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HUMAN CAPITAL INVESTMENT AND POVERTY

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1. Introduction

Human capital investment is a very powerful instrument to reduce poverty. As the rate of return on this type of investment is positive and in general, very high for the poor, an increase in human capital investment by poor families should result in better incomes and less poverty. On the other hand, recent models of economic growth, such as Romer (1986) and Lucas (1988), emphasize investment in human capital as an important factor contributing to economic growth. Thus, there is a consensus among economists and policy makers that policies should be directed to increase human capital investment and human capital accumulation by families in general, and by poor families in particular.

As the previous paragraph suggests, it is important for the objectives of the paper, to distinguish between human capital investment and human capital accumulation. Human capital accumulation is the amount of information the student is able to acquire, process and retain during the period he attends the school. It is basically the degree of qualification obtained by the student. Given the student capacity to learn, human capital accumulation depends on the quality of the educational system (or the school) and on the length of time spent by the student on the school. On the other hand, human capital investment is the amount of resources effectively spent by the family to accumulate human capital. This will include the time and the monetary costs of attending the school.

The empirical evidence shows that in most countries (and even more in developing countries), the rate of return on human capital investment is very high. Although this is so, drop out and repeater rates of poor families' children in the school system are also, in general, very high. As a result, poor families' children tend to have less education than the children of richer families.

In the paper, poor families are characterized by the fact that they do not save. This means that their choice behaviour is restricted. As their income level is low, they would like to borrow money in the present period, in order to reduce current poverty. However, they do not have access to a credit market, and will end up with less present consumption than the desired amount.

This aspect of the family decision regarding human capital investment is very relevant for poor countries. In these countries, the children of the poor tend to enter the labour market very early. Despite the fact that their wages are generally

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not high, these children may contribute significantly to the family income.[2] Therefore, low levels of human capital investment by the poor, may be linked to the very fact that these families are poor.

It will be shown that this characterization generates important insights about the decision of a poor family regarding human capital investment. This happens because, in this case, a trade-off between consumption and human capital investment will appear. An increase in human capital investment will necessarily bring a reduction in present consumption, as poor families' children could be in the labour market increasing current income. This will result in an increase in poverty today, as measured by the level of the families' consumption.

Thus if the objective is to use human capital investment and accumulation as an instrument to equalize opportunites and reduce poverty, policies should be implemented which induce these families' to increase their investment in human capital, even when the public education system is free.

The objective of this paper is to analyze three different policies which could affect human capital investment and accumulation by families, and compare them in terms of its relative efficiency to reach the objective. These policies are:

an increase in the quality of the educational system;
a subsidy for the family to keep their children in school;
and a pure income transfer to the families.

We analyse the decision on human capital investment in the context of a two period maximization of a single unified family utility function. This is consistent with the way microeconomic theory has traditionally considered household based economic decisions.[3] In the model, the family will invest in human capital in the current period because this will increase family income in the next period. We are therefore in the context of a parental altruism model such as Becker and Tomes (1976,1979), and Behrman, Pollak and Taubman (1982), in which the parents are concerned not only with their own consumption, but also with their children utility, which depends on the income they will earn when adults.[4]

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[2]see Almeida H. and Camargo J.M.(1993), for a survey of data regarding this aspect in Brazil.

[3]. See Behrman(1992) for a thorough review of household decision models,concerning human resource investments, and other economic decisions.

[4]. See also Blanchard and Fischer(1989),for a discussion of altruism models.

It is shown in the model that, if the objective is to increase human capital investment, then a subsidy for education is more effective than an income transfer to the family. This result is valid both when savings are positive and when they are not. It is also shown that an increase in the quality of the educational system has dubious effects on the level of human capital investment. This may be especially true if families are poor, that is, if they do not save.

In section 2, the basic model is presented, and the optimum choices of the "poor" and "non-poor" families analysed. In section 3, the policy problem is analysed in a general way. In section 4, the different policies are then carefully analized and compared. Finally, in section 5, the main conclusions are presented.

2.The Basic Model

Consider an economy where individuals live for two periods and die at the end of the second period. A family in this economy is composed of two types of individuals. Some of them are adults at period t and old at period t+1, and some are young at period t and adults at period t+1. Individuals are all supposed to die at period t+1. The former type of individuals will be called "parents", receiving exogenous income at period t and taking at this period all relevant decisions for the family. The latter type of individuals will be called "children" receiving education at period t and working at period t+1. Children income at period t+1 will depend on the amount of educational services accumulated in the previous period, which is a function of the time spent by children in school and on the quality of the school (the introduction of quality as an argument in the learning technology is consistent with Card and Krueger, 1992. See also Glomm and Ravikumar, 1992).

The assumptions that parents take all the relevant decisions freely at time t and the altruistic behaviour discussed at the introduction justify an analysis based on a single utility function for the family. Given that, the family problem can be formalized as follows:

$$\max\ U(c_t) + (1+\theta)^{-1}U(c_{t+1}) \qquad (1)$$

$$\text{s. to}\quad c_t + s_t + zh_t = y_t \qquad (2)$$

$$c_{t+1} = (1+r)s_t + w_{t+1}[e(q,h_t)] \qquad (3)$$

Where

c_t = family consumption in the first period
c_{t+1} = family consumption in the second period
s_t = family savings
r = interest rate on family's savings
θ = family discount rate
h_t = the time family's members attend school
z = the opportunity costs of being in school
y_t = family's income at time t

w_{t+1} = family's average wage at time t+1
e = educational services
q = quality of the educational system

The family chooses, at t, how much she will consume (c_t), how much she will save (s_t) and how many hours their children will spend on school (h_t), at the unit price z. The actual investment in human capital made in period t is $z.h_t$, the price per unit of time spent in school times the total time spent by family's children in school. Note that, if the school is a free public school, z is an opportunity cost. That is, the income the family would get if her young members were in the labor market, instead of in the school, plus the monetary costs with material, transport, etc. of attending the school. Thus, y_t should be interpreted as an exogenous potential income of the family, and not as actual monetary income[5].

Savings will accumulate at the rate r, per period, which will increase consumption in the next period, together with the income received by the family in this period (w_{t+1}). The income in period t+1 will depend on the amount of human capital the family has accumulated in the past (e). This will in turn depend on the quality of the educational system (q) and on the number of hours the family's members spent on school in the previous period (h_t). Note that, as all members are assumed to die at period t+1, everything saved or invested is consumed at period t+1.[6]

We make the following assumptions about the learning technology[7]:

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[5]. We can think of y* as being a function of the educational services accumulated by the parents at the previous period.As the accumulation of human capital by parents is not a variable within their control, it is effectively exogenous from the point of view of the optimal decisions made at time t.

[6]. The basic results would not change if we considered that the children would have to take decisions at t+1 in a similar fashion, as long as we consider that parents care about children income, and not consumption. Note that children income is totally exogenous from the point of view of the decisions they would have to face at t+1. The fact that the relevant variable for parents utility is children income is consistent with the literature (see Durlauf, 1993, and Behrman, 1992).

[7]. Subscript represents the derivative of the function in relation to the variable. If the function is univariate, its derivative in relation to this variable is represented by a ', and a " represents a second derivative in relation to the variable.

$e_q(h_t, q) > 0$ $\quad e_h(h_t, q) > 0$

$e_{qq}(h_t,q) < 0$ $\quad e_{hh}(h_t,q) < 0$ $\quad e_{qh}(h_t,q) > 0$

Given the quality of the educational system, an increase in human capital investment (here represented by h) increases the educational level (human capital accumulation), but at a decreasing rate. The positive cross derivative, e_{qh}, implies an important complementarity effect: an increase in the quality of the educational system increases the marginal accumulation of human capital for an increase in human capital investment.

The earnings function is assumed to be concave, that is.

$w'(e) > 0$ $\quad w''(e) \leq 0$

These assumptions imply decreasing returns for the investment in human capital, in terms of income. The marginal return of the investment in human capital in terms of income is given by:

$$\delta w/\delta h = w'(e).e_h(q,h),$$

which is a decreasing function of h, the investment in human capital[8].

By substituting the restrictions on the objective function, the problem can be re-written as:

$$\max_{h,s} U(y_t - z.h_t - s_t) + (1 + \theta)^{-1}.U\{(1 + r).s_t + w_{t+1}[e(q,h_t)]\} \qquad (4)$$
$$s_t \geq 0$$
$$h_t \geq 0$$

First order conditions for a maximum (assuming $s_t > 0$ and $h_t > 0$) can be stated as:

$$U'(c_{t+1})/U'(c_t) = (1 + \theta)/(1 + r) \qquad (5)$$
$$U'(c_{t+1})/U'(c_t) = z(1 + \theta)/w'(.)e_h(q,h) \qquad (6)$$

evaluated at s^*, h^*.

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[8]. This is consistent with Becker and Tomes,1976. However,there are empirical evidences for Brazil showing that investment in human capital might actually be subject to increasing returns (see J.C. dos Reis Carvalho, "Returns to Education in Brazil: a flexible functional form estimation", Série Seminários n. 16/93, IPEA, july, 1993). However, it must be considered that this result might arise from externalities related to education. If that is so, the empirical evidence does not contradict a model that is concerned with individual choice.

Condition (5) says that, at the optimum, the marginal rate of substitution between consumption in period t and t+1 must be equal to the ratio of the discount rate and the marginal return of financial investment. This is the increase in consumption which could be obtained in the second period due to the increase in income resulting from the savings of the previous period, (1 + r).

Condition (6) has a similar interpretation for the investment in human capital. The term $w'(.).e_h(q,h)$ is the increase in family's income in period t + 1 due to the investment in human capital (the marginal return of human capital investment). On the other hand, $z(1 + \theta)$ is an opportunity cost of the investment in human capital, which includes a direct opportunity cost and the discount rate. The ratio of these two must equal the marginal rate of substitution between consumption in period t and consumption in period t+1.

Conditions (5) and (6) can be put together as

$$(1 + \theta)/(1 + r) = z(1 + \theta)/ w'(.).e_h(q,h)$$
$$w'(.).e_h(q,h) = (1 + r).z \qquad (7)$$

Condition (7) means that families increase their demand for educational services up to the point where the marginal gains in doing so, $w'(.).e_h$, equals its marginal cost. The marginal cost is given by the reduction of one unit of savings in period t that would generate income (1 + r) in period t + 1, times the opportunity cost of being in school, z. This is just the conditon that the marginal return of human capital investment equals the marginal return of financial investment (times the opportunity cost z).[9]

The equilibrium in the model with an interior solution can be visualized in graphs 2.1 and 2.2. Graph 2.1 shows the optimal allocation of human capital investment, the choice of h^*. The downward sloping line is the marginal return of human capital investment, which decreases as h increases. Individuals increase h up to the point where the marginal return of human capital investment (divided by its opportunity cost, z) equals the marginal financial return, which does not depend on h.

Given the optimal choice of h, we can then determine the slope of the intertemporal budget constraint, which is:

$$(1 + r) = w'[e(q,h^*)].e_h(q,h^*)/z$$

Maximization of utility subject to this intertemporal constraint will then determine the optimal values of c_t and c_{t+1}, and therefore the optimal savings, s^*_t. This is represented in Graph 2.2.

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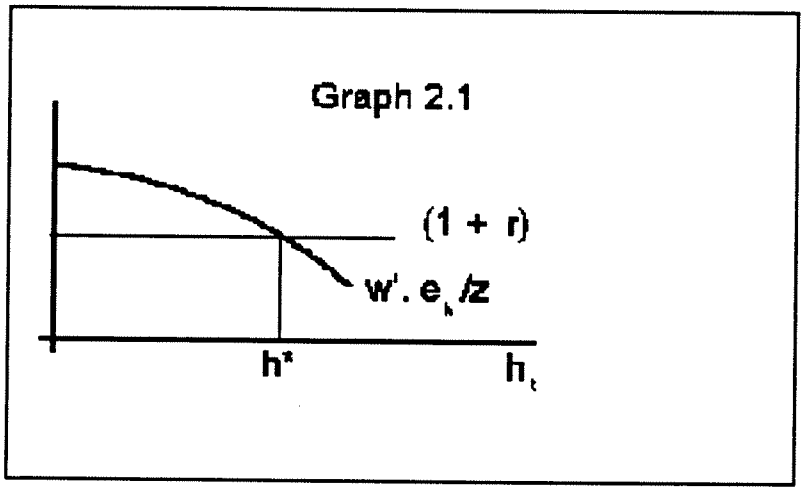
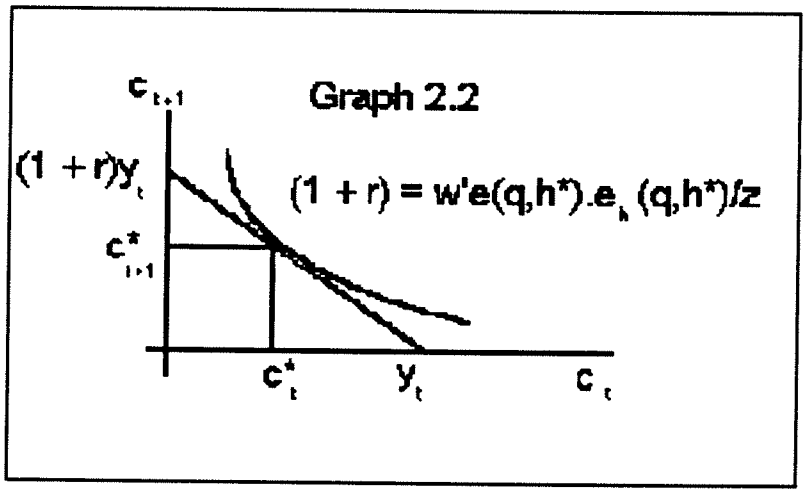
[9]. Again, this is consistent with Becker and Tomes,1976.


Graph 2.1
(1 + r)
w'. e_h /z
h*
h_t


c_t+1
Graph 2.2
(1 + r)y_t
(1 + r) = w'e(q,h*).e_h (q,h*)/z
c*_t+1
c*_t
y_t
c_t

Up to this point, we assumed that families must save a positive amount of their first period income. However, we can argue that, for poor families, this assumption is not adequate, since poor families do not save. They consume all their income, and would even be willing to borrow money in order to increase first period consumption, and reduce present poverty. So, for these families (if we assume that they do not have access to a credit market) , the optimum solution will be a corner solution, with $s_t = 0$[10]. In this case, the optimum conditions become[11].

$$U'\{w[e(q,h^{**})]\}/U'(y - zh^{**}) = z(1 + \theta)/w'(.).e_h(q,h^{**}) \quad (8)$$
$$U'\{w[e(q,h^{**})]\}/U'(y - zh^{**}) \leq (1 + \theta)/(1 + r) \quad (9)$$

These two conditions will now imply that:

$$w'(.)e_h(q,h^{**})/z \geq (1 + r) \quad (10)$$

By equation (10) it is clear that, if we have a corner solution with $s_t = 0$ and $h_t > 0$, it must be the case that, in the optimum, the return of human capital investment is greater than the return of financial investment. As families are choosing to transfer resources to the second period only through human capital investment, its return must be greater than the financial return. The graphical representation of the equilibrium is given by graphs 2.3 and 2.4.

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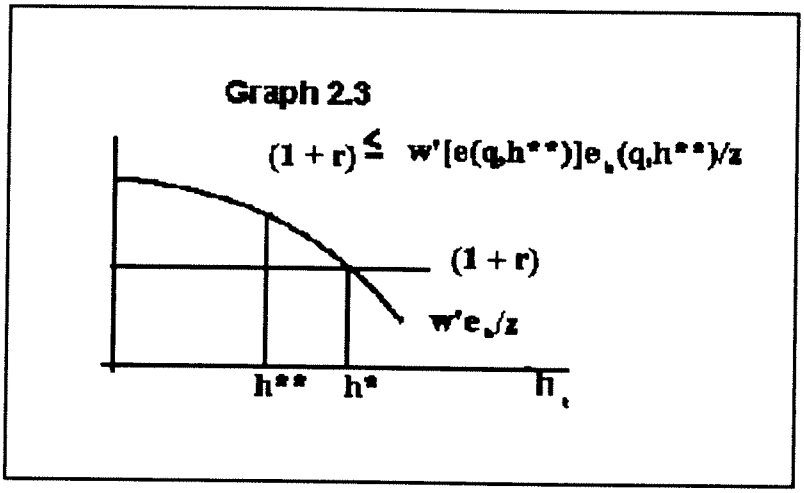
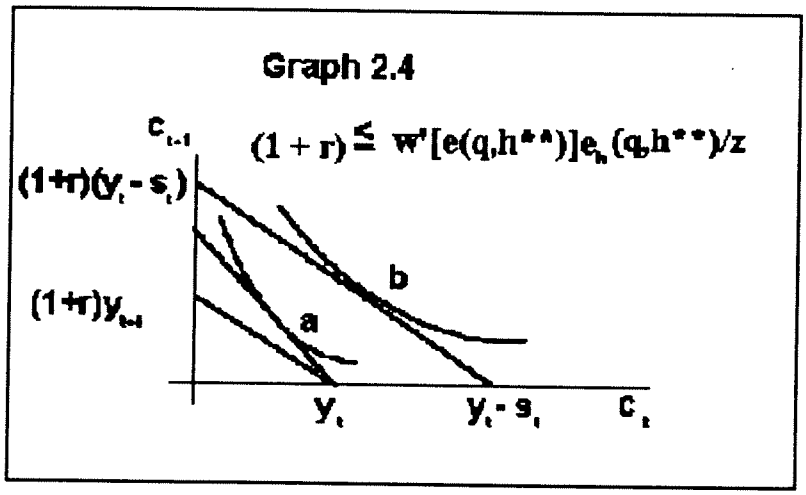
[10]. We are still assuming $h_t > 0$.

[11]. We denote the optimal solutions in this case with two asterisks,to contrast them with the former case,where $s_t > 0$.


Graph 2.3
$(1+r) \leq w'[e(q,h^{**})]e_h(q,h^{**})/z$
$(1+r)$
$w'e_h/z$
h^{**}
h^*
h_t


Graph 2.4
c_{t-1}
$(1+r) \leq w'[e(q,h^{**})]e_h(q,h^{**})/z$
$(1+r)(y_t - s_t)$
$(1+r)y_{t-1}$
b
a
y_t
$y_t - s_t$
c_t

Graph 2.3 shows the choice of h^{**}. For the solution to obey condition (10), given the decreasing returns of human capital investment, it must be the case that $h^{**} < h^{*}$, that is, individuals will invest less in human capital than in the unrestricted maximum (when we allow savings to be negative). If individuals have access to a credit market , they would save negatively in the first period in order to increase first period consumption. They would also increase the investment in human capital, until the return on human capital is equalized to (1 + r). This now represents the interest individuals must pay for the credit obtained in period t. In the graph, this represents a movement from h^{**} to h^{*}.

Given the choice of h^{**}, we can then determine the relevant budget constraint. Note that the slope of this constraint is now greater than (1 + r). The optimal consumption plan is then determined by constrained utility maximazation (point "a" in graph 2.4). If negative savings are allowed, the equilibrium will move to point "b" in graph 2.4. In the new equilibrium, the slope of the budget constraint is again equal to (1 + r), but the negative savings together with the greater return on human capital investment allow for an outward shift on the budget constraint, and a higher utility level (point "b"). Note that the higher utiliy level is achieved through an increase in first period consumption and in the investment on human capital.

These results can be given an interesting interpretation. As poor families have low income, they will have few resources to transfer to the future. These families value highly present consumption and therefore do not save, and invest little in human capital. In the next period, their children will also have low income, and will therefore be likely to face the same situation, with the same consequences. This suggests that poverty, per se, is an important determinant of the performance of the educational system.

The fact that poverty may be an important determinant of the efficacy of the educational system has been stressed elsewhere in the literature. In Durlauf (1993), for example, the marginal product of investment in human capital is a function of the distribution of income in a community. This marginal product increases as the distribution of income becomes more concentrated on higher incomes. A similar effect is present in Bénabou (1991), where rich neighborhoods are characterized by high human capital investment, and poor ones by low human capital investment. Durlauf (1992) shows that the dependence of the productivity of human capital investment on the income distribution of a community can imply the emergence and persistence of permanently poor communities.

However, our paper stresses a somewhat diferent point. Bénabou and Durlauf consider that poor and rich families face different returns for their human capital investment. This paper suggests that, even if the returns are the same, a family with low income is likely to invest less in human capital, due to the fact that it faces a relevant trade-off between investing in human capital and reducing current poverty. This result is extremely important for policy recommendations. If the returns on human capital investment for the poor are smaller than the returns for the non poor, an increase in the quality of the educational system which attend the poor could

increase human capital investment by them. However it will be shown in section 4 that, even if the increase in quality is guaranteed to increase the return on human capital, it is not certain that poor families will increase human capital investment. This happens precisely due to the trade off mentioned above.

3. Externalities, Human Capital Investment and Policy

In the previous section, it was shown that families that do not save, which have been defined as poor families, tend to invest less in human capital than families which save (defined as non-poor families). This would in principle open room for policies aimed at increasing human capital investment. However, it could convincingly be argued that the socially optimal level of human capital investment may be greater than the level which equals its private marginal return to its private marginal cost. This happens because education might be subject to important externalities.[12] Two important sources of externalities related to education are:

- externalities in the production functions of the economy. Working with more qualified workers may increase the productivity of a given worker. This is the idea implicit in the O-ring production function (see Kremer M., 1992.). Thus more investment in human capital would increase not only the productivity of the individual worker who became more qualified, but also the productivity of those who work with him;

- externalities in the learning tecnhology itself. Smarter classmates may increase the productivity of the time dedicated by an individual to school.

These ideas imply that it could be desirable to increase human capital investment beyond the level chosen by individuals. This would be true, in the context of our model, even for families that save.

It could be argued that human capital investment is not the relevant policy variable in education. If the objective is to reduce poverty levels, the policy should focus on the total qualification of workers, or what we called human capital accumulation. This could be obtained even without changing human capital investment. If the quality of the educational system increases, individuals could become more qualified even with less human capital investment. In the model, this is represented by the fact that, if q increases, e(q,h) could increase even with a decrease in h.

However, if there are in fact externalities involved in the education process, it will be always socially desirable to increase human capital investment, irrespective of the quality of the school (h would remain sub-optimal from a social point of view). More importantly, there may be other benefits coming from human capital investment, that are not directly linked to human capital accumulation. The education process is

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[12]. The fact that human capital investment may generate externalities has been stressed elsewhere in the literature. See, for an example, Lucas (1988).

a socializing process, important not only from an economic point of view. Society may value highly the fact that children are effectively at school, irrespective of the amount of human capital they accumulate.

These considerations suggest that human capital investment is in fact an important policy variable per se. This paper will thus focus on this variable for policy considerations.

There are two ways by which different policies could affect the family's decision regarding human capital investment:

- a reduction in the marginal cost of human capital investment in relation to its marginal return.
- a reduction in the importance of the trade-off between present consumption and human capital investment.

The first effect affects human capital investment both when savings are positive and when they are not. However, the second effect is only effective if savings are zero. This happens because, if savings are positive, the family can always increase human capital investment by reducing savings, without having to reduce present consumption. Consider equation (7). This equation, which determines the optimal allocation of human capital in the model when savings are positive, does not depend on present income, the discount rate, or any other variables that could affect the trade-off mentioned above. This point will be important in the discussion that follows.

4.Comparison of Policies

In the previous section, it was argued that investment in human capital is an important policy variable. This section will use the model developed in section 2 to compare the effectiveness of three different policies that could affect human capital investment by families. These policies are:

- an increase in the quality of the educational system;
- an income transfer, conditional on the family keeping all their children in school (a subsidy for education);
- a general income transfer, not attached to school attendance.

In terms of the model, the first policy will be represented just by an increase in q in the learning technology. The second policy will be represented by a reduction in z, the opportunity cost of going to school. For a given level of human capital investment, h, this means an increase in first period income by dz.h. This policy can be implemented if, for example, the family receives a transfer of s monetary units per time unit (hour, months, year) spent in school. The third policy is represented by an increase in y_t, first period income.

The sensitiveness of human capital investment (h), to each of the three policies can be determined by a differentiation of equations (7) and (8). These equations determine, respectively, the optimal allocation of human capital investment

when savings are positive and when they are not. If savings are positive :

$$\partial h/\partial q = - \{w'e_{qh} + e_h e_q w''\}/\{w'e_{hh} + (e_h)^2 w''\} \quad >< \quad 0 \qquad (11)$$

$$\partial h/\partial z = (1 + r)/\{w'e_{hh} + (e_h)^2 w''\} < 0 \qquad (12)$$

$$\partial h/ \partial y = 0 \qquad (13)$$

By condition (11) it can be said that an increase in the quality of the school has an ambiguous effect on the amount of investment in human capital, condition (12) shows that a subsidy to education increases human capital investment and condition (13) shows that a pure income transfer has no effect on human capital investment.

These results can be readily interpreted. As the income transfer acts only by alleviating the trade-off between human capital investment and first period consumption, it has no effect on h if savings are positive.[13] The reduction in the opportunity cost of going to school, on the other hand, reduces the marginal cost of h, and thus increases investment in human capital.

The quality policy affects the marginal return of human capital investment, and does so in an ambiguous way. Given h, an increase in the quality of the school increases qualification [e(q,h)]. If the income function, w(e), is stricly concave, this decreases the return of human capital investment. However, if there is complementarity in the learning technology ($e_{qh} > 0$), a better school means that an aditional unit of human capital investment becomes more productive. These counteracting effects are precisely the effects of the terms $w''e_h e_q$ and $w'e_{qh}$, in the numerator of equation (11). A sufficient condition for the quality policy to increase human capital investment is that $w'' = 0$, which eliminates the first effect.

If savings are zero, the results can be obtained if equation (9) is treated as an implicit function:

$$F(z,q,h,y) = U'(c_{t+1})/U'(c_t) - z(1 + \theta)/w'(.).e_h(q,h) = 0 \qquad (14)$$

Differentiating $F(z,\theta,q,h)$, we will have[14]:

$$\partial h/\partial y = - F_y/F_h > 0 \qquad (15)$$

$$\partial h/\partial z = - F_z/F_h < 0 \qquad (16)$$

$$\partial h/\partial q = - F_q/F_h >< 0 \qquad (17)$$

Thus, the effect of an increase in income and of a reduction of the opportunity cost of being in school unambiguously increase the amount of human

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[13]. See page 13 above.

[14]. See the appendix for the expressions of F_d, F_h, F_z, and F_q.

capital investment by the family. But the effect of an increase in quality is again ambiguous.

Again, these results are easily interpretable. If savings are zero, the trade-off between human capital investment and first period consumption becomes important. Thus, an income transfer in the first period allows the family to invest more in human capital. The subsidy policy, besides reducing the marginal cost of h, has also the effect of alleviating the trade-off between h and c_t. For a given level of h_t, a reduction in z implies that the family has an extra amount of left-over income to consume in the first period.

An increase in the quality of the school again has an ambiguous effect. However, in this case, even if it is guaranteed that the quality policy increases the return of human capital investment, its effect on h remains ambiguous. The condition for an increase in quality to result in more investment in human capital by families that do not save is that $F_q > 0$, in the numerator of (17). For this to be true, we must have :

$$w'e_{qh} > - \{[w'e_h]^2/(1 + \theta)z\}.\{U''(c_{t+1})w'e_q/U'(c_t)\} - w''e_he_q \qquad (18)$$

So, even if $w'e_{qh} > - w''e_qe_h$ we cannot guarantee that an increase in quality will increase human capital investment. The problem here is that, as savings are zero, for a given level of h, an increase in the quality of the school must imply an increase in consumption in period t + 1. As marginal utility of consumption is declining, the marginal utility of higher consumption in period t + 1 declines in relation to consumption in period t. As a result, there is a relative reduction in the demand for consumption in period t + 1 which, in the model, means a reduction in the demand for human capital investment. This effect can be seen through the expression:

$$U''(c_{t+1}).w'.e_q/U'(c_t) = \partial[U'(c_{t+1}/U'(c_t)]/\partial q < 0 \qquad (19)$$

Before the increase in the quality of the school, the family was already willing to trade c_{t+1} for c_t (saving negatively). So, if the quality of the school increases, families may use this increase to reduce the total costs of the human capital investment (z.h), free their children to enter the labour market and increase current income.

This means that, for families that do not save, there is an extra reason for which the effects of the quality policy are ambiguous. Even if better school quality policy increases the marginal return of human capital investment, the trade-off between first period consumption and human capital investment may lead these families to decrease this investment in response to increases in the quality of the school. If it is accepted that zero savings is a relevant characterization of poor families, this is an obviously important policy result. Policies that involve increasing the quality of the educational system should be evaluated with special care when

aimed at the poor[15].

It is important to stress that, as the quality of the school has increased, a reduction in h does not mean the family's children will become less qualified. It turns out that, in this model, although human capital investment may respond negatively to school quality improvement, human capital accumualation can be guaranteed to increase. The decrease on the time spent by children on school cannot be large enough so as to offset an increase in the quality of the educational system.

To see this, just diferentiate e(q,h) with respect to q, to get:

$$de(q,h)/dq = \partial e/\partial q + (\partial e/\partial h).(\partial h/\partial q) \qquad (20)$$

Expression (20) is valid for both models. Substituting for the value of $\partial h/\partial q$ when $s > 0$,we get:

$$de(q,h)/dq = e_q - e_h\{[w'.e_{qh} + e_h e_q w'']/[w'.e_{hh} + (e_h)^2.w'']\} \qquad (21)$$

Expression (21) can be rearranged to yield:

$$de(q,h)/dq = (e_q w' e_{hh} - w' e_h e_{qh})/[w' e_{hh} + w''(e_h)^2] > 0 \qquad (22)$$

Thus, he total derivative of human capital accumulation with respect to school quality is always positive. We show, in the appendix, that the same result is valid when we are in a corner solution, with $s = 0$. The subsidy policy (reduction in z) and the income transfer policy (in the case of zero savings) can also be guaranteed to increase human capital accumulation. Their effect on human capital accumulation is entirely due to their positive effect on human capital investment, and therefore is always positive.

However, it can be proved that, for a given level of government expenditure, the subsidy policy increases human capital investment more than the pure income transfer policy[16]. Formally, if :

$$dy = - h^{**}.dz \qquad (23)$$

Then :

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[15]. Note that the increase in quality does not affect the opportunity price of human capital investment. It could be argued that many policies that increase school quality also increase this opportunity cost, for example, if it is obtained through an increase in the time students must stay in the school or in the time they devote to school tasks at home. If that is so, the above negative effect is increased.

[16]. See apendix for a formal proof of this result.

$$(\partial h/\partial y)dy < (\partial h/\partial z)dz \qquad (24)$$

This result is due to the fact that, while the income transfer policy acts only by alleviating the trade-off between human capital investment and present consumption, the subsidy policy also reduces the marginal cost of human capital investment. If $dy = -h^{**}.dz$, the positive effect on the trade-off is the same for both policies. However, the subsidy policy still has the effect of inducing an increase on human capital investment by reducing its marginal cost.

5.Conclusions

This paper analyses the effect of a policy to increase the quality of the educational system as compared to a policy of subsidizing the time children spend on school, on human capital investment (time spent in school) and on human capital accumulation (educational level). The results of the simple model developed in this paper raise important points about the relationship between poverty and the performance of the educational system. If it is accepted that non-saving is a good characterization of poor families, then it was shown that poor families tend to underinvest in human capital, compared to non-poor families (families which save positively). this may cause a vicious circle of poverty. Poor families' children, with low level of human capital accumulation, will also tend to underinvest in human capital, and poverty may perpetuate.

In this context, policies aimed at increasing human capital investment are socially desirable. this may be true even for families that save, as the education process is subject to important externalities. Furthermore, in a less pecuniary sense, society may value highly the fact that children are effectively at school, irrespective of human capital accumulation.

Three policies were analysed, which could increase human capital investment. It was shown that a subsidy for education, by reducing the marginal cost of human capital investment, is always effective (both, when savings are zero and when ther are positive). In particular, this is a more effective policy than a pure income transfer, not linked to school attendance.

An increase in school quality has, in our model, an ambiguous effect on human capital investment. If savings are positive, this is due solely to the fact that this increase has an ambiguous effect on the marginal return of human capital investment, given the concavity of the income function. However, if savings are zero, even if it can be garanteed that the marginal return of human capital investment will increase, the ambiguity remanins. This happens because families that do not save face a relevant trade-off between human capital investment and first period consumption. They can only increase human capital investment at the cost of becoming poorer in the present. Therefore, they might choose to reduce human capital investment in response to an increase in school quality. This will not mean a reduction in the qualification level of these children in the future (what we call human capital accumulation), as we prove in the paper. However,if human capital investment is an important policy variable itself, this result has important policy implications.

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Appendix

Differentiating $F[z,y,q,h(z,y,q)] = 0$:

$$F_h = [U'(c_t)U''(c_{t+1})w'e_h + zU''(c_t)U'(c_{t+1})]/[U'(c_t)]^2 + [z(1+\theta)][w'e_{hh} + w''(e_h)^2]/[w'(e_h)]^2 < 0$$

$$F_y = -[U'(c_{t+1})U''(c_t)]/[U''(c_t)]^2 > 0$$

$$F_z = [U'(c_{t+1})U''(c_t)h]/[u'(c_t)]^2 - (1+\theta)/w'e_h < 0$$

$$F_q = [U''(c_{t+1})w'e_q]/U'(c_t) + [z(1+\theta)][w'e_{qh} + w''e_he_q]/[w'e_h]^2 \gtrless 0$$

And using the implicit function theorem, we have:

$$\delta h/\delta y = -[\delta F/\delta y]/[\delta F/\delta h] > 0$$

$$\delta h/\delta z = -[\delta F/\delta z]/[\delta F/\delta h] < 0$$

$$\delta h/\delta q = -[\delta F/\delta q]/[\delta F/\delta h] \gtrless 0$$

Thus, an increase in first period income or a decrease in the opportunity cost of being in school, increases the demand for hours of education. On the other hand, the effect of quality over the time spent on school is ambiguous.

Now, we prove that an increase in quality can be guaranteed to increase human capital acummulation, even when the savings constraint is binding.

Defining :

$g(q,h) = w[e(q,h)]$, we can write :

$$\delta h/\delta q = \{-U''(c_{t+1})U'(c_t)g_q(g_h)^2 - [U'(c_t)]^2z(1+\theta)g_{qh}\} / K \text{ , where:}$$

$$K = (g_h)^3U''(c_{t+1})U'(c_t) + U'(c_{t+1})U''(c_t)z(g_h)^2 + z(1+\theta)g_{hh}[U'(c_t)]^2 < 0$$

Now,using (20) we can write:

$$de/dq = \{ e_qU''(c_t)U'(c_{t+1})z(g_h)^2 + z(1+\theta)e_qg_{hh}[U'(c_t)]^2 - z(1+\theta)e_hg_{qh}[U'(c_t)]^2 \} / K \qquad (25)$$

For expression (25) to be always positive we need:

$$e_hg_{qh}z(1+\theta)[U'(c_t)]^2 > e_q(g_h)^2zU'(c_{t+1})U''(c_t) + e_qg_{hh}z(1+\theta)[U'(c_t)]^2 \qquad (26)$$

Using again the definition of g(.), we can rewrite condition (26) as:

$$[w'e_h e_{qh} - w'e_q e_{hh}]z(1+\theta)[U'(c_t)]^2 > e_q(g_h)^2 z U'(c_{t+1})U''(c_t) \qquad (27)$$

Condition (2730) is always satisfied. So, de/dq is always positive, irrespective of the binding constraint.

Now we prove that, for a given level of government expenditure, teh subsidy policy increases human capital investment by more than the pure income transfer policy. Defining:

$$dh = [\delta h/\delta y].dy$$
$$dh' = [\delta h/\delta z].dz$$

we will prove taht, for negarive dz, if (23) holds,

$$dh' - dh > 0.$$

Using the expressions avove, for $\delta h/\delta y$ and for $\delta h/\delta z$, we have:

$$dh = U'(c_{t+1})U''(c_t)/[U'(c_t)]^2\, dy \qquad (28)$$
$$dh' = [(1+\theta)/w'e_h F_h]dz - \{U'(c_{t+1})U''(c_t)/[U'(c_t)^2 F_h\}\, dy \qquad (29)$$

Remember that F_h stands for the derivative of (14) with respect to h. So,

$$dh - dh' - [(1+\theta)/w'e_h F_h]dz - \{U'(c_{t+1})U''(c_t)/[U'(c_t)]^2\} - \{U'(c_{t+1})U''(c_t)h^{**}/[U'(c_t)]^2 Fh\}dz \qquad (30)$$

If $h^{**}dz = -dy$, and $dz < 0$, the last two terms of expression (30) cancel out, and we have:

$$dh - dh' = (1+\theta)/w'e_h F_h\, dz > 0.$$

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