f(a)=f(b)=0$. Those conditions yield the following equations:

$$Me^{\gamma a}+N=0 \qquad \textbf{(A2)}$$

$$Pe^{\gamma b}+Q=0 \qquad \textbf{(A3)}$$

Continuity of the density function at c requires $f(c)^+=f(c)^-$, which results in:

$$Me^{\gamma c}+N=Pe^{\gamma c}+Q \qquad \textbf{(A4)}$$

Of course, the integral of the density function over the appropriate range should be equal to one. This gives the fourth equation:

$$\frac{M}{\gamma}(e^{\gamma c}-e^{\gamma a})+\frac{P}{\gamma}(e^{\gamma b}-e^{\gamma c})+N(c-a)+Q(b-c)=1 \qquad \textbf{(A5)}$$

Equations (A2-A5) determine the constants M,N,P,Q in (A1).

One-sided rules

The (suboptimal) one-sided is the limit of a two-sided rule when b tends to infinity. So, the density should have the following form:

$$f(z)=\begin{cases} 0; & z\le a \\ Me^{\gamma z}+N; & a\le z\le c \\ Pe^{\gamma z}+Q; & c\le z \end{cases} \qquad \textbf{(A6)}$$

Continuity at a and c yields:

$$Me^{\gamma a}+N=0 \qquad \textbf{(A7)}$$

$$Me^{\gamma c}+N=Pe^{\gamma c}+Q \qquad \textbf{(A8)}$$

The following additional condition should be satisfied because f is a density function:

$$\frac{M}{\gamma}(e^{\gamma c}-e^{\gamma a})+\frac{P}{\gamma}(e^{\gamma b}-e^{\gamma c})+N(c-a)+Q(b-c)=1 \qquad \textbf{(A5)}$$

TABLE 1: Numerical exercises

		η=-0.1 σ=0.1 ρ=2.5% k=0.01	η=-0.1 σ=0.1 ρ=2.5% k=0.05	η=-0.2 σ=0.2 ρ=2.5% k=0.01	η=-0.2 $\sigma=0.1\sqrt{2}$ ρ=2.5% k=0.01
TWO SIDED	a	-0.14	-0.21	-0.20	-0.16
	c	0.05	0.11	0.05	0.07
	b	0.20	0.32	0.25	0.24
	$\theta_a(c)$	1.82	3.12	1.12	1.15
	$\theta_b(c)$	30.42	216.06	8.24	32.57
	$\theta_a(0)$	1.35	2.06	0.92	0.81
	$\theta_b(0)$	31.16	217.77	8.54	33.11
	$C_2(0)$	0.400	1.050	0.732	0.603
ONE SIDED	a	-0.14	-0.21	-0.21	-0.17
	c	0.04	0.10	0.02	0.06
	$C_1(0)$	0.42	1.05	0.91	0.62
	$\lambda(0)$	6.6%	0.9%	24.8%	3.1%

Note: we assumed ρ=2.5%

TABLE 2: Numerical exercises based
on real economy data

		INTER	U.S.	COLOMBIA
		m=0.149 s=0.125	m=0.073 s=0.062	m=0.184 s=0.104
TWO SIDED	a	-0.15	-0.11	-0.14
	c	0.06	0.06	0.08
	b	0.22	0.17	0.23
	$\theta_a(c)$	1.39	2.25	1.50
	$\theta_b(c)$	29.03	154.38	77.22
	$\theta_a(0)$	1.02	1.48	0.95
	$\theta_b(0)$	29.65	155.62	78.02
	$C_2(0)$	0.51	0.29	0.53
ONE SIDED	a	-0.16	-0.11	-0.14
	c	0.05	0.06	0.08
	$C_1(0)$	0.54	0.29	0.53
	$\lambda(0)$	5.1%	0.8%	0.4%

Note: we assumed $k=0.01$ and $\rho=2.5\%$.

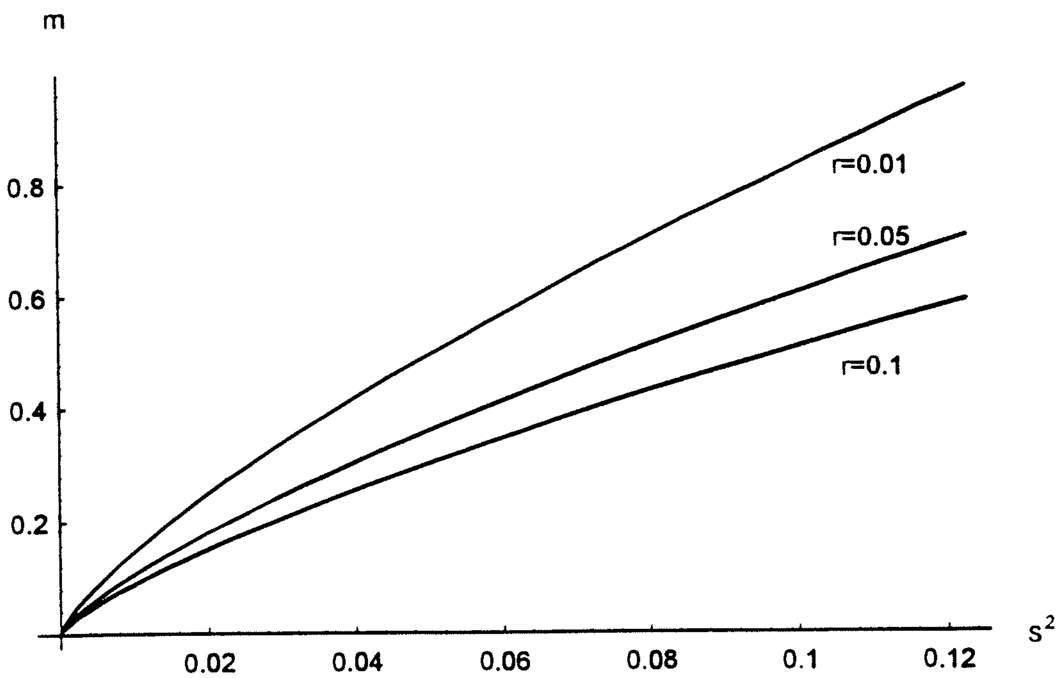

Figure 1

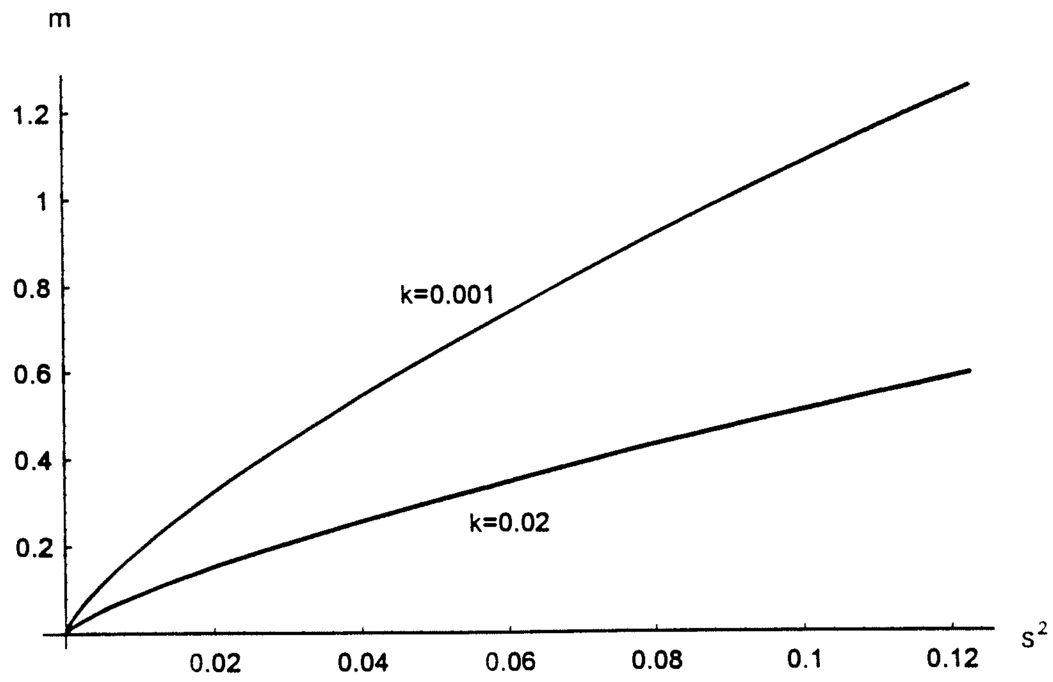

Figure 2

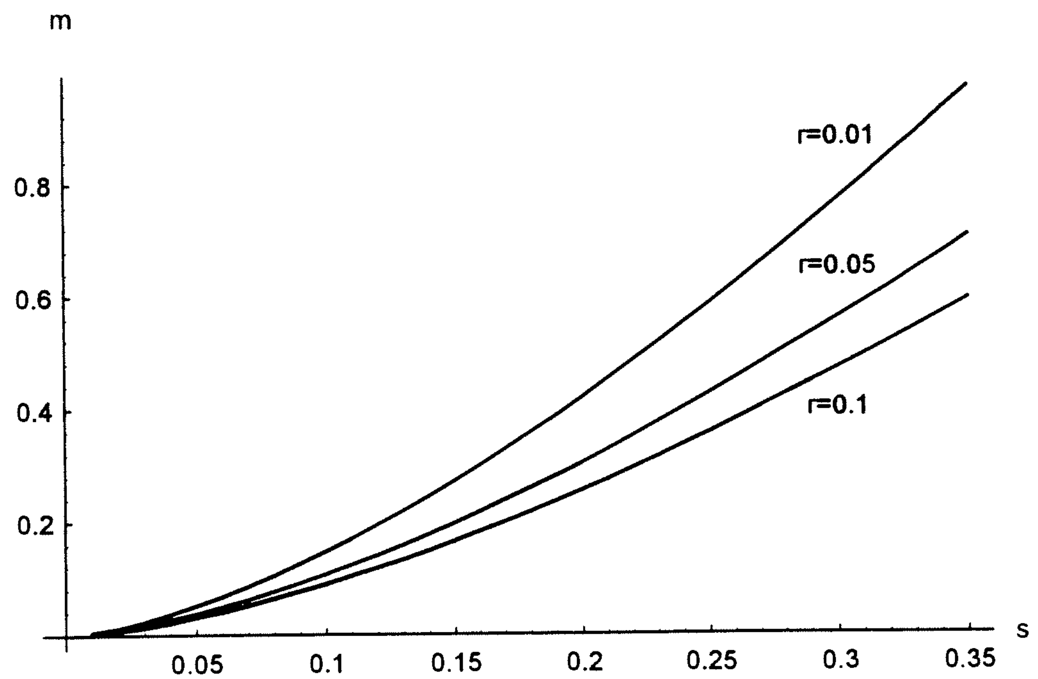

Figure 3

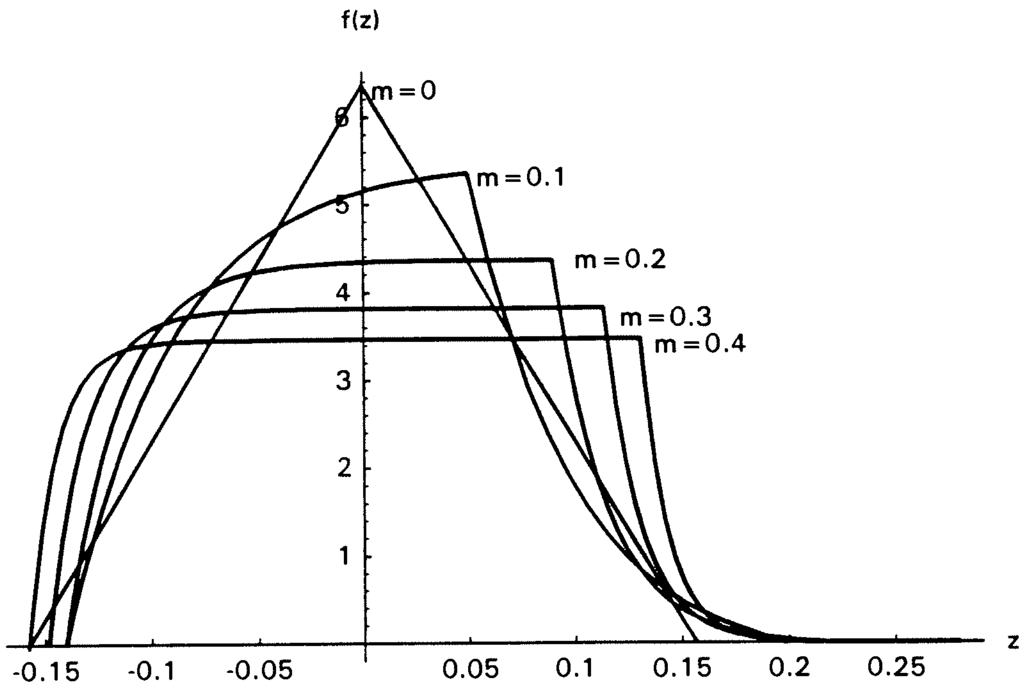

Figure 4

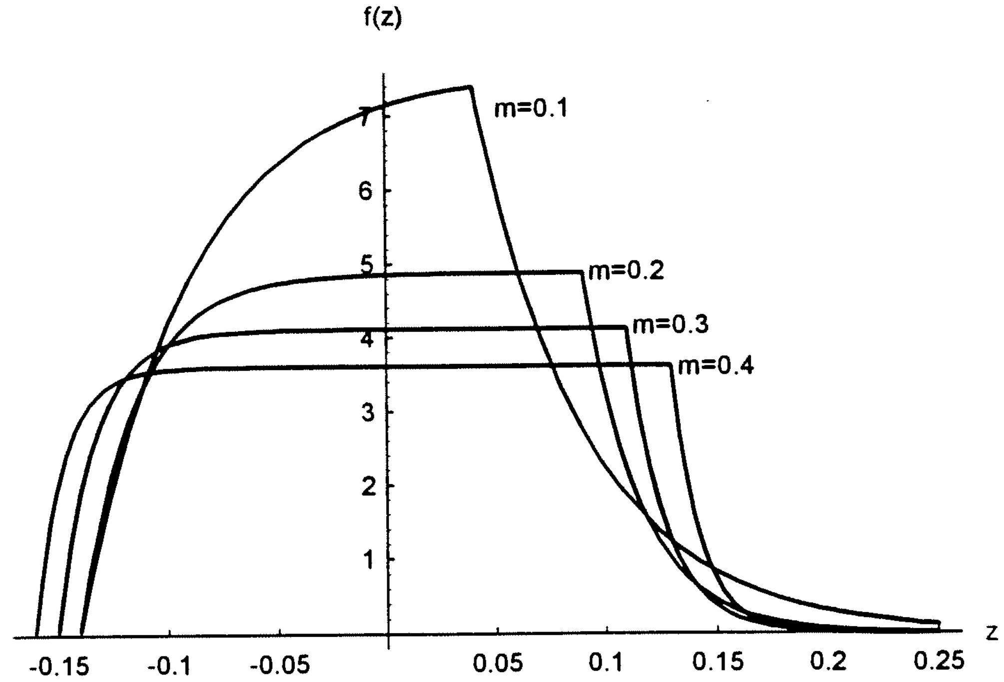

Figure 5

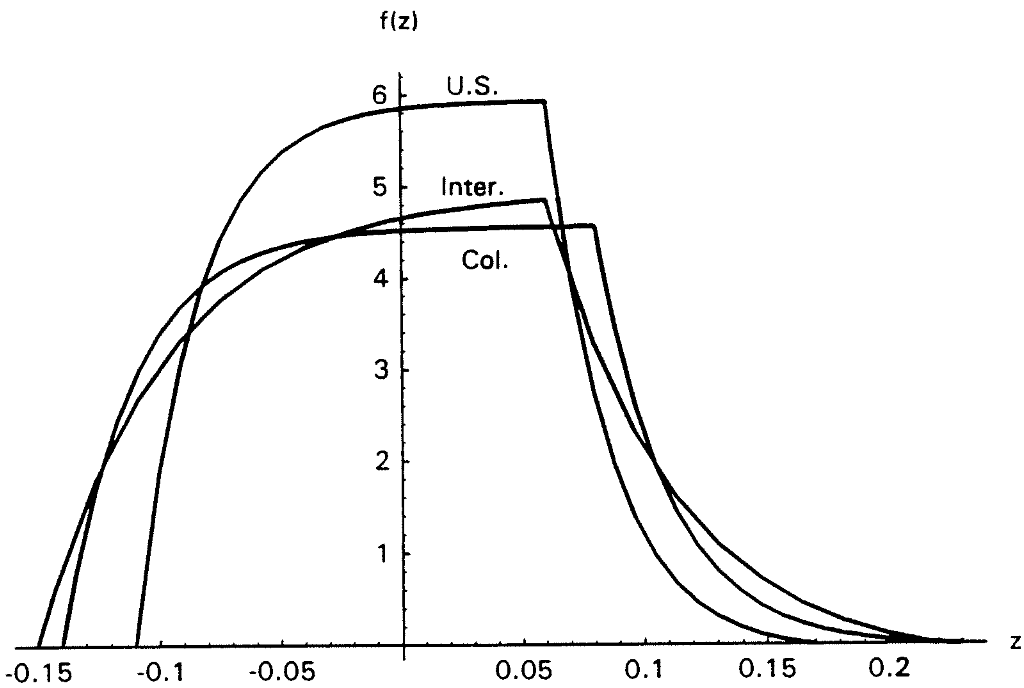

Figure 6

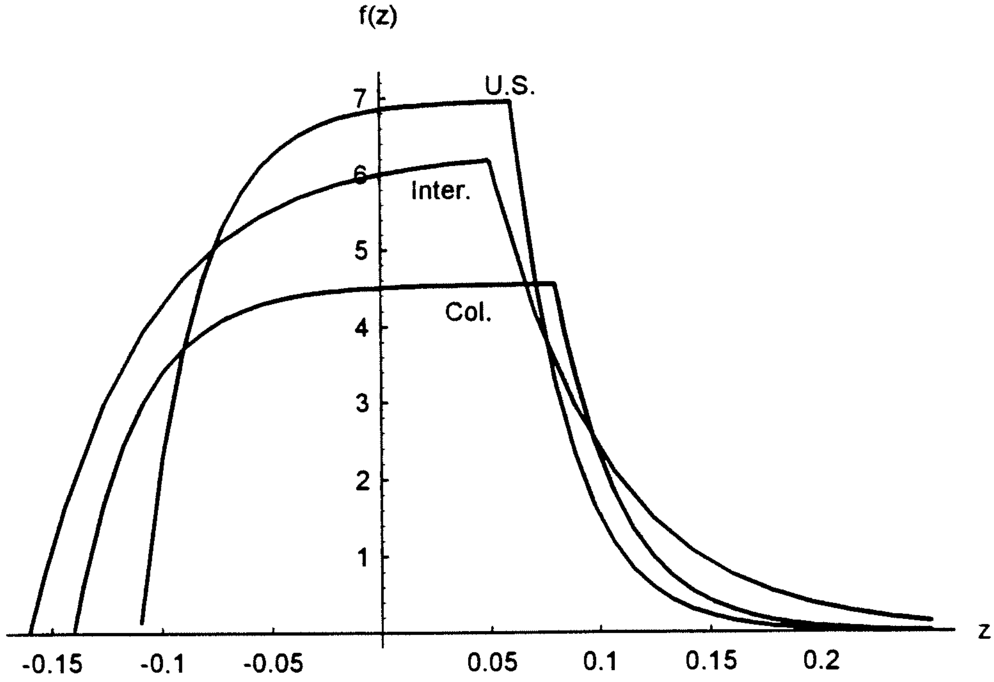

Figure 7

Textos para Discussão:

296. Abreu, M.P. "Brazil-US economic relations and the enterprise for the Americas Initiative". jan. 1993

297. Garcia, M. G. P. "Política monetária e formação das expectativas de inflação: Quem acertou mais, o governo ou o mercado futuro?". mar. 1993

298. Bacha, E.L. "Selected international policy issues on private market financing for developing countries". feb. 1993

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305. Abreu, M.P. "A dívida pública externa brasileira em francos franceses," 1888-1956"

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307. Carneiro, D.D.; Werneck, R.L.F.; Garcia, M.G.P. "Strengthening the financial sector in the Brazilian economy"

308. Bonomo, M.; Garcia, R. "Disappointment aversion as a solution to the equity premium and the risk-free rate puzzles"

309. Gonzaga, Gustavo M. "Assymmetric employment cycles at the firm level: a dymanic labor demand model and some empirical evidence"

310. Amadeo, E. J. "An Economist's poltical view of democratization in Brazil"

311. Abreu, M. P. "O Brasil na Rodada Uruguai do GATT: 1982-1993"

312. Amadeo, E. J."Distributive and welfare effects of inflation and stabilization"