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MULTIPLE EQUILIBRIA AND PROTECTIONISM

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Multiple Equilibria and Protectionism

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Abstract:

This paper provides a novel perspective on the dynamics of infant industry protection. In a normative approach, trade policies are analyzed when the industrial sector generates positive externalities in production, and there are adjustments costs to changing production between sectors. If the government is able to fully commit to its tariff schedule for the future, the welfare maximizing policy is to maintain a positive tariff forever. However, if the government is not able to commit, the only time consistent policy is zero tariff always. In the intermediary case, i.e., when the government can commit for a limited period of time, the time consistent optimal tariff will be positive but lower than the "full-commitment" tariff. This result indicates that some institutions that have always been considered sources of inefficiency, such as protectionist lobbying, may in fact be welfare improving in some cases!

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Índice

1. Introduction

The infant industry argument for protection has been in the literature since the 18th Century. The basic classical argument states that the private rate of return on investment in the industrial sector is below the social rate, therefore the investment level is less than optimal. If industrialization improves productivity in the sector so that the private and social rates of return become equal, then infant industry protection may be welfare enhancing. If industrialization does not equalize private and social marginal returns, protection would have to be permanent to ensure a welfare maximizing investment level, rather than as an infant industry one.

A revival of interest in this topic occurred in the 1950's, with the introduction of two new arguments for infant industry protection. The first one is based on the idea of external economies. Each investment may affect the profitability of other investments, but this is not internalized in the investment decision process. Here, again, the level of investments will not be socially optimal, and infant industry protection could enhance welfare, provided that once the economy reaches its new equilibrium the removal of the tariffs would not have efficiency distorting effects. This can be viewed as a revision of the basic classical argument, where a structure is given to the difference between social and private rates of return on investment.

The second argument relies on a dual economy approach, in which the economy has two sectors: agriculture and industry. Initially, most labor is in the agricultural sector. Wages paid in industry would have to be greater than wages in agriculture to attract workers to the former sector, because workers live in the rural areas and would be reluctant to move to the cities. Thus, the wage differential between the sectors would not reflect the difference in their marginal value product of labor. Hagen (1958) maintains that tariffs or subsidies are necessary to ensure that the economy will not remain in the (socially inferior) agricultural equilibrium. Kenen (1963) argues that in a dynamic perspective one can show that tariffs or subsidies can lead to a faster transition to the industrial equilibrium, but the lack of them does not necessarily mean that the economy specializes in agriculture permanently.

The model presented in this paper incorporates the arguments stated in the two previous paragraphs. There is an economy with two sectors, industry and agriculture, and external economies make the social and private cost of factors in industry diverge. The externality enters through workers' labor allocation decisions: each worker perceives only her private benefit and does not internalize the effects of her decision on the income of the others. Therefore, although workers are rational, the rate of labor movement between sectors will differ from the socially optimum one. The dynamics of the economy are derived from the introduction of labor adjustment costs, which make labor movement sluggish, rather than instantaneous. The first best policy would be to introduce a production subsidy that makes the workers internalize the benefits from moving into the industrial sector. The subsidy should be paid by lump sum transfers from the consumers. In the absence of this policy instrument, trade policy can be used as a second best policy to correct the problem. Similarly to Kenen (1963), this paper shows that, for some initial allocations, the economy may reach the long run industrial equilibrium with no government intervention, but tariffs can make the economy move at a (welfare improving) faster rate.

This paper analyses trade policy in a dynamic perspective and under different assumptions about the government's commitment capability with respect to future policy. The main new result is that under the conditions that the earlier literature claimed would require infant industry protection, it may actually be a second best policy to maintain import tariffs or export subsidies forever, even after the industrialized equilibrium is reached. This is true in a dynamic setting where agents have free access to financial markets and trade policy announced by the government for the whole future has full credibility from the private sector. The government's commitment capability and the infinitely lived agents' free access to financial markets are crucial to this result. The combination of these assumptions allows the setting of a tariff schedule to be aimed at smoothing consumption over the whole future. In the other extreme case, when the government's announcement has no credibility at all, the only time consistent policy is a zero tariff in all periods.[1] Only in the intermediate case, when the government can commit to its policy for a limited number of periods, does a sort of "infant industry protection" turns out to be the second best policy.

The results of this paper can also be applied to a more general situation when the government could use subsidies, but not lump sum transfers to pay for it. If the government could only use income tax, for instance, it would still face the same trade off: introduce subsidies to deal with the externality problem and bear the distorting taxes, or no subsidies and, therefore, no distorting taxes.

Section 2 introduces the model. In subsection 2.1 the production side of the economy is described, subsection 2.2 analyses the first best policy to deal with the externality problem, and subsection 2.3 presents the consumption side of the economy. In section 3 the optimal tariff schedule is analyzed in full commitment, no commitment, and limited commitment environments. Section 4 presents the conclusions.

2. The model

A country that is small in the international goods and financial markets is considered. Goods prices are exogenous, and all economic agents may borrow or lend freely at the exogenous interest rate r. Each individual in this economy is a producer and a consumer. Her decision problem is analyzed in two steps: as a producer she decides how to allocate her labor endowment among sectors, and then, as a consumer, she chooses how much of each good to consume each period. For simplicity, all economic agents are assumed identical, i.e., they have the same per period labor endowment $\bar{L}$, and they face the same decision problems. The production side of the economy will be studied first.

[1] Karp and Paul (1993) reaches, independently, this same conclusion that with no credibility the best time consistent policy is zero tariffs in all periods in a situation similar to the one presented in this model, but with no financial markets.

2.1. Production

The basic framework of the production side of the economy is based on Krugman (1991). There is an economy with two sectors, agriculture and industry, which produce goods A and N, respectively. The only factor used in the production of both goods is labor. The agricultural sector presents constant returns to scale, and units are chosen so that one unit of the agricultural output requires one unit of labor. The industrial sector exhibits increasing returns to scale, external to the firm. For simplicity, the externality function is presented as being linear in total labor allocated to the sector ($L*$).

$$N = E(L*)L = (\alpha + \beta L*)L \tag{1}$$

where N and L are the industrial sector output and labor allocation, respectively, of a representative agent, and α and β are constants. Note that, because workers are identical, $L* = nL$, where n is the total number of individuals.

Prices are normalized so that the prices of agricultural and industrial goods are, respectively, $p_A = 1$ and $p_N = p$. It is straightforward to see that if inequalities (2) hold there will be multiple long-run equilibria in this economy.

$$pE(n\overline{L}) > 1 \text{ and } pE(0) < 1 \tag{2}$$

When workers allocate all their time to the industrialized sector, the value of the marginal product of labor in that sector will be larger than in the agricultural sector, and workers will not want to change their allocation. An analogous situation holds when everyone works only in the agricultural sector. A third possibility is that the labor allocation is such that $pE(n\,L) = 1$. This will also be an equilibrium, but an unstable one: any variation in the labor allocation will make the economy go to one of the other two equilibria. If there were no cost of adjustment to labor, there would be another set of equilibria where the economy alternates among periods in each of the three equilibria. The only constraint is that the labor must move at the same time to the same equilibrium.

Dynamics are added to the system by introducing some cost for the labor to move from one sector to the other. Labor will not relocate instantaneously, but will follow some law of motion, governed by the adjustment cost combined with the externality. Following Krugman (1991), the cost is taken to be quadratic in the per period adjustment size:[2]

$$\frac{\left(L_{\tau+1}-L_{\tau}\right)^2}{2\gamma} \tag{3}$$

where γ is a constant and L_τ is the amount of labor per worker allocated to industry at time τ.

Each worker chooses how to allocate her per period labor endowment between the two sectors maximizing the present value of output,

$$\max \sum_{\tau=0}^{\infty}\left(\frac{1}{1+r}\right)^{\tau}\left[pE\left(L_{\tau}^{*}\right)L_{\tau}+\left(\bar{L}-L_{\tau}\right)-\left(u_{\tau}\right)^{2}/2\gamma\right] \tag{4}$$

$$\text{subject to } u_\tau = L_{\tau+1}-L_\tau \tag{4.1}$$

$$0 \le u_\tau + L_\tau \le \bar{L} \tag{4.2}$$

where u_τ is the control variable, and $\bar{L}-L_\tau$ represents the amount of labor the worker allocates in agriculture.

When making their labor allocation decision, workers do not perceive the effect of their decision on the productivity of labor in the industrial sector, which, in turn, will affect the income of every worker. Appendix A presents the solution to the maximization problem above. Let ξ_τ denote the Lagrange multipliers for constraints (4.2). They will assume the value zero when the constraints are not binding, and some positive value otherwise. To solve the problem a future value co-state variable, λ_τ, is introduced. The expression $\frac{\lambda_\tau}{(1+r)^{\tau-1}}$ represents the value today of one more unit of labor in the industrial sector at time τ, and corresponds to the multipliers for constraints (4.1). Along the optimal path, the change in the co-state variable is equal to the

[2]Mussa (1978) justifies this form of adjustment cost by introducing a moving industry which requires resources to move labor from one sector to the other and presents decreasing returns to scale. Here, the movement cost is motivated by "educational costs". In each sector the individual has to perform different tasks. The longer she works in a sector, the more tasks she has to perform. There could be introduced an educational industry which requires resources to teach tasks to the workers and presents decreasing returns to scale.

difference between its rate of return and the private value of one more unit of labor in the industrialized sector.

$$\lambda_{\tau+1} - \lambda_{\tau} = r\lambda_{\tau} - d_{\tau} \tag{5}$$

where $d_{\tau} = p(\alpha + n\beta L_{\tau}) - 1$.

Until an equilibrium is reached, labor will move at any time τ proportionally to the value of the co-state variable.

$$L_{\tau+1} - L_{\tau} = \gamma\lambda_{\tau+1} \tag{6}$$

Depending on the initial value of labor allocation and the parameters of the economy, equilibrium conditions given by the two equations of motion above and transversality conditions, derived in appendix A, may lead the economy either to the equilibrium with specialization in industry, or the one with specialization in agriculture. Equation (7) presents the terminal condition for the co-state variable if the economy is heading to the industrialized equilibrium, and equation (8) gives the terminal condition if it is heading to the agriculture one.[3]

$$\lambda^{N} = \frac{p(\alpha + n\beta\bar{L}) - 1}{r} \tag{7}$$

$$\lambda^{A} = \frac{p\alpha - 1}{r} \tag{8}$$

It is straightforward to see that the labor movement resulting from a free market is different from that chosen by a central planner, because of the externality in production. The workers underestimate the benefit from moving into the industrial sector, because they do not account for the effect of their movement on the productivity of the sector, and therefore on the income of every worker. The central planner's solution to the maximization of income implies labor movement as

[3]See appendix A for the derivation of equations (7) and (8). The terminal conditions here are different from those derived in Benabu and Fukao (1993) for Krugman's model. The difference is due to their interpretation of the Krugman model as having each worker producing either the industrial or the agricultural good (not both, as in the present model). The moving cost, then, depends on the total number of workers changing sectors, so that it becomes another externality source in the model.

the same function of the co-state variable as in equation (6), but the movement of the co-state variable is given by equation (9) instead of (5).

$$\lambda_{\tau+1} - \lambda_{\tau} = r\lambda_{\tau} - d_{\tau}^{P} \qquad (9)$$

where $d_{\tau}^{P} = p(\alpha + 2n\beta L_{\tau}) - 1$ is the difference between the value of the marginal product of labor in the two sectors as perceived by the central planner. Note that the value for the central planner of one more unit of labor in industry at each period (d_{τ}^{P}) is greater than the value for the worker (d_{τ}).

The central planner's terminal conditions for the co-state variable if the economy is heading to the industrialized or agricultural equilibria are given by equation (10).

$$\lambda^{N} = \frac{p(\alpha + 2n\beta \overline{L}) - 1}{r} \qquad (10)$$
$$\lambda^{A} = \frac{p\alpha - 1}{r}$$

To help the comparison between the free market and central planner solutions to the problem, a graphical interpretation of the dynamics is presented.

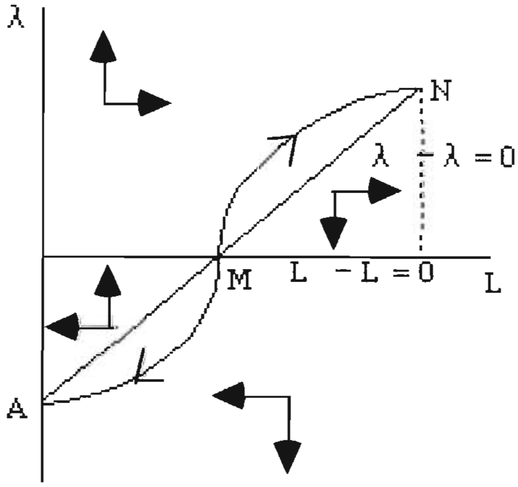

Figure 1

Figure 1 shows the shape of the paths leading to the long run equilibria, derived from the system of difference equations defined by the first order conditions from the workers' decision (equations (5) and (6)), and the terminal conditions (equations (7) and (8)). The variables will follow discrete points along the paths. The continuous path is the limit as the time interval between periods (which is taken as being '1') goes to zero.

The two roots of the system, $\mu_1 = \frac{r + \sqrt{r^2 - 4\gamma\beta p}}{2}$ and $\mu_2 = \frac{r - \sqrt{r^2 - 4\gamma\beta p}}{2}$, are positive; therefore the system will diverge from its only singular point, represented by M in the figures. Point M is a source, that is, all the characteristics of the system originate from that point, moving away from it. The labor boundary conditions create two stable equilibria: A and N. At N, the value of co-state variable is positive (from (7)), that is, the value today of one additional unit in the industrialized sector at the time the economy is specialized in its production is positive. Hence, the workers would be willing to allocate more of their labor endowment to that sector. As they have no more labor available, they do not move from that point. Point A is analogous, with a negative value of the co-state variable. These points are denoted "corner" solutions.

For an initial labor allocation, the value of the co-state variable λ_τ will determine the path the economy will follow. In other words, the value of one more unit of labor in the industrial sector (λ_τ) will determine how much labor moves, and, together with the current amount of labor in this sector, it will determine how much its own value changes. The shape of the path in the figure represents the dynamics when the roots of the system are real.[4]

For initial labor allocations to the right of point M the productivity in industry is high enough so that λ_1 assumes a positive value, and the economy heads to the industrialized equilibrium. The opposite is true for initial labor allocations to the left of point M.

Now the central planner's solution is compared to the free market dynamics. The $L_{\tau+1} - L_\tau = 0$ schedule is the same in the two cases, since it must coincide with $\lambda = 0$ in both cases. But the value of one more unit of labor in the industrial sector is greater for the central

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[4]Krugman (1991) presents an interesting interpretation of the dynamics when the roots of the system are imaginary.

planner for each allocation of labor. Therefore the $\lambda_{\tau+1} - \lambda_{\tau} = 0$ line for the central planner is located up and to the left compared to the market one.

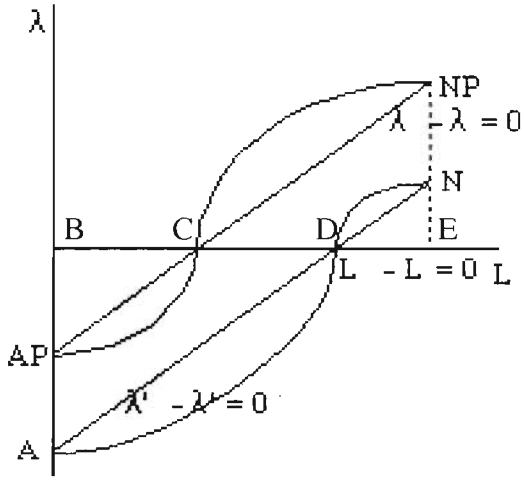

Figure 2

In figure 2, jP and j, for j=A, N, represent the equilibrium points where the economy specializes in sector j resulting from the central planner and free market dynamics, respectively.

The marginal product of labor in industry for the central planner includes the effect of labor movement on the productivity of every worker, which is not taken into account by the individual worker. Therefore for each labor allocation the value of the marginal product of labor in industry is higher for the central planner than for the workers. If the initial labor allocation laid between points C and D in figure 2, then this difference would be decisive to determine which long run equilibrium were to be reached. The central planner would lead the economy to the industrialized equilibrium, whereas the free market would specialize in agriculture.

For initial labor allocation between points B and C, there is so little labor in industry that even the central planner considers not worth moving into that sector. Both the central planner and the free market would go to the equilibrium with specialization in agriculture, however the central planner would do so at a slower rate.

Finally, if the initial labor allocation were to the right of point D, central planner and the free market would head to the industrialized equilibrium. The central planner would move at a faster rate due to his higher valuation of the marginal product of labor in industry for each labor allocation.

2.2. First Best Policy

The first best economic policy to implement the central planner's solution is to introduce a production subsidy paid by lump sum transfers from the consumers. The value of the subsidy at each point in time should be the one that makes the producer follow the central planner's equilibrium path, given the initial labor allocation. The subsidy would make the workers' perception of the value of shifting labor between sectors equal to that of the central planner, and it would have no other effect in the rest of the economy. Comparing the equations that define the dynamics of the equilibrium path for the free market (equations (5), (6), (7) and (8)), and those for the central planner (equations (6), (9) and (10)), it is straightforward to check that the subsidy at some time τ is such that the price faced by the producers is the one represented in equation (11).

$$p^P = p\left(1 + \frac{\beta L_\tau^*}{\alpha + \beta L_\tau^*}\right) \tag{11}$$

For the range of initial labor allocation within which the free market would lead the economy to the agricultural equilibrium, whereas the central planner would follow the path to the industrial equilibrium, economic policy would increase the present value of income not only by setting the first best labor movement each period, but also by *reversing* the direction of labor movement and leading the economy to the better long run equilibrium. If the economy were already heading to the industrialized equilibrium with no policy intervention, the subsidy would make it move at a faster rate. On the other hand, if the initial labor allocation were so far away from the industrialized equilibrium that even the central planner would prefer to move towards the agriculture equilibrium, the subsidy would cause it to happen at a (welfare improving) slower rate than under the free market.

If the initial labor allocation lies between points B and C, the value of the subsidy will decrease until it reaches zero when the agricultural equilibrium is reached. For initial labor allocations to the right of point C, the value of the subsidy increases as the industrial equilibrium approaches, and remains positive and constant at the long run equilibrium point. This last result relies on the fact that the labor allocation at any point in time depends on the whole path of subsidy values. As shown in appendix A, the value of one more unit of labor in industry next period, $\lambda_{\tau+1}$, can be written as:

$$\lambda_{\tau+1} = \sum_{i=\tau+1}^{\infty} \frac{d_i}{(1+r)^{i-\tau}} \tag{12}$$

where $d_i = p(1+s_i)(\alpha + n\beta L_i) - 1$, and s_i is the subsidy.

Each d_i is a function of the tariff and labor allocation during period i. Hence, each $\lambda_{\tau+1}$ can be written as a function of all tariffs and labor allocations from period τ on. Thus, using equation (6), each period's labor movement can also be written as a function of all tariffs and labor allocations from period τ on.

$$L_{\tau+1} - L_\tau = \gamma \sum_{i=\tau+1}^{\infty} \frac{d_i}{(1+r)^{i-\tau}} \tag{13}$$

This means that a change in the value of the subsidy even after the economy has reached the equilibrium point would imply in a change of the whole path taken (if predicted, of course). The equilibrium conditions require that at the industrialized equilibrium the subsidy is positive and constant, so as to make the terminal condition for the market's co-state variable equal to the central planner's one. However, the subsidy can be set to zero when time goes to infinity. As proven in appendix B, $\frac{\partial PVI}{\partial s_{T+\tau}} = (1+r)^{-\tau} \frac{\partial PVI}{\partial s_T}$, where PVI is the present value of income and T is the time the industrialized equilibrium is reached, therefore $\frac{\partial PVI}{\partial s_{T+\tau}} \to 0$ when $\tau \to \infty$, so that the first order condition for the maximization of income is satisfied for any value of subsidy set as time goes to infinity.

This paper considers the situation in which trade policies are the only instruments available to the policy makers. An import tariff and export subsidy of equal magnitude to the production subsidy described above would mimic its effect from the producers' point of view, but it would also affect the consumption decision, causing utility diminishing distortions. Therefore, to derive the optimal trade policy, it is necessary to study its effects on the welfare of the economy as a whole, not only on its production side.

2.3. Consumption

The government will set some import tariff/export subsidy schedule, taking as given the production and consumption decisions of the individuals. Workers and consumers face the price $p(1+v_\tau)$ for the industrial good, where v_τ is the value of the ad valorem import tariff/export subsidy imposed at time τ. Workers choose how to allocate their labor between the two sectors, and with the income they receive they decide how much to consume of each good. Each worker is taken to be identical, as they face the same decision problem, and therefore make the same decisions. They are taken to be identical consumers as well, by assuming they have the same utility function. Due to the symmetry of the problem, maximizing welfare is equivalent to maximizing utility for the representative consumer, which is represented by:

$$U^i = U = \sum_{\tau=0}^{\infty} \frac{1}{(1+\theta)^\tau}\left[a\log C_\tau^N + b\log C_\tau^A\right] \tag{14}$$

where C_τ^j is the consumption of good j at time τ, and θ is the discount rate.

The consumer has free access to the international financial market at the world interest rate r. In each period the increment to her asset holdings equals the interest on assets held at the beginning of the period, plus the difference between her total income and her total expenditures, plus any transfer received from, or given to, the government. No assets are held at the start of the initial period, by assumption.

$$f_{\tau+1} - f_\tau = rf_\tau + I_\tau^P - p(1+v_\tau)C_\tau^N - C_\tau^A + R_\tau$$
$$f_0 = 0 \tag{15}$$

where total income received from production is $I_\tau^P = p(1+v_\tau)N_\tau + A_\tau - \frac{\gamma(\lambda_{\tau+1})^2}{2}$, f_τ is the amount of assets the consumer holds at the start of period τ, and R_τ is the lump sum transfer from the government at each period. The government's balanced budget requires that $\sum_{i=0}^{\infty}(1+r)^{-i}\left[pv_i(C_i^N - N_i) - R_i\right] = 0$.

The following Hamiltonian is used to solve the problem:

$$H = \left(a\log C_\tau^N + b\log C_\tau^A\right)(1+\theta)^{-\tau} + + \bar{y}_{\tau+1}\left[rf_\tau + I_\tau^P - p(1+v_\tau)C_\tau^N - C_\tau^A + R_\tau\right] \tag{16}$$

where $\bar{y}_{\tau+1}$ is the co-state variable, which is interpreted as the value today of having one more unit of asset at time $\tau+1$.

The first order conditions for the maximization above are:

$$\frac{a}{C_\tau^N} - y_{\tau+1}p(1+v_\tau) = 0 \tag{17.a}$$

$$\frac{b}{C_\tau^A} - y_{\tau+1} = 0 \tag{17.b}$$

$$y_{\tau+1} - y_\tau = \left(\frac{\theta - r}{1+r}\right)y_\tau \tag{17.c}$$

$$f_{\tau+1} - f_\tau = rf_\tau + I_\tau^P - p(1+v_\tau)C_\tau^N - C_\tau^A + R_\tau \tag{17.d}$$

where $y_{\tau+1} \equiv \bar{y}_{\tau+1}(1+\theta)^\tau$ is the future value co-sate variable.

Equation (17.c) shows that the value of y_τ will change over time if, and only if, the interest rate is different from the rate of time preference. As having this variable changing over time would not add any insight to the analysis, $r = \theta$ is assumed from now on. This assumption implies perfect consumption smoothing in both goods.

Furthermore, the transversality condition states that as time goes to infinity either the present value of one more unit of future asset (the co-state variable) goes to zero, or no assets are being held.

$$\lim_{\tau \to \infty} \frac{y_\tau}{(1+\theta)^\tau} f_\tau = 0 \tag{18}$$

Now the model is complete: the decision problem of consumers and producers is solved, and the best tariff schedule can be derived.

3. Optimal Tariff

Three different environments will be considered: the first is when the government is able to commit to its tariff schedule for the future, the second is the opposite case, when the government cannot commit to any future tariff, and, finally, the third situation is when the government has limited commitment, i.e., it can commit only for a certain number of periods. In each case the indirect utility function that the policy maker maximizes when setting tariffs, which is the indirect utility function for the representative individual, is derived.

3.1. Full Commitment

When the government is able to fully commit to its tariff plan, it will decide in the initial period on a tariff schedule for the whole future, maximizing the indirect utility function over that period of time, i.e., forever. First this indirect utility function will be derived.

Equation (19) is derived from equation (17.d) and the fact that no assets are held at the initial time:

$$f_\tau = \sum_{i=0}^{\tau} (1+r)^{\tau-i} \left[I_i^P - p(1+v_i) C_i^N - C_i^A + R_i \right]. \tag{19}$$

Using the government's budget constraint and substituting equations (17.a) and (17.b) into equation (19), the transversality condition (equation (18)) yields:

$$y = \frac{S}{PVI}, \tag{20}$$

where $S = \sum_{i=0}^{\infty}(1+r)^{-i}\left[\frac{a}{1+v_i}+b\right]$ can be interpreted as a measure of the present value of the consumption distortion caused by the tariff, and $PVI = \sum_{i=0}^{\infty}(1+r)^{-i}I_i$ is the present value of income.

The indirect utility function in equation (21) is derived by substituting the value of y into equations (17.a) and (17.b) to get the levels of consumption, and then substituting these into the utility function:

$$V_0(L_\tau, v_\tau) = K + \frac{(1+\theta)(a+b)}{\theta}(\log PVI - \log S) - a\sum_{i=0}^{\infty}(1+\theta)^{-i}\log(1+v_i), \tag{21}$$

where K is a constant term.

Before proceeding with the maximization of the equation above, the effect of tariffs on the present value of income needs to be studied. The present value of income is a function of the labor allocation in each period. Equation (13) shows how each period's labor movement depends on future tariffs and labor allocations. That leads to the following expression for each period's labor allocation, given that the initial labor allocation is L_0:

$$L_\tau = L_0 + \gamma\sum_{j=1}^{\tau}\left[\sum_{i=j}^{\infty}\frac{d_i}{(1+r)^{i-j+1}}\right] \quad \text{for } 1 \le \tau < T \tag{22}$$

From (22), the path of future tariffs affects the labor allocation today. At time T an equilibrium is reached, such that:

$$L_{T+s} = L_0 + \gamma\sum_{j=1}^{T}\left[\sum_{i=j}^{\infty}\frac{d_i}{(1+r)^{i-j+1}}\right] = \begin{cases}\bar{L} & \text{if the upper boundary is reached}\\ 0 & \text{if the lower boundary is reached}\end{cases} \tag{23}$$

for $s \ge 0$

The system of equations represented in equation (22) has a unique solution for a set of tariff schedules. The government will choose the one that maximize welfare.

Given the values of the initial and final labor allocations, L_0 and L_T, and the value of T, the system of equations represented in equation (22) can be solved to get the entire labor allocation stream as a function of the tariffs schedule. The indirect utility function can be written as a function of the tariff schedule only.

$$V_0(L_\tau, v_\tau) = V\left(L_\tau\left(\{v_i\}_0^\infty\right), v_\tau\right) \tag{24}$$

To set the conditions for the welfare maximizing tariff plan, all we need is the tariff schedule that makes the derivative of the indirect utility function equal zero, i.e., that satisfies:

$$\frac{(1+\theta)(a+b)}{\theta PVI^*}\frac{\partial PVI}{\partial v_\tau} = \left[1 - \frac{(1+\theta)(a+b)}{\theta S^*(1+v_\tau^*)}\right]\frac{a}{(1+v_\tau^*)(1+r)^\tau} \qquad \forall \tau \geq 0 \tag{25}$$

where v_τ^* is the optimal tariff at time τ, and PVI^* and S^* represent the value for those variables when the optimal tariff plan is followed.

The left hand side of the equation above represents the effect of the tariff on the present value of income. Producers' decisions determine the present value of income, but those decisions are not optimal from the central planner's point of view because the producers do not consider the effect of their decision on the productivity of industry (review the difference between equations (5) and (9)). A positive value for the tariff will bring the producers' decisions closer to the central planner's solution, thereby increasing the present value of income.

The right hand side represents the loss in utility brought about by the tariff due to the consumption distortion. It depends not only on the current tariff, but on the whole stream of tariffs through S^*. This means that if, for some reason, only the tariff at some particular time t affected the PVI, (i.e., if the l.h.s. were different from zero only in period t), nevertheless the optimal tariff would be different from zero at all times. Because of the concavity of the utility function, consumers would prefer to spread the consumption distortion over the time, instead of having it concentrated in only one period.

The solution to the problem as stated above yields the utility maximizing tariff schedule given that the labor frontier is to be reached in T periods. The problem can then solved for all possible values of T, and the welfare maximizing tariff schedule is the one with a value for T that achieves the highest utility.

The optimal tariff plan is stated in proposition 1.

Proposition 1: Under full commitment $v_\tau^* > 0 \,\forall \tau \geq 0$, and $v_{T+s}^* = v_T^* \,\forall s \geq 0$, where T is the time at which the labor boundary is reached.

Proof: $\frac{\partial PVI}{\partial v_\tau} > 0 \,\forall \tau > 0$, hence equation (25) is satisfied if and only if $v_\tau^* > 0 \,\forall \tau \geq 0$. As for the second statement of proposition 1, Appendix B proves that:

$$\frac{\partial\, PVI}{\partial\, v_{T+s}} = (1+r)^{-s} \frac{\partial\, PVI}{\partial\, v_T} \qquad \forall s \geq 0. \tag{26}$$

Therefore, for the optimal tariff after the labor boundary is reached, equation (25) can be rewritten as :

$$\frac{(1+\theta)(a+b)}{\theta PVI^*} \frac{\partial\, PVI}{\partial\, v_T} = \left[1 - \frac{(1+\theta)(a+b)}{\theta S^*\left(1+v_{T+s}^*\right)}\right] \frac{a}{\left(1+v_{T+s}^*\right)(1+r)^T} \qquad \forall s \geq 0. \tag{27}$$

For the r.h.s. to be positive it is necessary that $1 - \frac{(1+\theta)(a+b)}{\theta S^*\left(1+v_{T+s}\right)} > 0$, which is equivalent to $1 + v_{T+s} > \frac{(1+\theta)(a+b)}{\theta S^*}$. The expression $\frac{(1+\theta)(a+b)}{\theta S^*}$ would equal 1 if the tariff were zero at all periods, and would be greater than 1 for positive tariffs. Therefore $v_{T+s} > 0$. Equation (27) must hold for all v_{T+s}, and the other variables in the equation are constant $\forall\, s \geq 0$, except for v_{T+s}. Therefore, the tariff must be constant over this period of time. Moreover, the left hand side of the equation is constant and strictly greater than zero, so that the tariff must be positive for the right hand side to be also positive.

Q.E.D.

Proposition 1 states that the optimal import tariff/export subsidy will be always positive, moreover it will be constant after the labor boundary is reached.[5]

The result that the optimal tariff will be constant after the boundary is reached relies on the fact that tariffs from that moment on will not affect the present value of income, PVI, by affecting present or future labor allocations, because labor will remain static at the frontier (i.e., $\frac{\partial L_{T+\tau}}{\partial v_{T+s}} = 0 \ \forall \tau \geq 0$ and $\forall s \geq 0$). They affect the present value of income only through labor allocation decisions prior to reaching the boundary. Therefore the effect of tariffs in different periods after time T will be equal except for the different discount applied to each one, i.e., the result in equation (26). If this were not a "corner" solution, this result would not hold, i.e., tariffs at the equilibrium point would be able to affect the position of the economy so that $\frac{\partial L_{T+\tau}}{\partial v_{T+s}} \neq 0 \ \forall \tau \geq 0$ and $\forall s \geq 0$. Tariffs would still be positive at equilibrium, but they would not necessarily be constant.

3.2. No Commitment

When the government is not able to commit to a tariff schedule for the whole future, he will choose at each moment in time the tariff that maximizes welfare from that moment on. The optimal tariff plan under full commitment is not time consistent. Equation (13) shows that at any period τ the decision on how much labor to move into industry for the next period depends only on *future* tariffs. On the other hand, the consumption distortion created by the tariff is contemporaneous to it. Therefore, the government will have an incentive to announce the tariff consistent with the desired rate of labor adjustment, but by the time of its implementation the labor movement would already have taken place, and the best thing to do would be to set zero tariffs. Proposition 2 presents the time consistent solution.

[5]This result refers to the tariff schedule that maximizes welfare. If one seeks for a tariff schedule that just ensures industrialization, many others would do the job, including an infant industry protection.

Proposition 2: With no commitment $v_\tau^* = 0\ \forall\tau \geq 0$.

Proof: First the optimal tariffs schedule after the labor boundary is reached will be derived. The indirect utility function the government will be facing at time T is:[6]

$$V_T(L_\tau, v_\tau) = \bar{K} - \frac{(1+\theta)(a+b)}{\theta}\log S_T - a\sum_{i=T}^{\infty}(1+\theta)^{T-i}\log(1+v_i), \tag{29}$$

where $S_T = \sum_{i=0}^{\infty}(1+r)^{-i}\left[\frac{a}{(1+v_{T+i})} + b\right]$.

The only variable in this new indirect utility function is the tariff. There is no labor movement after the boundary is reached, and therefore tariff changes will not affect the present value of income. The first order conditions for maximization are:

$$\left[1 - \frac{(1+\theta)(a+b)}{\theta S_T^*(1+v_{T+s}^*)}\right]\frac{a}{(1+v_{T+s}^*)(1+r)^s} = 0 \quad \forall s \geq 0. \tag{30}$$

The first term of the product above must equal zero for the condition to be satisfied. The value of the tariff is the only variable in this term, which means that the value of the tariff that satisfies the condition is the same over time, i.e., $v_{T+s} = v_T\ \forall s \geq 0$. Hence, using the assumption that $\theta = r$, the tariff must satisfy $\frac{(1+\theta)bv_T^*}{\theta(1+v_T^*)} = 0$, which will be true if and only if $v_T^* = 0$. Thus, the tariff will be set equal to zero when the boundary of labor supply is reached, regardless the previous tariff schedule.

Taking one step back, the optimal tariff to be set one period before the boundary is reached is derived. At time $T-1$ workers decide how much labor to move into industry based on the value of λ_T (see equation (6)), which in turn will be given by equation (12), and only tariffs after time $T-1$ enter this equation. It is then clear that the value of the tariff at time $T-1$ will not affect labor allocation in that or any future period, and hence will not affect income either. Therefore, the

[6] See the derivation of this equation in appendix C.

condition for maximization of welfare from period's $T-1$ perspective will be analogous to equation (30):

$$\left[1-\frac{(1+\theta)(a+b)}{\theta S_{T-1}^{*}\left(1+v_{T-1}^{*}\right)}\right]\frac{a}{\left(1+v_{T-1}^{*}\right)}=0 \tag{31}$$

where $S_{T-1}^{*}=\frac{a}{1+v_{T-1}^{*}}+\frac{a}{r}+b\left(\frac{1+r}{r}\right)$, given the result above that $v_{T+s}=0\ \forall s\geq 0$.

Again, equation (31) will be satisfied if, and only if, $v_{T-1}=0$. Going backwards in time the same situation arises: at each period the best trade policy from that period's perspective is a zero tariff.

Q.E.D.

The optimal tariff plan in this no commitment case yields a third best outcome. The government lacks the policy instrument that could make possible the achievement of the first best outcome: subsidies. The use of "surprise", or diverging from the pre-announced policy, works as an additional instrument to try to improve on the second best outcome. However, the agents predict this temptation, and act accordingly: they make their decisions expecting zero tariffs in the future. The only time consistent plan is the one with zero tariffs forever.

3.3. Limited Commitment

Instead of the two extreme cases discussed above, it may be more realistic to think of the government as having the ability to commit to a policy for a limited period of time. In a democracy, the government changes periodically. It can try to create rules that make it difficult for the next government to change its policies, but it cannot fully commit to an economic policy after its term in office is over.

I thus ask which would be the best time consistent trade policy under the current model if the government can commit to its policy for some number of periods h, where $h>0$. The dynamics work as follows: at the start of the initial period the government sets a tariffs schedule for

h periods, before the end of the h^{th} period, but after labor allocation decisions are made in period $h-1$, it sets the tariffs for the next h periods, and so on. Proposition 3 summarizes the optimal tariff schedule for this situation.

Proposition 3: If the government is able to commit to its policy plan for h periods, so that each h periods it announces the policy for the next h periods, then $v_\tau^* > 0$ for $\tau < kh$, $k \in N$, $k-1 < T/h$ and $v_\tau^* = 0$ for $\tau \geq kh$, $k \geq T/h$. Moreover, $v_\tau^* > v'^*_\tau$ where v_τ^* is the optimal tariff if the commitment interval is h periods, and v'^*_τ if the interval is h' periods, for $h' < h$, $\tau < \min\{kh, k'h'\}$, and $\max\{kh, k'h'\} < T$.

Proof: To prove proposition 3 the same strategy as in the "no commitment" case will be used: work backwards in time in the model. Given that the long run equilibrium is reached at time T, the indirect utility function which the government will maximize every h periods to choose the tariff schedule for the h-period interval is:[7]

$$V_t(L_\tau, v_\tau) = \overline{\overline{K}} + \frac{(1+\theta)(a+b)}{\theta}(\log PVI_t - \log S_t) - a\sum_{i=t}^{t+h}(1+\theta)^{-i}\log(1+v_i) \tag{32}$$

where $PVI_t = \sum_{i=t}^{T}(1+r)^{-(i-t)}I_i + \frac{I^E}{r(1+r)^{T-t}}$, with I^E the (constant) value of the per period income at one of the long run equilibria, $S_t = \sum_{i=t}^{\infty}(1+r)^{-(i-t)}\left(\frac{a}{1+v_i}+b\right)$, and $\overline{\overline{K}}$ is a constant.

The first order conditions for maximization are:

$$\frac{(1+\theta)(a+b)}{\theta PVI_t^*}\frac{\partial PVI_t}{\partial v_\tau} = \tag{33}$$

$$= \left[1 - \frac{(1+\theta)(a+b)}{\theta S_t(1+v_\tau^*)}\right]\frac{a}{(1+v_\tau^*)(1+r)^{\tau-t}} \quad \forall \tau, t \leq \tau < t+h$$

For $k \geq T/h$ the l.h.s. of equation (33) is equal to zero. Therefore, using the same argument as in the previous section, $v_\tau^* = 0$ for $\tau \geq kh$, $k \geq T/h$. For $k-1 < T/h$ the l.h.s. of

[7]Equation (32) is derived similarly to equation (27). At some time t future utility is maximized, taken as given stocks accumulated until that time, and, here, also given the tariffs after time $t+h$.

equation (33) will be positive, hence positive tariffs will be required for the equation to be satisfied. Finally, the larger h, more elements will be included in the l.h.s. sum, therefore the larger will be the tariff each period over the interval to satisfy equation (33).

Q.E.D.

The welfare resulting from the limited commitment time consistent policy is higher than the no commitment one. Because of the possibility of some commitment to future policy, the government can use, at least partially, trade policy to diminish the loss from the externality problem. But the results still worse compared to the full commitment case, when trade policy can be explored fully to deal with the externality.

4. Conclusion

This paper develops a model in which there are two sectors: agriculture and industry. The industrial sector presents positive externalities in production, and there are adjustment costs to changing production from one sector to the other. The model shows that, although the equilibrium with specialization in the industrial good is strictly better than the equilibrium with specialization in agriculture, the initial labor allocation may have so little labor in industry that the present value of income is maximized with the economy following the path to the agricultural equilibrium. The externality in the production of the industrial good distorts the incentives for the workers to shift labor into that sector. Therefore, the thread point which determines whether the economy should move towards specialization in agriculture or industry is different for the market equilibrium and the central planner's solution, and so is the rate of labor movement. There is a labor allocation range over which the central planner would take the economy to the industrial equilibrium, while the market on its own would head toward agriculture. Thus, economic policy could not only make workers shift their labor at a income improving rate, but could also lead the economy to a different equilibrium than it would go to on its own. The paper analyses how trade policy should be used in this setting.

Perhaps the most striking result of this paper is the one presented in proposition 1, which states that if the government can make credible, indefinite commitments, the first best trade policy is to keep protection forever, even after the long run equilibrium is reached, and even if the equilibrium reached is the agricultural one! The intuition behind this result is the following. Although trade policy can improve incentives on the production side of the economy, it also causes welfare diminishing distortions on the consumption side. The optimal import tariff/export subsidy finds the best balance between the two effects. As our infinitely lived agents can borrow and lend at a given interest rate, and they have strictly concave utility functions, they would like to smooth their consumption. Therefore, instead of higher tariffs in the beginning that fade away with time, as in the subsidy prescription, the best policy here would be lower tariffs during the transition and constant positive tariffs after equilibrium is reached. In this way the proper incentives to the workers would be provided, but the "consumption cost" would be spread over time.

The "tariffs forever" result relies on some strong assumptions that are not generally observed in the real world. Even if the assumption that agents live forever could be justified by saying that each agent cares about his successor's utility, it is hard to believe that all agents would have free access to financial markets at a given rate as established in the paper. Moreover, who knows of a government that can fully commit to its policy plan for the indefinite future? Therefore, one should not expect that consumption smoothing throughout all time is what agents are doing, or are able to do...

Proposition 2 highlights the importance of precommitment. It shows that if the government has no commitment capability the tariff will be always zero, i.e., the government will not be able to use trade policy to achieve a higher income level.

The result from proposition 3 seems to be more realistic. It states that when the government has limited commitment to its future policy tariffs will be lower than with full commitment, and it will eventually be zero after equilibrium is reached. The result is still worse for our economy, compared to the case the government has full credibility.

An interesting development of these results is that institutions that give credibility to a government's long term policy plan could be welfare improving. Take, for instance, protectionist lobbying. Given that it may be a credible guarantor of protection over some period of time, its presence may be welfare improving if the government does not have much policy credibility. A word of caution is necessary here: if the lobbying is not believed to be effective over the medium term, but is able to raise trade barriers, then the worst result of all arises. Workers will make their labor allocation decisions given that they expect no protection in the future, so that the (unexpected) protection does not improve labor allocation, but still causes consumption distortions.

What about a developing country that inherits trade barriers on industrial goods, and is on its way to the industrialized equilibrium? If the government is to restructure its trade policy, the first best alternative is to choose import tariffs/export subsidies so as to maximize the indirect utility function, as in section 3.1. Given that the government can commit to its policy for some number of periods h, this would imply positive tariffs, even for some time after complete industrialization. However, one has to inquire the effect of changing current trade policy on the government's credibility vis-a-vis the new tariff plan it announces. A result like preposition 2 may arise.

Appendix A

In this appendix the producer's problem is solved, which is the maximization the present value of income (equation (4)) subject to the constraint on the movement of the state variable (equation (4.1)), and the per period constraints on the state variables (equation (4.2)). Equation (4.2) yields two possible cases: the lower boundary is binding, in which case the constraint is $\breve{G}(L,u) = -u_\tau - L_\tau \leq 0$; or the upper boundary is binding, in which case it is $\hat{G}(L,u) = u_\tau + L_\tau - \overline{L} \leq 0$.

The following Hamiltonian is used to solve the producer's maximization problem:

$$H(L,u,\lambda,\tau) = \left[d_\tau L_\tau + \overline{L} - u_\tau^2/2\gamma\right](1+r)^{-\tau} + \overline{\lambda}_{\tau+1} u_\tau \qquad \text{(A1)}$$

where $d_\tau = p(\alpha + n\beta L_\tau) - 1$, and $\lambda_{\tau+1} \equiv \overline{\lambda}_{\tau+1}(1+r)^\tau$.

The first order conditions necessary and sufficient[8] for maximization are:

(**a**) the control variable (u_τ) at each period must be chosen to maximize the function $H(L,u,\lambda,\tau)$, subject to constraint $\breve{G}(L,u)$ or $\hat{G}(L,u)$, depending on which one is binding;

(**b**) the state and co-state variables must change over time according to the following equations:

$$\begin{aligned} \overline{\lambda}_{\tau+1} - \overline{\lambda}_\tau &= -H_L^*(L_\tau, \lambda_{\tau+1}, \tau) \\ L_{\tau+1} - L_\tau &= H_{\overline{\lambda}}^*(L_\tau, \lambda_{\tau+1}, \tau) \end{aligned} \qquad \text{(A2)}$$

where $H^*(L_\tau, \lambda_{\tau+1}, \tau)$ is the value of the Hamiltonian after u_τ is optimally chosen.

There are two different sets of conditions for part (a), depending on which constraint is binding, i.e., whether the economy will reach the agricultural or industrialized equilibrium in the long run.

[8]The following conditions are sufficient as well as necessary because the Hamiltonian maximized with respect to the control variable is a concave function of the state variable. (See Intrilligator (1971), p.366, fn. 5)

If the economy is heading to the agricultural equilibrium, the Lagrangian $\Im = H(L,u,\lambda,\tau) + \breve{\xi}_\tau \breve{G}(L,u)$ is used, and the following condition from part (a) is derived:

$$\frac{u_\tau}{\gamma} - \lambda_{\tau+1} - (1+r)^\tau \breve{\xi}_\tau \le 0 \ , \quad -u_t \ge 0 \ \text{ with complementary slackness} \ , \qquad \text{(A3)}$$

$$u_\tau + L_\tau \ge 0 \ , \quad \breve{\xi}_\tau \ge 0 \ \text{ with complementary slackness} \ .$$

And if the economy is heading to the industrialized equilibrium, the Lagrangian used is $\Im = H(L,u,\lambda,\tau) + \hat{\xi}_\tau \hat{G}(L,u)$, and the conditions:

$$-\frac{u_\tau}{\gamma} + \lambda_{\tau+1} - (1+r)^\tau \hat{\xi}_\tau \le 0 \ , \quad u_\tau \ge 0 \ \text{ with complementary slackness} \ , \qquad \text{(A4)}$$

$$u_\tau + L_\tau - \overline{L} \ge 0 \ , \quad \hat{\xi}_\tau \ge 0 \ \text{ with complementary slackness} \ .$$

Finally, the equations for part (b) are:

$$\lambda_{\tau+1} - (1+r)\lambda_\tau = -d_\tau \qquad \text{(A5)}$$

$$L_{\tau+1} - L_\tau = u_\tau \qquad \text{(A6)}$$

The transversality conditions also need to be satisfied. These conditions state that as time goes to infinity either the value of moving one more unit of labor into a sector is zero, or there is no labor in that sector. Equation (A7) presents the transversality conditions for the industrialized and agricultural equilibria, respectively.

$$\lim_{\tau \to \infty} \frac{\lambda_{\tau+1}}{(1+r)^\tau} L_\tau = 0 \quad \text{or}$$
$$\lim_{\tau \to \infty} \frac{-\lambda_{\tau+1}}{(1+r)^\tau} \left(\overline{L} - L_\tau\right) = 0 \qquad \text{(A7)}$$

Equations (A3)-(A7) completely determine the solution to the problem. Using equations (A3) and (A4), labor movement will follow:

$$L_{\tau+1} - L_\tau = \gamma\lambda_{\tau+1} , \qquad \text{(A8)}$$

when the labor frontier is not binding

When one of the frontiers is reached the control variable is set to zero due to equation (A6), $u_\tau = 0$, and either $(1+r)^\tau \breve{\xi}_\tau \geq -\lambda_{\tau+1}$ or $(1+r)^\tau \hat{\xi}_\tau \geq \lambda_{\tau+1}$, depending on whether the lower or upper boundary is reached, respectively.

From equation (A5), and using the transversality conditions:

$$\lambda_{\tau+1} = \sum_{i=\tau+1}^{\infty} \frac{d_i}{(1+r)^{i-\tau}}. \tag{A9}$$

The value of the co-state variable when the economy is at the long run equilibrium is derived using the equation above:

$$\lambda^k = \frac{d^k}{r} \quad \text{for } k = N, A \tag{A10}$$

where $d^N = p(\alpha + n\beta \overline{L}) - 1$ and $d^A = p\alpha - 1$.

Appendix B

In this appendix the validity of equation (26) will be proved. The value of equation (B1) will be derived.

$$\frac{\partial PVI}{\partial v_{T+s}} = \sum_{i=1}^{T-1} \frac{\partial PVI}{\partial L_i} \frac{\partial L_i}{\partial v_{T+s}} + \sum_{i=1}^{T} \frac{\partial PVI}{\partial \lambda_i} \frac{\partial \lambda_i}{\partial v_{T+s}} \tag{B1}$$

First the focus will be on the second term of the sum in the equation above. Combining equations (4.1) and (6) yields $u_\tau = \gamma \lambda_{\tau+1}$, and substituting this into equation (4), it is straightforward to check that $\frac{\partial PVI}{\partial \lambda_i}$ is the same for all v_{T+s}, $s \geq 0$:

$$\frac{\partial PVI}{\partial \lambda_i} = (1+r)^{-(i-1)} \gamma \lambda_i \tag{B2}$$

Using equation (A9) from appendix A:

$$\frac{\partial \lambda_i}{\partial v_{T+s}} = p(\alpha + n\beta L_T)(1+r)^{-(T+s-i-1)} = (1+r)^{-s} \frac{\partial \lambda_i}{\partial v_T} \quad \text{for } 1 \leq i < T \tag{B3}$$

where L_T is the (constant) value of labor at one of the long run equilibria.

Now turning to the first term of the sum in equation (B1). Equation (22) can be rewritten as:

$$L_\tau - \gamma \sum_{j=1}^{\tau} (1+r)^j \left[\sum_{i=j}^{T-1} \frac{d_i}{(1+r)^{i+1}} \right] = L_0 + \gamma \sum_{j=1}^{\tau} (1+r)^j \left[\sum_{i=T}^{\infty} \frac{d_i}{(1+r)^{i+1}} \right] \tag{B4}$$

for $1 \leq \tau < T$.

Equation (B4) represents a system of $T-1$ linear equations, which can be written in matrix form as $A\vec{L} = \vec{b}$:

$$\begin{pmatrix} a_{11}(v_1) & \cdots & a_{1,T-1}(v_{T-1}) \\ \vdots & \ddots & \vdots \\ a_{T-1,1}(v_1) & \cdots & a_{T-1,T-1}(v_{T-1}) \end{pmatrix} \begin{pmatrix} L_1 \\ \vdots \\ L_{T-1} \end{pmatrix} = \begin{pmatrix} b_1\left(\{v_i\}_1^\infty\right) \\ \vdots \\ b_{T-1}\left(\{v_i\}_1^\infty\right) \end{pmatrix} \tag{B5}$$

where $b_i = L_0 + K_i\left(\{v_k\}_1^{T-1}\right) + \gamma (1+r)^i \sum_{k=T}^{\infty} \frac{d_k}{(1+r)^{k+1}}$, K_i is a function of tariffs previous to period T, $a_{ii} = 1 - \frac{\gamma p \beta v_i}{(1+r)^i} \left(\sum_{k=0}^{i-1} (1+r)^k \right)$, and $\underset{i \neq j}{a_{ij}} = -\frac{\gamma p \beta v_j}{(1+r)^j} \left(\sum_{k=0}^{\min\{i-1, j-1\}} (1+r)^k \right)$.

For this system of equations to have a solution, matrix A has to be non singular. The system was constructed imposing the condition that a specific long run equilibrium will be reached at period T. If matrix A turns out to be singular for any tariff schedule, it means the equilibrium is not achievable in that time frame from the initial position. Limiting the values for T as suggested in section 3.1 overcomes this problem.

Using Cramer's rule to solve for the value of each labor allocation, we have:

$$L_\tau = \frac{D\left(A_\tau^b\right)}{D(A)} \tag{B6}$$

where $D(A)$ represents the determinant of matrix A, and A_τ^b is a matrix obtained from A by replacing its τ^{th} column by the vector $\vec{b}$.

We want to know the value of the derivative $\frac{\partial L_\tau}{\partial v_{T+s}}$ $\forall s \geq 0$. The denominator of the function that determines L_τ (equation (B6)) does not depend on the value of the tariffs after time T. Therefore:

$$\frac{\partial L_t}{\partial v_{T+s}} = \frac{1}{D(A)} \frac{\partial D\left(A_\tau^b\right)}{\partial v_{T+s}} \tag{B7}$$

$D\left(A_\tau^b\right)$ can be expanded using the τ^{th} column, yielding:

$$D\left(A_\tau^b\right) = (-1)^{1+\tau} b_1 D\left(A_{1\tau}\right) + \cdots + (-1)^{(T-1)+\tau} b_{T-1} D\left(A_{T-1,\tau}\right)$$

where $A_{i\tau}$ is a matrix obtained from A_τ^b by excluding row i and column τ from it.

Now equation (B7) can be rewritten as:

$$\frac{\partial L_\tau}{\partial v_{T+s}} = \frac{\frac{\partial b_1}{\partial v_{T+s}}(-1)^{1+\tau} D\left(A_{1\tau}\right) + \cdots + \frac{\partial b_{T-1}}{\partial v_{T+s}}(-1)^{(T-1)+\tau} D\left(A_{T-1,\tau}\right)}{D(A)} \tag{B8}$$

We know that:

$$\frac{\partial b_i}{\partial v_{T+s}} = \gamma(1+r)^i \frac{p(\alpha + n\beta L_T)}{(1+r)^{T+s+1}} = (1+r)^{-s} \frac{\partial b_i}{\partial v_T} \tag{B9}$$

Combining equations (B8) and (B9), we finally get:

$$\frac{\partial L_\tau}{\partial v_{T+s}} = (1+r)^{-s} \frac{\partial L_\tau}{\partial v_T} \tag{B10}$$

Substituting equation (B3) and (B10) into (B1) the proof is finished:

$$\frac{\partial PVI}{\partial v_{T+s}} = \sum_{i=1}^{T} \frac{\partial PVI}{\partial L_i}(1+r)^{-s} \frac{\partial L_\tau}{\partial v_T} + \sum_{i=1}^{T} \frac{\partial PVI}{\partial \lambda_i}(1+r)^{-s} \frac{\partial \lambda_\tau}{\partial v_T} = (1+r)^{-s} \frac{\partial PVI}{\partial v_T} \tag{B11}$$

Appendix C

The problem for the policy maker after the long run equilibrium is reached is to maximize the indirect utility function from that moment on. First the function must be derived for this time frame.

In period T consumers will maximize:

$$\sum_{\tau=T}^{\infty}\frac{1}{(1+\theta)^{\tau}}\left[a\log C_{\tau}^{N}+b\log C_{\tau}^{A}\right] \tag{C1}$$

subject to the following budget constraints, observing that the quantity of assets held at the beginning of period T is exogenously given from that period's point of view:

$$f_{\tau+1}-f_{\tau}=rf_{\tau}+I_{\tau}^{P}-p(1+v_{\tau})C_{\tau}^{N}-C_{\tau}^{A}+R_{\tau},\quad \tau\geq T$$
$$f_{T}=\sum_{i=0}^{T}(1+r)^{T-i}\left[I_{i}^{P}-p(1+v_{i})C_{i}^{N}-C_{i}^{A}+R_{i}\right] \tag{C2}$$

The first order conditions for the maximization above are analogous to those derived previously, i.e., equations (17.a-d); and the transversality condition in equation (18) can be rewritten as:

$$\lim_{\tau\to\infty}\frac{y}{(1+\theta)^{\tau}}f_{T+\tau}=0 \tag{C3}$$

Using equation (17.d) and the initial value for f_T, it follows that:

$$f_{T+\tau}=(1+r)^{\tau}f_{T}+\sum_{i=0}^{\tau}(1+r)^{\tau-i}\left[I_{T+i}^{P}-p(1+v_{T+i})C_{T+i}^{N}-C_{T+i}^{A}+R_{T+i}\right] \tag{C4}$$

Using the government's budget constraint, and substituting equations (17.a) and (17.b) into equation (C4), the transversality condition yields:

$$y=\frac{S_{T}}{PVI_{T}+\phi_{T}} \tag{C5}$$

where $S_{T}=\sum_{i=0}^{\infty}(1+r)^{-i}\left[\frac{a}{(1+v_{T+i})}+b\right]$, $\phi_{T}=\sum_{i=0}^{T}(1+r)^{T-i}\left[I_{i}-pC_{i}^{N}-C_{i}^{N}\right]$ and $PVI_{T}=\sum_{i=0}^{\infty}(1+r)^{-i}I_{T+i}$.

Substituting equation (C5) into (17.a) and (17.b), and then substituting these optimal consumption decisions into the utility function, gives the following indirect utility function for time T:

$$V_{T}(L_{\tau},v_{\tau})=\overline{K}-\frac{(1+\theta)(a+b)}{\theta}\log S_{T}-a\sum_{i=T}^{\infty}(1+\theta)^{T-i}\log(1+v_{i}), \tag{C6}$$

where $\overline{K}$ is a constant term.

Note that PVI_T is now included in the constant term due to the fact that the labor allocation is constant from period T on. $PVI_T = I^E \frac{(1+r)}{r}$, where I^E is the per period income at the equilibrium point.

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