TEXTO PARA DISCUSSÃO

No. 612

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Anna Carolina dos Reis

Eduardo Zilberman

DEPARTAMENTO DE ECONOMIA
www.econ.puc-rio.br

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Anna Carolina Dos Reis — Banco Modal

Eduardo Zilberman — PUC-Rio

July 2013

Abstract

A public job can be seen as a source of insurance against income risk. Indeed, many public employees have job stability, which is compounded with less volatile and more compressed wages. Hence, by increasing its number of public employees, the government enhances the overall degree of insurance in the economy. In this paper, we introduce public employment in a standard incomplete markets model with overlapping generations. The aim is to explore the welfare gains or losses due to a larger government, accounting for this extra source of insurance. In a model economy calibrated to Brazil, where public employment is around 13.5 percent of the workforce, we find that if the government relies on consumption taxes to balance its budget, the optimal size of public employment is nearly flat, ranging from 8 to 12 percent of the workforce. However, if the public employment is reduced from 12 to 8 percent, welfare losses due to a reduction in the degree of insurance are 2 percent, which are compensated by welfare gains due to level and inequality effects. This insurance effect is robust to a missepecification of the production technology associated with the public sector.

Keywords: pubic employment, insurance, incomplete markets, optimal policy.

JEL Classification: D31, E24, H11.

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*Dos Reis: annacsaba@gmail.com. Zilberman: zilberman@econ.puc-rio.br. We thank without implicating Tiago C. Berriel, Vinícius Carrasco, Pedro C. Ferreira, Gustavo M. Gonzaga, Marcelo R. dos Santos, Rodrigo R. Soares, and participants at the 2013 Midwest Macroeconomic Meeting, 2013 REAP Meeting and 2013 SED Meeting for insightful comments and discussions.

1 Introduction

In most countries, public sector jobs offer some advantages over private sector jobs. In particular, governments usually provide protection against dismissals for public workers. In Brazil, for instance, job stability is a right guaranteed by constitution for those that, after entering the public sector, have stayed at the job for at least three years.[1]

In a similar vein, many empirical studies have found that wages in the public sector are more compressed and less volatile than their counterparts in the private sector.[2]

Job stability compounded with a more compressed and less disperse wage distribution can be interpreted as a source of insurance against income risk. Indeed, whoever enters the public sector is exchanging a more volatile, but potentially higher, income for a less volatile one. Hence, by increasing its number of public employees, the government enhances the overall degree of insurance in the economy.[3]

The aim of this paper is to explore the welfare gains or losses due to a larger government. The novelty is to properly account for the aforementioned source of insurance. To do so, we introduce public employment in a standard incomplete markets model with overlapping generations (e.g. Huggett [1996]). In particular, the size of the government, defined by the number of agents employed in the public sector, affects not only the degree of insurance in the economy, but also the distribution of consumption. Hence, from an utilitarian perspective, whether a larger government increases or decreases welfare is an empirical question.

In a model economy calibrated to Brazil, we find that if changes in the public wage bill associated with changes in public employment are financed with consumption taxes,

[1] High public job security is also present among other countries as noted in OECD [2008]: "A stronger protection against dismissals and other forms of termination of the employment is also normally a part of the special arrangements [of government employment]. This would traditionally guarantee employment for life with dismissal only possible for misconduct."

[2] This pattern holds in several countries. See Gregory and Borland [1999] for a review.

[3] Notice that this source of insurance might not be available to everyone. If earnings dynamics in the public sector are too generous, there will be a larger number of candidates than public vacancies. Hence, a set of rules is necessary to match candidates and vacancies. In Brazil, for instance, most public servants are selected based on merit through a public exam. In particular, each exam is designed to test the knowledge necessary to perform a specific job.

the optimal size of public employment is nearly flat, ranging from 8 to 12 percent of the workforce. However, if the public employment is reduced from 12 to 8 percent, welfare losses due to a reduction in the degree of insurance are around 2 percent, which are compensated by welfare gains due to level and inequality effects. If changes in the wage bill are financed by capital taxes instead, the optimal size of public employment is 6 percent of the workforce, which is associated with welfare losses of 5.9 percent due to a worse degree of insurance than in the benchmark calibration. Importantly, these insurance effects are robust to a missepecification of the production technology associated with the public sector.

The model has three main ingredients. First, we consider an overlapping generations model with heterogeneous agents. In particular, heterogeneity regards their income profiles that vary with age, human capital, and an uninsurable idiosyncratic risk (i.e. productivity shock).

Second, we consider a competitive economy with incomplete markets in the sense that borrowing-constrained agents can only save trough risk-free bonds.

Third, there are two sectors: public and private. The private sector combines effective labor and capital to produce a single good. The public sector employs effective labor and capital to produce public goods. On the one hand, since we consider a closed economy, the production of public goods crowds out private production. On the other hand, public goods enhance total factor productivity in the private sector. Hence, the overall effect of public goods on aggregate output is ambiguous.

During their life-cycle, agents choose whether to work in the private sector or to apply for a public job. In line with the aforementioned evidence, we assume that public workers cannot be fired, but they may quit. Similarly, once in the public sector, risk becomes less volatile at the expense of a more compressed distribution of wages. Finally, we assume that income profiles also vary across sectors.

For each level of human capital, the government opens a given number of vacancies it is willing to fill. Depending on the model's parameters, the public wage scheme might attract a larger number of candidates than open vacancies. If this is the case, in order to fill vacancies, the government only hires the most productive candidates. Notice that this selection mechanism emulates a public exam in which performance is positively associated with productivity. Finally, as some agents with a high income profile in the private sector might not apply for a public job, the effects of a larger government on the overall distribution of income, wealth and consumption are ambiguous. In our benchmark calibration, for instance, only agents with intermediate and relatively high levels of productivity shocks are hired by the government.

The optimal size of public employment maximizes an ex-ante utilitarian welfare criterion. Following Conesa et al. [2009], we consider only the welfare of newborn agents. In particular, the overall welfare effect associated with a given policy is defined by how much lifetime consumption has to increase uniformly across newborn agents in the benchmark economy in order to equalize welfare measures across stationary equilibriums.

By adapting the methodology from Flodén [2001] to an environment with overlapping generations, we decompose the overall welfare effect of a change in public employment into three categories: (i) the level effect associated with changes in aggregate consumption; (ii) the inequality effect associated with changes in the distribution of consumption; and (iii) the uncertainty effect associated with changes in the degree of insurance in the economy.

Instrument	optimal public employ. (%)	total welfare effect (%)	level effect (%)	inequality effect (%)	uncertainty effect (%)
consumption taxes	8 to 12	0.5	1.7 to 0.7	0.7 to -0.5	-1.8 to 0.2
capital taxes	6	2.6	7.9	1.1	-5.9
lump-sum taxes	2 to 4	11.5	0.7 to 2.5	10.6 to 8.2	0.2 to 0.6

Table 1: Summary of the main results.

Table 1 anticipates some of the results in this paper. It reports the optimal level of public employment. In the benchmark economy, for instance, public employment is calibrated at 13.5 percent of the workforce. Since different sizes of public employment imply changes in the wage bill, we assume that these changes are financed with a single policy instrument. In particular, we consider capital, consumption and lump-sum taxes.

Finally, Table 1 also reports the welfare effects from moving from the benchmark economy to the economy associated with the optimal policy.

If the single instrument used to balance the government budget is a linear tax on consumption, the optimal size of public employment is nearly flat, ranging from 8 to 12 percent of the workforce. In particular, total welfare gains are only 0.5 percent in this range. However, if public employment is reduced from 12 to 8 percent, losses due to the uncertainty effect are around 2 percent, which are compensated by welfare gains due to level and inequality effects. If a linear tax on capital is considered instead, the optimal size of public employment is around 6 percent of the workforce, which is associated with total welfare gains of 2.6 percent. These gains come from both inequality and level effects. In contrast, losses due to the uncertainty effect are 5.9 percent. Importantly, the insurance effect is remarkably robust to alternative calibrations of the technology associated with the public sector. Hence, we conclude that public employment is an important source of insurance in this economy.

If lump-sum taxes are considered, the optimal size of public employment ranges from 2 to 4 percent of the workforce, which is associated with a total welfare effect of 11.5 percent. Notice that these large welfare gains result from a large inequality effect. Intuitively, a large public sector benefits individuals with intermediate and relatively high levels of productivity shocks. Once the size of the government becomes smaller, the extra resources obtained from the reduction in the public wage bill are converted into lump-sum taxes, which particularly improves the welfare of those agents at the bottom of the consumption distribution.

Finally, we also decompose welfare effects by human capital levels. In our calibration, we proxy human capital θ by the level of schooling. We find that larger governments benefit mostly individuals with the highest level of human capital, which is college education. In particular, our calibration to the Brazilian economy implies that public wages represent a more effective insurance scheme to college graduates.

The paper is structured as follows. Section 2 presents a brief review of the literature.

Section 3 presents the model. Section 4 presents the quantitative analysis, including the calibration procedure, results and sensitivity analysis. Section 5 concludes.

2 Related Literature

This paper relates to a vast literature studying different aspects of public policy and its welfare implications within an incomplete markets framework with heterogeneous agents and idiosyncratic risk.[4] Flodén and Lindé [2001] and Alonso-Ortiz and Rogerson [2010], for example, study the optimal level of public insurance in an economy with distortive taxes. Public insurance, for instance, is achieved trough lump-sum transfers.

Flodén and Lindé [2001], in particular, provide a strong motivation to account for public employment in this framework. They calibrate a model without public employment to both Sweden and the US economies. Given that wages are more persistent and volatile in the US than in Sweden, their model concludes that taxes and transfers (i.e. the degree of public insurance) should be higher in the US than in Sweden. However, these results would be biased if large transfer programs require a sizeable government to operate them. In particular, a sizeable government would further improve public insurance as public wages are less uncertain, which in turn would call for less generous transfers.[5] Our paper properly accounts for this extra source of insurance associated with the size of government.

Other papers study the role of policy instruments, other than lump-sum transfers, to improve welfare. To the best of our knowledge, none of them consider public employment policies. Aiyagari and McGrattan [1998] and Flodén [2001], for instance, consider the role of public debt.[6] Domeij and Heathcote [2004], Nishiyama and Smetters [2005], Conesa and Krueger [2006] and Conesa et al. [2009] study the effect of a variety of consumption,

[4] See Heathcote et al. [2009] and Guvenen [2011] for recent reviews of this framework.

[5] Flodén and Lindé [2001] acknowledge but do not address this possibility: "... although we look at wages before taxes and transfers, the relatively low degree of wage risk in Sweden may be a result of the big government sector. For example, a large fraction of the population work in the government sector and wage setting there seems to imply a significant amount of risk sharing."

[6] To be precise, Flodén [2001] studies the interaction of lump-sum transfers and public debt.

income and capital tax schedules. Berriel and Zilberman [2011] emphasize the role of targeted transfers to the poor. Imrohoroglu et al. [1995], Conesa and Krueger [1999], Huggett and Ventura [1999] and Storesletten et al. [1999] focus on the role of different social security arrangements. Finally, Hansen and Imrohoroglu [1992] explore the role of unemployment insurance.

In a different context, Rodrik [1998] and Rodrik [2000] explore a related idea to this paper. These articles argue that bigger governments might be an endogenous response to a higher level of external risk. As Rodrik [2000] points out:

> "... relatively safe government jobs represent partial insurance against undiversifiable external risk faced by the domestic economy. By providing a larger number of "secure" jobs in the public sector, a government can counteract the income and consumption risk faced by the households in the economy."

Also related is Jetter et al. [2011], who develop a model to study the effect of wage volatility on growth. The crucial assumption is that public wages are not volatile, but their counterparts in the private sector are. If volatility increases, both precautionary savings and the size of government increase for insurance reasons, affecting economic growth ambiguously.

Several papers study the implications of public wage and employment policies in macroeconomic workhorse models. Finn [1998] and Pappa [2009], for example, introduce public employment in standard real business cycle and new-Keynesian frameworks, respectively. Horner et al. [2007] and Quadrini and Trigari [2008] integrate public wage and employment policies into models with search and matching. However, we are not aware of any paper that introduces public employment in an incomplete markets model that follows in the tradition of Imrohoroglu [1989], Huggett [1993], Aiyagari [1994] and Huggett [1996]. This paper bridges this gap.

3 Model

We incorporate public employment in an overlapping generations framework with incomplete markets similar to Huggett [1996] and Imrohoroglu et al. [1999]. In particular, we consider a public sector, in which the government opens a given number of vacancies every period. Agents can choose to apply for these jobs or to work in the private sector. Candidates who are not hired by the public sector work in the private sector. The aim is to study the welfare implications of public employment policies.

3.1 Demographics, Preferences and Endowments

The economy is populated with overlapping generations whose decisions follow a well-defined life-cycle structure. At any point in time there is a measure one of agents indexed by age $t \in \{1, ..., T\}$, who face an age-dependent probability π_t of surviving up to age t conditional of surviving up to age $t-1$. Once they reach age T, death is certain so $\pi_{T+1} = 0$. We assume an equal measure of agents is born at every period, such that the age distribution remains stationary. Thus, at every period, agents at age t constitute a constant fraction $\mu_t \in (0,1)$ of the population, such that $\sum_t \mu_t = 1$.

At $t = 1$, agents have identical preferences over streams of consumption $\{c_t\}_{t=1}^T$, given by

$$\mathbb{E} \sum_{t=1}^{T} \beta^{t-1} \left(\prod_{i=1}^{t} \pi_i \right) u(c_t), \quad \text{with } u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \gamma > 0,$$

where $\beta \in (0,1)$ is the discount factor. Notice we assume there is not altruism, so bequests are accidental and distributed lump-sum to all agents alive.

Agents are not endowed with assets when they enter the labor market at $t = 1$ (i.e. when they are born). However, they are endowed with one unit of labor, which is supplied inelastically until the age of $t = T_r < T$, when they are forced to retire. Moreover, each agent experiences an productivity profile that determines the value of this unit of labor over time. In particular, this productivity profile depends on: (i) the experience at the labor market, which is equal to age t in our model; (ii) a fixed level of human capital

$\theta \in \{\theta_1, \theta_2, ..., \theta_m\}$ drawn by nature at the time the agent is born from a distribution in which each θ has mass μ_θ and $\sum_\theta \mu_\theta = 1$; and (iii) an uninsured idiosyncratic risk z (i.e. productivity shock) that follows a finite state Markov chain with transition probabilities $\Pi(z', z) = \text{Prob}(z_{t+1} = z'|z_t = z)$, where $z, z' \in \{z_1, z_2, ..., z_n\}$.[7]

Let $s \in \{g, y\}$ be the sector an agent is working in, where g stands for the public sector while y stands for the private sector.[8] We assume that the productivity profile, which may vary across sectors, is given by:

$$q_s(t, \theta, z) = \exp\{\gamma_1^s \cdot (t-1) + \gamma_2^s \cdot (t-1)^2 + \gamma_3^s(\theta) + \gamma_4^s(z)\}, \;\; s \in \{g, y\}.$$

Notice that γ_1^s and γ_2^s are parameters whereas $\gamma_3^s(\cdot)$ and $\gamma_4^s(\cdot)$ are functions to be specified in the next section. Importantly, these objects may depend on the sector $s \in \{g, y\}$ the agent is working in. We assume that in the private sector, $\gamma_4^y(z) = z$, but as we discuss later, it is not clear how one's productivity shock is affected by being employed in the public sector.

3.2 Private Production

There is a representative firm that produces consumption goods with a Cobb-Douglas function augmented with public goods,

$$Y = G^\xi K_y^\alpha H_y^{1-\alpha}, \;\; \alpha, \xi \in (0, 1),$$

where K_y and H_y are aggregate capital and efficient labor units, respectively, employed at the private sector. Each period capital K_y depreciates at rate δ_y. Finally, we assume that public goods G, which are produced by the government, enhance total factor productivity in the private sector.

[7] We rule out aggregate risk by assuming that this stochastic process is independent and identically distributed across agents.

[8] Since the public sector produces public goods G and the private sector produces consumption goods Y, we choose g and y, respectively, to denote these sectors throughout the paper.

3.3 Markets Arrangements

There are no insurance markets for the idiosyncratic risk z. In particular, markets are incomplete in the sense that agents can only accumulate wealth trough risk-free bonds. Moreover, agents are subject to a no-borrowing constraint.

We consider a closed economy with competitive markets. Hence, at every period, the interest rate r and the private wage rate w_y clear the markets for capital and efficient labor units, respectively.

Finally, accidental bequests are distributed lump-sum to all agents alive.

3.4 Public Sector

We assume that the government taxes linearly labor income (τ_h), financial income (τ_a), consumption (τ_c) and bequests (τ_{beq}) in order to finance its consumption (C_g), investment in public capital (I_g), lump-sum transfers (Υ) and payroll bill ($w_g H_g$), where w_g is the public wage rate set by the government. The government can also issue public debt D, at the equilibrium interest rate r, to finance its deficit.

The government also produces public goods G with efficient labor units H_g and capital K_g, which depreciates at a rate δ_g.[9] In particular, we assume a Cobb-Douglas production function:

$$G = A_g K_g^{\eta} H_g^{1-\eta}, \;\; \eta \in (0,1),$$

where A_g is the total factor productivity in the production of public goods. Since we normalize A_g to match the steady-state ratio G/Y we observe in the data, this formulation is general enough to accommodate a public sector in which only a fraction of public employment is used in productive activities.[10]

Notice that public sector production has opposing effects on aggregate output. Since we consider a closed economy, it crowds out private production. In contrast, it also

[9] In a stationary equilibrium, the law of motion of public capital implies that $\delta_g = I_g/K_g$. Thus, given an investment decision I_g, K_g is determined endogenously.

[10] Indeed, if ω is the fraction of efficient labor units employed to produce public goods, $G = \tilde{A}_g K_g^{\eta}(\omega H_g)^{1-\eta} = A_g K_g^{\eta} H_g^{1-\eta}$, where $A_g = \tilde{A}_g \omega^{1-\eta}$.

enhances total factor productivity in the private sector.

Finally, the government also runs a pay-as-you-go pension system. In particular, workers of both sectors contribute a fraction τ_{ss} of their labor income, while retired agents receive a flat benefit b. Since we calibrate the model economy to Brazil, where pension schemes are in deficit, we include the pension system in the government budget constraint, which reads

$$\tau_a r(K_y + D) + \tau_c C_y + (\tau_h + \tau_{ss})(w_y H_y + w_g H_g) + \tau_{beq} beq = C_g + I_g + \Upsilon + rD + w_g H_g + B$$

in a stationary equilibrium. Notice that *beq* stands for accidental bequests and B stands for the aggregate level of pension benefits b.

We assume that tax instruments, public debt, pension benefits, and investment are exogenously set, in the sense that we calibrate them to capture how fiscal policy is conducted in Brazil. The government consumption C_g is the policy variable used to balance its budget. It remains to discuss how employment is chosen and wages are set in the public sector.

3.4.1 Admission Policy

At every period, for each level of human capital $\theta \in \{\theta_1, ..., \theta_m\}$, the government is willing to employ $\lambda(\theta)$ workers. Hence, it opens the number of vacancies necessary to accomplish this goal. Agents choose to either apply for a public job or work in the private sector. For simplicity, we assume an agent can only apply for vacancies assigned to her level of human capital. In our calibration, we proxy human capital θ by the level of schooling, which is observable by the government. In practice, depending on the complexity of the job, the government requires a minimum degree of schooling from candidates.

Depending on the model's parameters, public jobs may attract a larger number of candidates than open vacancies. If this is the case, in order to fill vacancies, the government only hires the most productive candidates.[11] Notice that this selection mechanism emu-

[11] Since labor is inelastically supplied, candidates work in the private sector if they are not hired by

lates a public exam in which performance is positively associated with the productivity shock. Admissions to public jobs trough public exams are widely spread across countries. In Brazil, for instance, most of the vacancies are filled with agents who perform well in a public exam designed to test the knowledge necessary to perform a specific job.

Although the age t also affects the productivity profile $q_s(t, \theta, z)$, $s \in \{g, y\}$, it is not clear how age t affects performance in a public exam. On one hand, older agents have more time to prepare themselves for the exam. On the other hand, performing well in an exam may require a specific skill that tends to depreciate over time, especially for those agents who have spent some years working in the private sector. Hence, we assume that admission to the public sector depends only on human capital θ and productivity shock z.

In a stationary equilibrium, the selection mechanism we explain above implies that, for each level of θ, there is a threshold $\underline{z}(\theta)$ such that open vacancies, necessary to keep $\lambda(\theta)$ workers in the public sector, are filled with type-θ agents who experience $z \geq \underline{z}(\theta)$. Importantly, not necessarily all type-θ agents with $z \geq \underline{z}(\theta)$ apply for a public job. Indeed, the private sector might be more attractive for some of them.

Finally, as we observe in practice, we assume public workers cannot be fired, but they may quit if the private sector becomes more attractive.

3.4.2 Wage Setting

Let w_y and w_g be the wage rates paid in the private and public sectors, respectively. Recall that productivity profile is given by:

$$q_s(t, \theta, z) = \exp\{\gamma_1^s \cdot (t-1) + \gamma_2^s \cdot (t-1)^2 + \gamma_3^s(\theta) + \gamma_4^s(z)\}, \;\; s \in \{g, y\}.$$

Since we assume that the private sector behaves competitively, the productivity profile $q_y(t, \theta, z)$ has a dual role. First, $q_y(t, \theta, z)$ is employed to produce consumption goods. Second, $w_y q_y(t, \theta, z)$ is the wage schedule in the private sector. Hence, by using data at

the government.

the individual level on wages, experience and human capital, one can estimate γ_1^y, γ_2^y and $\gamma_3^y(\cdot)$ and, thus, calibrate the productivity profile in the private sector.

However, even in a competitive equilibrium, the government may choose to not remunerate productivity competitively. In this case, $w_g q_g(t,\theta,z)$ might not be the wage schedule in the public sector. Hence, we define a wage setting rule in the public sector denoted by $w_g \hat{q}_g(t,\theta,z)$, where

$$\hat{q}_g(t,\theta,z) = \exp\{\hat{\gamma}_1^g \cdot (t-1) + \hat{\gamma}_2^g \cdot (t-1)^2 + \hat{\gamma}_3^g(\theta) + \hat{\gamma}_4^g(z)\}.$$

In a similar fashion, we can use data on public workers to estimate $\hat{\gamma}_1^g$, $\hat{\gamma}_2^g$ and $\hat{\gamma}_3^g(\cdot)$,[12] and thus, calibrate the wage setting rule in the public sector.

We postpone to the next section the discussion on how we set $q_y(t,\theta,z)$, $q_g(t,\theta,z)$ and $\hat{q}_g(t,\theta,z)$ to solve numerically the model.

3.5 Recursive Equilibrium

In this paper, we focus on the properties of a stationary competitive equilibrium in which the measure of agents, defined over an appropriate family of subsets of the individual state space, remains invariant over time.

3.5.1 Agents' Problem

Agents make two types of decision during their lives. First, they choose how to allocate their disposable income between consumption and risk-free bonds. Second, they decide whether to work in the private or public sector. Once hired by the public sector, workers cannot be fired but they may quit. Finally, as mentioned above, not all candidates have the option to work in the public sector as their productivity shock may not be high

[12] Many empirical studies estimate these objects for both sectors and find substantial differences across them (e.g. Braga et al. [2009]). There are two possible complementary explanations for this discrepancy. First, the productivity profile varies across sectors. Second, productivity plays a minor role when setting public wages.

enough.[13]

In this context, there are five individual state variables: age t, a fixed level of human capital θ, the idiosyncratic risk z, the previous sector s one works, and the amount of assets a accumulated. We assume that $s = y$ for those agents at the age of $t = 1$. Given our assumptions on the hiring and firing of government employees, the agent's problem prior to retirement, i.e. for $t < T_r$, is given by:

$$V_t(a, s, z; \theta) = \max_{c, s', a'} \left\{ u(c) + \beta \pi_{t+1} \sum_{z'} \Pi(z', z) V_{t+1}(a', s', z'; \theta) \right\},$$

subject to

$$(1 + \tau_c)c + a' \leq [1 + (1 - \tau_a)r]a + (1 - \tau_h - \tau_{ss}) w_{s'} \hat{q}_{s'}(t, \theta, z) + \Upsilon + (1 - \tau_{beq}) beq,$$

$$c \geq 0, \;\; a' \geq 0,$$

$$s' \in \begin{cases} \{y\} & \text{if } z \leq \underline{z}(\theta) \text{ and } s = y \\ \{g, y\} & \text{otherwise} \end{cases},$$

$$V_{T_r}(a', s', z'; \theta) = \tilde{V}_{T_r}(a'), \;\text{ for all } s', z', \theta,$$

where $\tilde{V}_{T_r}(a')$ is the value of retiring at the age of $t = T_r$. Notice we implicitly define $\hat{q}_y(t, \theta, z) = q_y(t, \theta, z)$, for all t, θ, z, so we can write a single problem for all agents.

After retiring, i.e. for $T_r \leq t < T$, the agent's problem is a cake-eating one:

$$\tilde{V}_t(a) = \max_{c, a'} \left\{ u(c) + \beta \pi_{t+1} \tilde{V}_{t+1}(a') \right\}$$

subject to

$$(1 + \tau_c)c + a' \leq [1 + (1 - \tau_a)r]a + b + \Upsilon + (1 - \tau_{beq}) beq,$$

$$c \geq 0, \;\; a' \geq 0,$$

$$\tilde{V}_T(a') = 0 \text{ for all } a'.$$

By solving the problems above, one obtains decision rules for consumption $c_t(a, s, z; \theta)$, savings $a'_t(a, s, z; \theta)$, and job sector $s'_t(a, s, z; \theta)$ along the life-cycle $t = 1, ..., T$.

[13] Recall that for a given θ, government only hires those θ-type agents with $z \geq \underline{z}(\theta)$.

3.5.2 Definition and Policy Experiment

The definition of stationary competitive equilibrium is standard, except for the role the government has in hiring workers. In particular, (i) given prices and fiscal policies, agents solve their problems; (ii) given prices and fiscal policies, the representative firm maximizes profits; (iii) accidental bequests are distributed lump-sum to all agents alive; (iv) the private wage rate w_y and the interest rate r clear the labor and capital markets, respectively; (v) the government produces public goods and chooses fiscal policy objects, which remain invariant over time, subject to a balanced budget constraint and the law of motion for public capital; (vi) for each θ, the government specifies a threshold $\underline{z}(\theta)$ such that it employs $\lambda(\theta)$ workers; finally, (vii) for each age t and human capital θ, there is a stationary measure $\psi_{t,\theta}$ defined over an appropriate family of subsets of the individual state space.[14] A formal definition is provided in Appendix A.

We are interest in welfare properties of the stationary equilibrium. In particular, we study the welfare implications of different levels of public employment, which is given by

$$L_g = \sum_{t<T_r} \mu_t \sum_{\theta} \mu_\theta \int \mathbf{I}_{\{s_t'(a,s,z;\theta)=g\}} d\psi_{t,\theta}(a,s,z) = \sum_{\theta} \lambda(\theta),$$

where $\mathbf{I}$ is the indicator function.[15] The policy experiment we study is to increase or decrease $\lambda(\theta)$ proportionally for all θ.[16] In this case, public employment L_g increases or decreases, at the same time that the proportion of public workers across human capital levels remains the same.

[14] The individual state space is the cartesian product of the spaces associated with the individual state variables, i.e., a, s, z.

[15] Notice that L_g is not equal to H_g, which is the aggregate level of efficient labor units employed at the public sector. In particular,

$$H_g = \sum_{t<T_r} \mu_t \sum_{\theta} \mu_\theta \int \mathbf{I}_{\{s_t'(a,s,z;\theta)=g\}} q_g(t,\theta,z) d\psi_{t,\theta}(a,s,z).$$

[16] For each θ, $\underline{z}(\theta)$ also has to adjust so public vacancies can be filled.

3.5.3 Welfare Criterion

The optimal size of public employment maximizes an ex-ante utilitarian welfare criterion in a stationary equilibrium. Following Conesa et al. [2009], we consider only the welfare of newborn agents. Thus, social welfare reads

$$\sum_{\theta} \mu_{\theta} \int V_1(a, s, z; \theta) d\psi_{1,\theta}(a, s, z).$$

Throughout the paper, we report welfare effects in terms of consumption equivalence. In other words, the welfare effect associated with a given policy is defined by how much lifetime consumption would have to increase uniformly across newborn agents in the benchmark economy in order to equalize social welfare measures across stationary equilibriums.

By adapting the methodology from Flodén [2001] to this environment, we decompose the overall welfare effect of a change in public employment into three categories: (i) the level effect associated with changes in aggregate consumption; (ii) the inequality effect associated with changes in the distribution of consumption; and (iii) the uncertainty effect associated with changes in the degree of insurance in the economy. See Appendix B for more details.

Finally, we also consider a conditional welfare criterion. In particular, for each θ, we calculate the aforementioned welfare effects considering

$$\int V_1(a, s, z; \theta) d\psi_{1,\theta}(a, s, z).$$

The aim is to study how welfare effects vary across groups with different levels of human capital.

4 Quantitative Analysis

This section assesses quantitatively the equilibrium effects of public employment on welfare. The algorithm used to solve numerically for the stationary recursive equilibrium is standard. We use value function iterations to solve the household problem and a variant of the algorithm suggested by Imrohoroglu et al. [1999], augmented with an extra loop to pin down, for each θ, the value of $\underline{z}(\theta)$ that implies $\lambda(\theta)$ type-θ public employees.

4.1 Calibration

We calibrate the model to match some characteristics of the Brazilian economy. Whenever we calibrate a parameter to target a specific aggregate variable, we consider its annual average for the periods between 2000 and 2009. We also use the 2005 *Pesquisa Nacional por Amostra de Domicílios* (PNAD) – an annual cross-sectional household data survey – to calibrate the parameters associated with the distributions of workers and wages. In Appendix C we describe the sample of workers we use to tabulate these distributions.

4.1.1 Demography

We assume agents are born (i.e. enter the labor market) with 25 years old. They may live up to the age of 80, when death is certain. Each period corresponds to a five years interval, so that $T = 12$. The agents retire at the age of 65, that is $T_r = 9$. We calculate the age-dependent probability of survival, π_t, from mortality data provided by the *Instituto Brasileiro de Geografia e Estatística* (IBGE) – the government department responsible for collecting data and processing official statistics.[17]

4.1.2 Productivity and Public Wage Setting

In order to specify the productivity profile, one must proxy the level of human capital θ with an observable variable. In particular, we proxy θ by the degree of education an

[17] In particular, $\pi_t \in \{1, 0.991, 0.990, 0.987, 0.982, 0.975, 0.964, 0.948, 0.927, 0.895, 0.844, 0.775\}$.

individual acquired before entering the job market. We consider three levels of θ: (1) at most 10 years of schooling, which includes basic education and incomplete secondary education; (2) between 11 and 14 years of schooling, which includes secondary education and incomplete college education; and (3) at least 15 years of schooling, which includes college education.[18]

The distribution of θ is obtained from the PNAD. In particular, we calculate the share of workers in each education group: $\mu_{\theta_1} = 0.59$ (basic or no education), $\mu_{\theta_2} = 0.31$ (secondary education), and $\mu_{\theta_3} = 0.10$ (college education).

Recall that the productivity profile in the private sector is given by:

$$q_y(t, \theta, z) = \exp\{\gamma_1^y \cdot (t-1) + \gamma_2^y \cdot (t-1)^2 + \gamma_3^y(\theta) + z\}.$$

Under the assumption that markets behave competitively, by using data at the individual level on wages, experience in the labor market and schooling, obtained from the PNAD, we estimate $\gamma_1^y = 0.124$, $\gamma_2^y = -0.009$, $\gamma_3^y(\theta_1) = 0$, $\gamma_3^y(\theta_2) = 0.53$, and $\gamma_3^y(\theta_3) = 1.47$. The estimation procedure is described in Appendix C.

However, even in a competitive equilibrium, the government may not remunerate productivity competitively. Hence, an analogous estimation procedure for public workers does not represent their productivity profile. Instead, we interpret it as the wage setting rule in the public sector, given by:

$$\hat{q}_g(t, \theta, z) = \exp\{\hat{\gamma}_1^g \cdot (t-1) + \hat{\gamma}_2^g \cdot (t-1)^2 + \hat{\gamma}_3^g(\theta) + \hat{\gamma}_4^g(z)\}.$$

In particular, $\hat{\gamma}_1^g = 0.048$, $\hat{\gamma}_2^g = -0.005$, $\hat{\gamma}_3^g(\theta_1) = 0$, $\hat{\gamma}_3^g(\theta_2) = 0.54$, and $\hat{\gamma}_3^g(\theta_3) = 1.24$. It remains to specify $\hat{\gamma}_4^g(\cdot)$, to which we turn later.

In the absence of a good strategy to estimate the productivity profile in the public sector, we suppose that productivity profiles are the same in both sectors but the gov-

[18] In Brazil, depending on the job description, the government may require basic, secondary or college education from a candidate to fill a vacancy. Hence, we consider only these three levels of schooling.

ernment does not remunerate productivity competitively. That is, $q_g(t, \theta, z) = q_y(t, \theta, z)$. We acknowledge this is an extreme assumption. Hence, we check sensitivity by reporting results when productivity profile varies across sectors and government remunerates productivity competitively. That is, $q_g(t, \theta, z) = \hat{q}_g(t, \theta, z)$. In practice, reality should be in between these extremes scenarios.

4.1.3 Idiosyncratic Risk

The Markov process $\Pi(z', z)$ follows from an approximation of an AR(1) process:

$$z' = \rho z + \varepsilon, \quad \text{where } \varepsilon \sim N(0, \sigma^2)$$

In Brazil, due to the lack of a household panel data survey, such as the Panel Study of Income Dynamics in the U.S., we cannot estimate ρ properly. As an alternative strategy, we set $\rho = 0.82$ based on evidence for the U.S. economy,[19] and then, we use the distribution of residual wages in the private sector to estimate $\sigma^2 = 0.17$. The estimation procedure is described in Appendix C. Importantly, since we use residual wages, the idiosyncratic risk does not absorb some permanent components of the actual productivity profile that are not properly modeled in this paper, but included in the estimated wage equation.

We use Rouwenhorst [1995]'s algorithm with 17 states to approximate this AR(1) process using a Markov chain. We assume that the initial distribution of the idiosyncratic risk is the invariant distribution associated with this Markov chain.

The Rouwenhorst [1995] method has a property that is useful to define $\hat{\gamma}_4^g(\cdot)$, i.e. the function that maps productivity shock z into public wages. In particular, the transition matrix associated with the Markov chain does not depend on the variance of the AR(1) process. Hence, by reducing σ, the values of the states get more compressed, but the transition probabilities remain the same.

[19] The literature estimates this process to be very persistent. Flodén and Lindé [2001], for example, estimate $\rho = 0.91$, whereas French [2005] estimates $\rho = 0.98$ using annual data. Since a period in the model encompasses five years, we set $\rho = 0.96^5$.

Many empirical studies have found that wages in the public sector are more compressed and less dispersed than their counterparts in the private sector. Hence, these empirical regularities can be captured by associating $\hat{\gamma}_4^g(z_i)$, $i = 1, ..., n$, with the i-th state generated by the Rouwenhorst [1995]'s algorithm applied to an AR(1) process with the same persistence $\rho = 0.82$ but a smaller standard deviation than σ, say $\hat{\sigma}$.[20] As the states get more compressed, whoever draws a low (high) z would be paid more (less) in the public than in the private sector. Hence, the possibility to enter the public sector is a source of insurance in this economy.

As described in Appendix C, we use the distribution of residual wages in the public sector to estimate $\hat{\sigma}$. In particular, we find that $\hat{\sigma}^2 = 0.12$, which corresponds to 71 percent of its counterpart in the private sector, σ^2.

4.1.4 Preferences and Private Production

We set the coefficient of relative risk aversion γ at 2.5, which is within the range used in the literature. In addition, we set β to match the annual ratio of capital to output of 3, which is obtained from national accounts provided by the IBGE.

The capital share α in Brazil is around 0.4 (e.g. Paes and Bugarin [2006]). The productivity of public goods ξ is set to 0.1. In the absence of a consensus on the magnitude of this coefficient, with estimates ranging from zero (e.g. Holtz-Eakin [1994]) to 0.2 (e.g. Lynde and Richmond [1993]), we perform sensitivity analysis on ξ. Finally, δ_y is set to match the annual ratio of investment to capital of 0.05, obtained from national accounts provided by the IBGE.

4.1.5 Public Sector

The production function in the public sector is calibrated as follows. We set δ_g to match the annual ratio of public investment to public capital of 0.04, which is obtained from national accounts provided by the IBGE. Since public goods are not tradeable in the

[20] In an extreme scenario in which $\hat{\sigma} = 0$, $\hat{\gamma}_4^g(\cdot)$ becomes constant.

market, their value are proxied by the IBGE through information on production costs. In particular, the ratio of public goods to output is 0.14. We normalize A_g to match this figure.

In the absence of information on η, which is the parameter in the public production technology, we set it equal to its counterpart in the private sector α, which is 0.4. We perform sensitivity analysis on η.

We follow Pereira and Ferreira [2010] to calibrate some tax instruments. In particular, by using data on tax revenues and macroeconomic variables, we calculate the average consumption, labor income and capital tax rates, which are $\tau_c = 0.23$, $\tau_h = 0.21$ and $\tau_a = 0.14$, respectively. We follow the tax code to set the tax rate on bequests τ_{beq} at 0.04 and the contribution to the pension system τ_{ss} at 0.11, whereas the flat benefit b is set to match the pension deficits as a percentage of output, obtained from the *Ministério da Previdência e Assistência Social* – the government branch responsible for managing the pension system.

The ratios of public investment I_g to output, lump-sum transfers Υ to output and debt D to output are set to 2.2, 8.4 and 47 percent, respectively. These figures are provided by the IBGE and the National Treasury. Note that public consumption C_g is left free to balance the government budget.

Finally, we consider parameters related to public employment and wage policies. The public wage rate is set to match the ratio of the public wage bill to the private wage bill, i.e. $w_g H_g / w_y H_y = 0.3$, provided by the IBGE. Recall that $\lambda(\theta_1) + \lambda(\theta_2) + \lambda(\theta_3)$ is the share of public workers, which is 13.5 percent according to the PNAD. Hence, it remains to calibrate $\lambda(\theta_1)$ and $\lambda(\theta_2)$ to match the shares of public workers with basic or no education (i.e. 27 percent) and secondary education (i.e. 45 percent), respectively. These figures are also obtained from the PNAD.

4.1.6 Summary

Table 2 summarizes the values assigned to internally calibrated parameters.

parameters	target variable	data	model
$\beta = 0.85$	annual K_y/Y	3	2.96
$\delta_y = 0.23$	annual I_y/K_y	0.05	0.05
$A_g = 0.74$	G/Y	0.14	0.13
$\delta_g = 0.18$	annual I_g/K_g	0.04	0.04
$w_g = 0.44$	$w_g H_g / w_y H_y$	0.30	0.31
$b = 0.29$	pension deficits$/Y$	0.014	0.014

Table 2: Internally calibrated parameters.

The model is also able to generate some statistics, other than targeted variables, that represent the Brazilian economy during the 2000s. The Gini coefficient for earnings, for instance, is 0.48 in the calibrated model, which is close to 0.53, calculated with data from the PNAD. The ratio of the average wage paid in the public sector to the average wage paid in the private sector is 1.99 in the calibrated model, which is close to 1.77, also calculated with data from the PNAD.[21] The calibrated model also generates a high Gini coefficient for wealth, 0.72, and a high annual interest rate, 7.6 percent, which characterize the Brazilian economy during the 2000s.

4.2 Results

This section reports the results. First, we discuss whether the model is able to replicate some dimensions of the distribution of public workers across age and education groups. Second, we study the welfare implications of different public employment policies. Finally, we perform some sensitivity analysis.

4.2.1 Public Employment

The main objective of this paper is to study the welfare effects of public employment accounting for its role in improving the insurance degree in the economy. Hence, it

[21] In the data, these ratios are 1.28 for the basic or no education group, 1.23 for the secondary education group and 0.9 for the college education group. In the calibrated model, these figures are 2.2 for the basic or no education group, 2.1 for the secondary education group and 0.6 for the college education group. Hence, the model replicates the fact that, on average, individuals with college education earn more in the private sector, but individuals with a lower degree of education earn less in the private sector.

is desirable that the model replicates some features in the data associated with public employment. Due to data availability, we only consider the distribution across age and education groups.[22]

In Figure 1, we compare the distribution of public workers across age groups in the model against the data, tabulated from the PNAD.

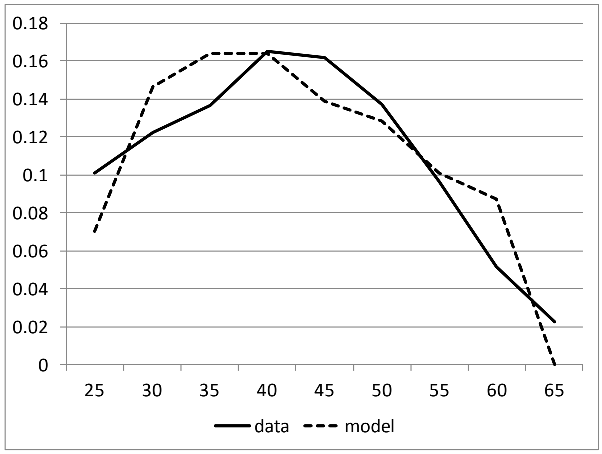

Figure 1: Distribution of public workers across age groups.

The share of workers increases up to a certain age, 35 in the model but 40 in the data, remains nearly flat for one period, and then, declines. Hence, the model can replicate the general pattern of the distribution. However, the model predicts higher shares of both young and old workers in the public sector. We conclude that, albeit imperfectly, this model can replicate some dimensions of the distribution of public employment across age groups.

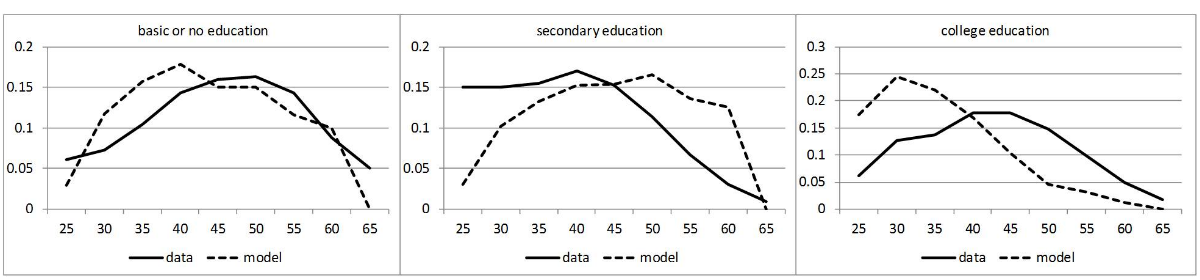

Figure 2: Distribution of public workers across age and education groups.

[22]As each period in the model encompasses five years, we fit the data to age intervals. For the age of 25, for example, we group agents who have between 21 and 25 years of age; for the age of 30, we group the agents who have between 26 and 30 years of age; and so on.

Figure 2 plots de distribution of public workers across age groups for each level of human capital. Notice that the higher share of young workers in the model is partially due to those with college education, whereas the higher share of old workers is due to those with secondary education.

We conjecture that these discrepancies are due to three facts: (1) the model does not allow retirement at earlier ages, so that the share of old workers are higher than in the data; (2) agents have a more generous pension benefit if they stay longer in the public sector, so that the share of middle-aged workers is higher in the data; and (3) some public jobs require more previous training and experience than others, which might explain a smaller share of young workers with college education in the data.

It is feasible to incorporate some features that may help the model to match these distributions. For example, we may properly model the pension system and retirement choice to account for points (1) and (2). During the 2000s, the pension schemes for public and private workers differed in contribution rates and benefit payments. Moreover, public workers might retire earlier than private workers if they wanted so.[23] Since we would like isolate the role of the public sector as an insurance provider through a less uncertain wage schedule, rather than a more generous pension scheme, we abstract from these Brazilian specificities that would further complicate the model and exposition.

Similarly, in order to account for point (3), we may add a cost to occupy public vacancies that increases with human capital. Hence, workers would have to accumulate a bit before entering the public sector. In the absence of information on this cost function, we choose to not include it in the model.

Finally, Figure 3 plots the distribution of private and public workers across productivity shocks for each level of education. In particular, it plots the distribution across the indexes, $i = 1, ..., 17$, of the productivity shock rather than levels $z_1, z_2, ..., z_{17}$. The thresholds $\underline{z}(\theta)$ to enter the public sector associated with basic or no education, secondary

[23] See Gloom et al. [2009] for a description of the convoluted Brazilian pension system during the 2000s. This paper develops a macroeconomic model to study a reform that induces civil servants to retire later.

education and college education are indexed by $i = 12, 11$ and 8, respectively.

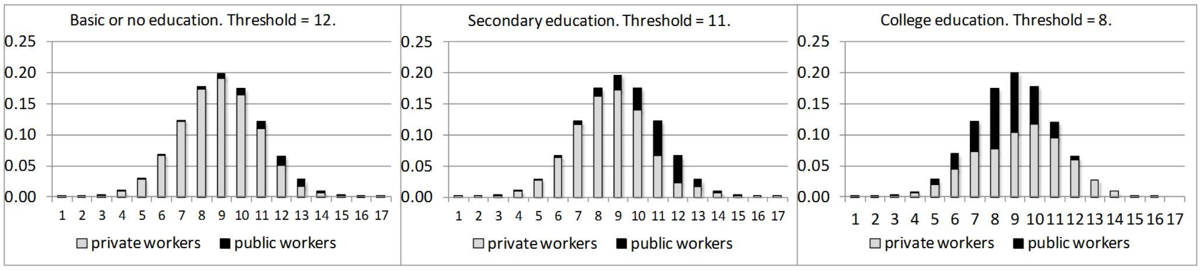

Figure 3: Distribution of public workers across productivity shocks.

Notice that public employment is effective in increasing the welfare of intermediate and relatively high types. Indeed, most of the agents with very high shocks prefer to work in the private sector, whereas those with low shocks cannot enter the public sector. Moreover, a sizable number of individuals benefits from the fact that they cannot be fired, i.e. their productivity shocks fall below the threshold levels and, thus, they benefit from insurance provided by the government.

4.2.2 Welfare Effects

This section shows our main results. In particular, we report the welfare implications of different sizes of public employment. Once the government changes the size of public employment, it affects the public wage bill and, thus, has to adjust its fiscal policy in order to balance its budget. We consider three types of policy adjustments: (1) consumption taxes τ_c; (2) capital taxes τ_a and (3) lump-sum transfers Υ.[24]

Results considering a lump-sum tax adjustment should be read with caution. As we argue above, lump-sum transfers capture the role of large welfare programs which require public workers to operate them. Hence, in practice, exchanging public employment for lump-sum transfers might not be feasible. In contrast, a simple change in the capital or consumption tax rate could be designed without an effective change in public employment.

[24] We do not consider income taxes τ_h because labor is supplied inelastically. Hence, adjustments in τ_h are not distortive.

Recall that in our benchmark calibration, public employment is set to 13.5 percent of the workforce. Figure 4 plots the welfare gains (y-axis) against the size of public employment ranging from 2 to 16 percent of the workforce (x-axis). If the government tries to hire more than 16 percent of the workforce, it would not be able to fill all open vacancies that require college education.

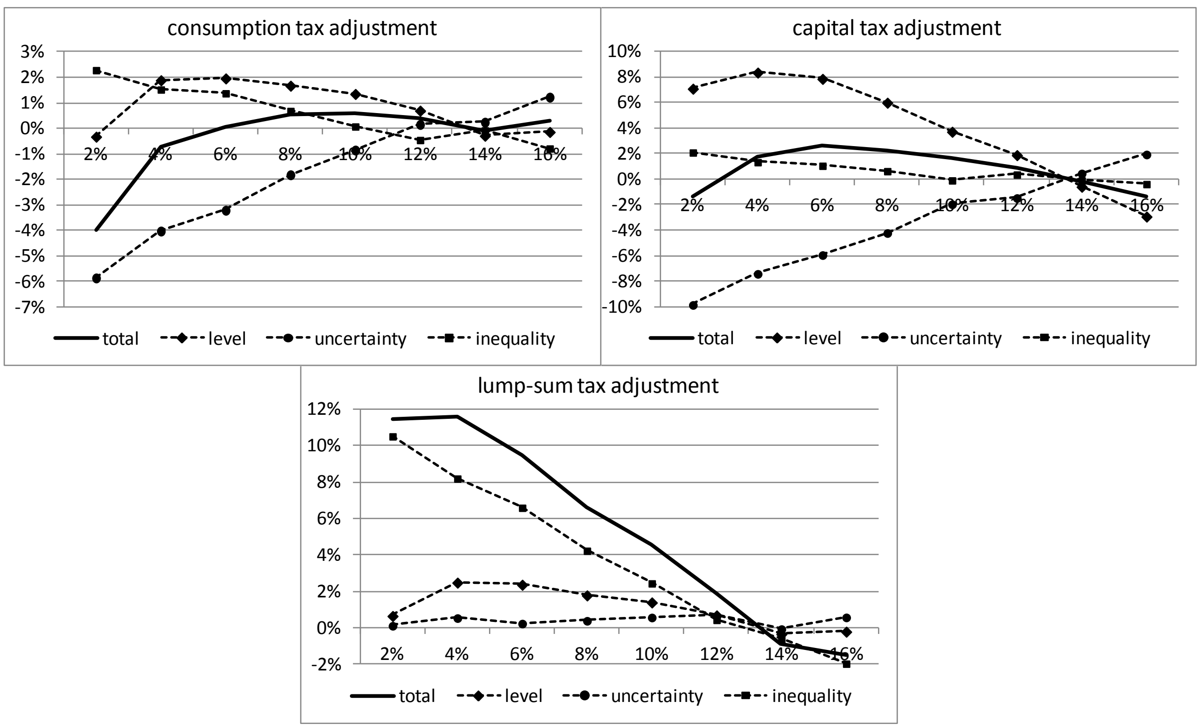

Figure 4: Welfare implications (x-axis: share of public workers; y-axis: welfare effects).

First, consider the experiment with consumption tax adjustment (top-left plot). Social welfare is maximized at any share of public employment ranging from 8 to 12 percent, which is associated with a total welfare effect of nearly 0.5 percent. However, welfare losses due to uncertainty increase at a fast pace as public employment drops. In particular, if the government reduced public employment to 8 percent, welfare losses due to uncertainty would be 1.8 percent. These losses are counteracted by welfare gains of 1.7 and 0.7 percent due to level and inequality, respectively, effects.

Notice that the uncertainty effect increases monotonically with public employment. Hence, a larger government is associated with a higher degree of insurance in the economy. In contrast, the inequality effect decreases monotonically with public employment. Intuitively, as Figure 3 highlights, a sizeable government benefits mostly individuals with intermediate or relatively high levels of productivity shocks. Hence, consumption inequality tend to increase with the size of public employment.[25]

Second, results considering a capital tax adjustment (top-right plot) are qualitatively similar. Quantitatively, the optimal public employment is 6 percent of the workforce, which represents total welfare effects of 2.6 percent. However, the optimal policy generates welfare losses of 6 percent due to a worse degree of insurance in the economy.

Notice that welfare gains due to the level effect and losses due to the uncertainty effect are amplified in comparison with the previous case, in which consumption taxes adjust to balance the budget. If the government reduced public employment from 13.5 percent to 8 percent of the workforce, for example, welfare gains due to the level effect and losses due to the uncertainty effect would be 6 percent and 4.2 percent, respectively.

Finally, welfare gains can be considerably high if the government is allowed to exchange public employment for lump-sum transfers (bottom plot). In this case, the optimal level of public employment ranges from 2 to 4 percent of the workforce, which represents welfare gains of nearly 11.5 percent. These gains are due mainly to the inequality effect. Intuitively, a large public sector benefits mostly individuals with intermediate levels of productivity shocks. Once the size of the government becomes smaller, the extra resources obtained from the reduction in the public wage bill are distributed lump-sum, which particularly improves the welfare of those agents at the bottom of the consumption distribution. Hence, consumption is distributed from intermediate to low types, increasing social welfare.

[25] We also consider a forth scenario – not presented in the paper, but available upon request – in which the government adjusts its own consumption, C_g, instead of a tax instrument. Both uncertainty and inequality effects are quantitatively the same independent on whether the government adjusts C_g or τ_c. However, since adjustments in C_g are not distortive, in this case, the level effect increases monotonically with public employment.

Notice that the uncertainty effect remains fairly constant and close to zero for all shares of public employment. Intuitively, lump-sum transfers are effective to increase the overall degree of insurance in the economy. Hence, the reduction in the degree of insurance due to a smaller government is compensated by an increase due to a higher level of lump-sum transfers.

4.2.3 Welfare Effects by Education Groups

In this section, we decompose the welfare effects by education groups. In particular, for each education group, we calculate total welfare and uncertainty effects. The level effect reported in Figure 4, for instance, does not vary across education groups.

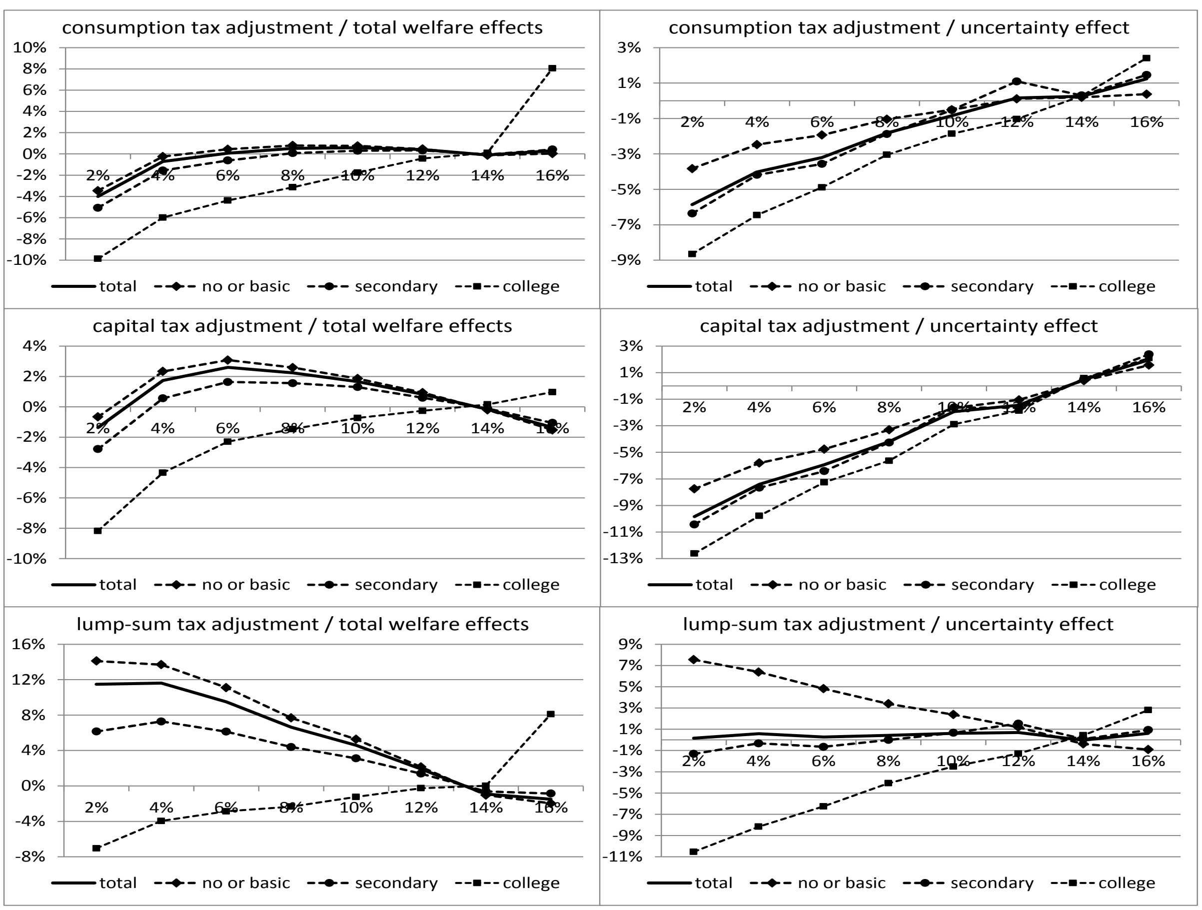

Figure 5: Welfare implications by education groups (x-axis: share of public workers; y-axis: welfare effects).

Figure 5 plots total welfare and uncertainty effects (y-axis) conditional on different levels of θ against the size of public employment, ranging from 2 to 16 percent of the workforce (x-axis).

Independent of the tax instrument used to balance the budget, total welfare gains for individuals with college education increase monotonically with the size of the government (left-plots). Moreover, this education group is the most benefited one by an increase in public employment. A sizeable part of these welfare gains are due to the insurance effect.

The insurance scheme provided by the government for college graduates is particularly effective for two complementary reasons. First, the government hires proportionately more college graduates. Indeed, 37 percent of the workers with college education work in the public sector, whereas only 20 percent and 6 percent of the workers with secondary and basic education, respectively, are public servants. Second, the government hires college graduates with relatively lower realizations of the productivity shock z. Indeed, the threshold for a college graduate to enter the public sector is the 8^{th} highest possible realization of the idiosyncratic risk z, whereas this threshold for an individual with no or basic (secondary) education is the 12^{th} (11^{th}) highest possible realization.[26]

4.3 Sensitivity Analysis

In this section, we check whether our results are sensitive to: (i) a different productivity profile in the public sector; (ii) different values of ξ, which captures the productivity of public goods; (iii) different values of η, which captures the productivity of public capital. In all cases, we recalibrate the model to match the targets in Table 2.

4.3.1 Different Productivity Profiles ($q_g(t, \theta, z) = \hat{q}_g(t, \theta, z)$)

In the absence of a good strategy to estimate the productivity profile in the public sector, our benchmark results consider an extreme case in which productivity profiles are the same in both sectors but the government does not remunerate productivity compet-

[26]Recall that we consider $n = 17$ possible realizations of the idiosyncratic risk.

itively. That is, $q_g(t, \theta, z) = q_y(t, \theta, z)$ for all t, θ, z. In this section, we assume that the productivity profile varies across sectors and government remunerates productivity competitively. That is, $q_g(t, \theta, z) = \hat{q}_g(t, \theta, z)$ for all t, θ, z. In practice, reality should be in between these extremes scenarios.

In a stationary equilibrium, $q_g(t, \theta, z)$ only affects the aggregate level of efficient labor units employed at the public sector H_g. Indeed, since we normalize the public total factor productivity A_g to match the ratio of public goods to product G/Y we observe in the data, any reduction in H_g is absorbed by an increase in A_g.[27] Hence, except for A_g and H_g, the stationary equilibrium has the same properties in both this and the benchmark cases.

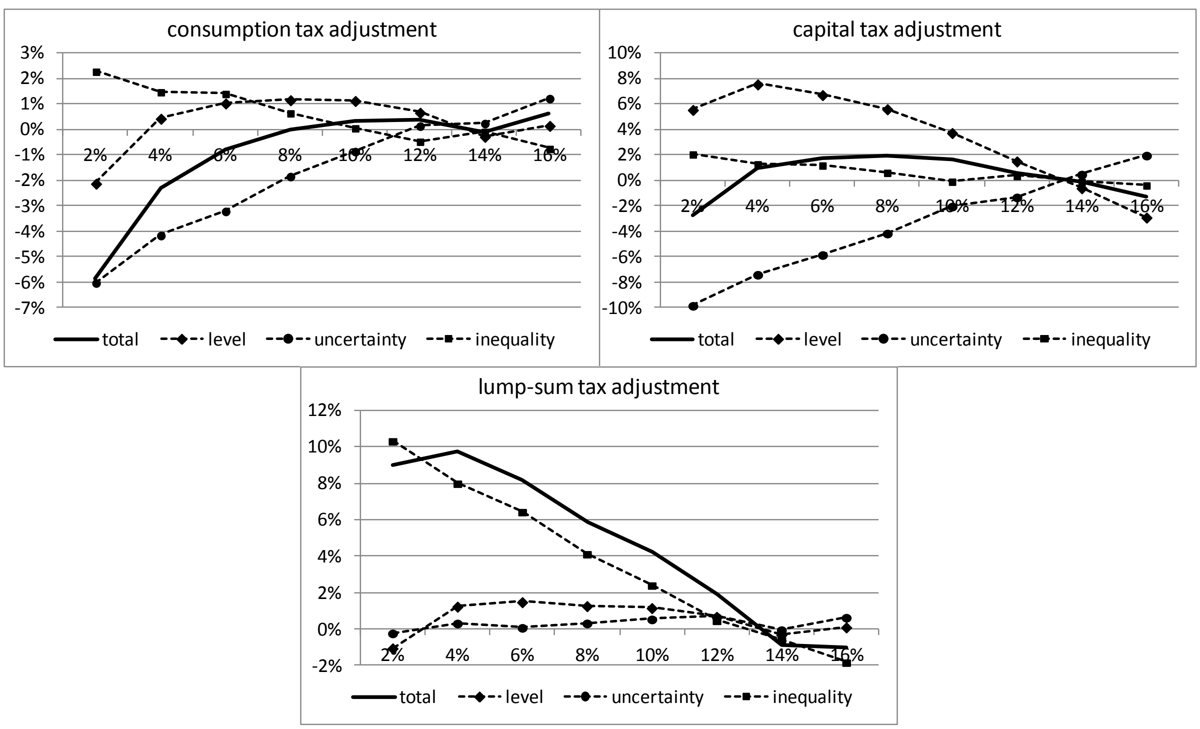

Figure 6: Welfare implications (x-axis: share of public workers; y-axis: welfare effects). Alternative productivity profile.

[27]In particular A_g increases from 0.74 to 0.90.

Figure 6 shows the welfare implications under this alternative productivity profile. It plots the welfare gains (y-axis) against the size of public employment, ranging from 2 to 16 percent of the workforce (x-axis).

A comparison between Figures 4 and 6 shows that the welfare implications due to both uncertainty and inequality effects are almost the same as in the benchmark case. Intuitively, since agents are remunerated according to $w_g \hat{q}_g(t, \theta, z)$ when working in the public sector, the productive profile $q_g(t, \theta, z)$ does not enter directly in their optimization problems. However, they depend indirectly on $q_g(t, \theta, z)$, as it may affect prices r and w and thresholds $\underline{z}(\theta)$ through A_g and H_g. Hence, we conclude that general equilibrium and selection effects are not strong enough to modify the welfare implications due to uncertainty or inequality.

In contrast, welfare gains due to the level effect are smaller in this case. Intuitively, once the government reduces public employment, workers leave a relatively more productive public sector than in the previous case, which mitigates welfare gains. Nonetheless, the difference of level effects is not that large. For example, if the government reduced public employment to 8 percent of the workforce, the level effect would be nearly 0.5 percent higher in the benchmark case independent of the tax instrument used to balance its budget.

4.3.2 Productivity/Production of Public Goods (ξ and η)

In this section, we analyze the role of ξ, which governs the productivity of public goods, and η, which governs the productivity of public capital.

Figure 7 plots total welfare and uncertainty effects (y-axis) for different values of ξ against the size of public employment ranging from 2 to 16 percent of the workforce (x-axis). In particular, we consider $\xi = 0$, $\xi = 0.05$ and $\xi = 0.2$. Recall that the benchmark value of ξ is set at 0.1.

Figure 8 plots total welfare and uncertainty effects (y-axis) for different values of η against the size of public employment ranging from 2 to 16 percent of the workforce (x-axis). In particular, we consider $\eta = 0.3$ and $\eta = 0.5$. Recall that the benchmark value of η is set at 0.4.

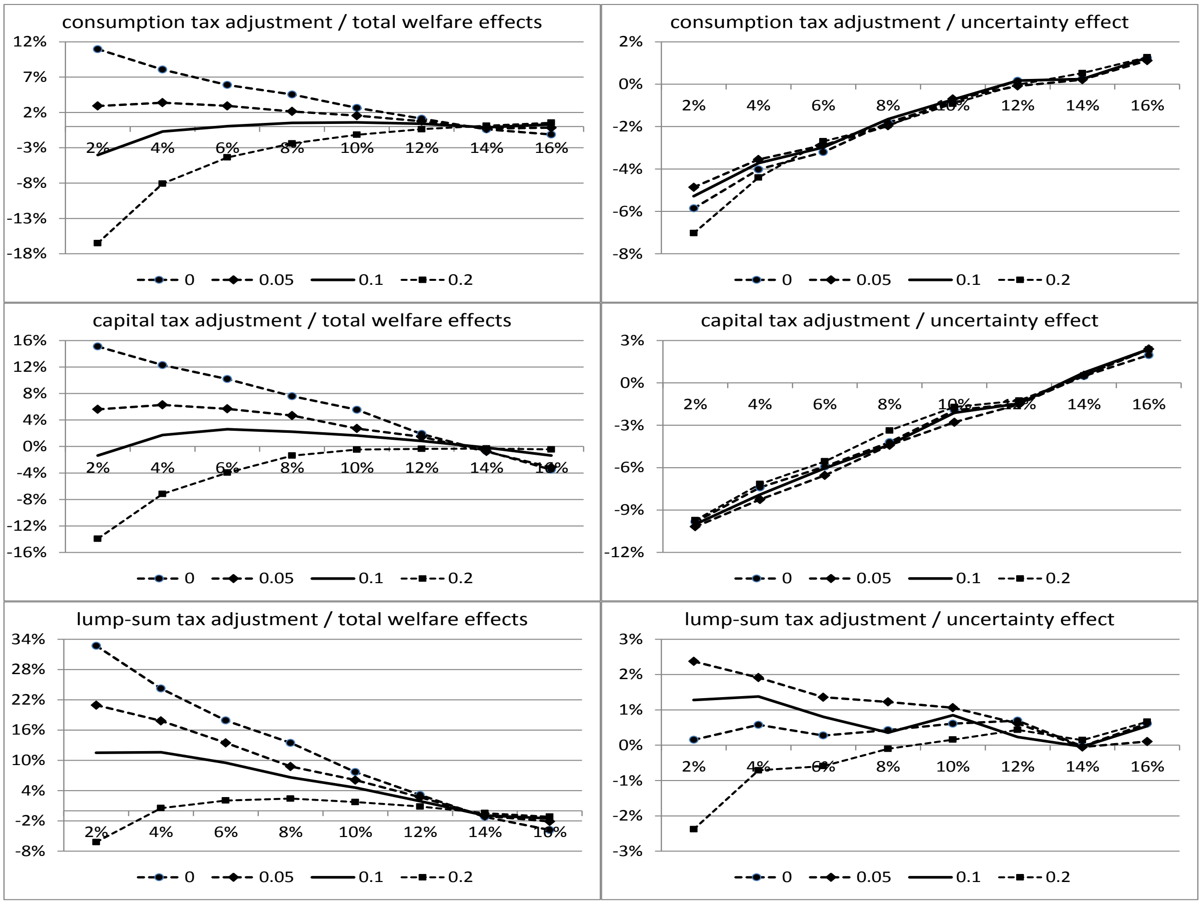

Figure 7: Welfare implications (x-axis: share of public workers; y-axis: welfare effects). Alternative values for ξ.

As in the previous section, the level effect explains most of the changes in welfare gains due to different values of ξ or η (not shown in Figures 7 and 8). A similar intuition applies. Moreover, as Figures 7 and 8 highlight, except for the lump-sum tax adjustment case, the uncertainty effect due to a smaller size of the government does not change much as we vary ξ or η.

We conclude from these sensitivity analyses that, although a misspecification of the technology associated with the public sector might bias social welfare evaluations, the uncertainty effect is fairly robust to misspecification.

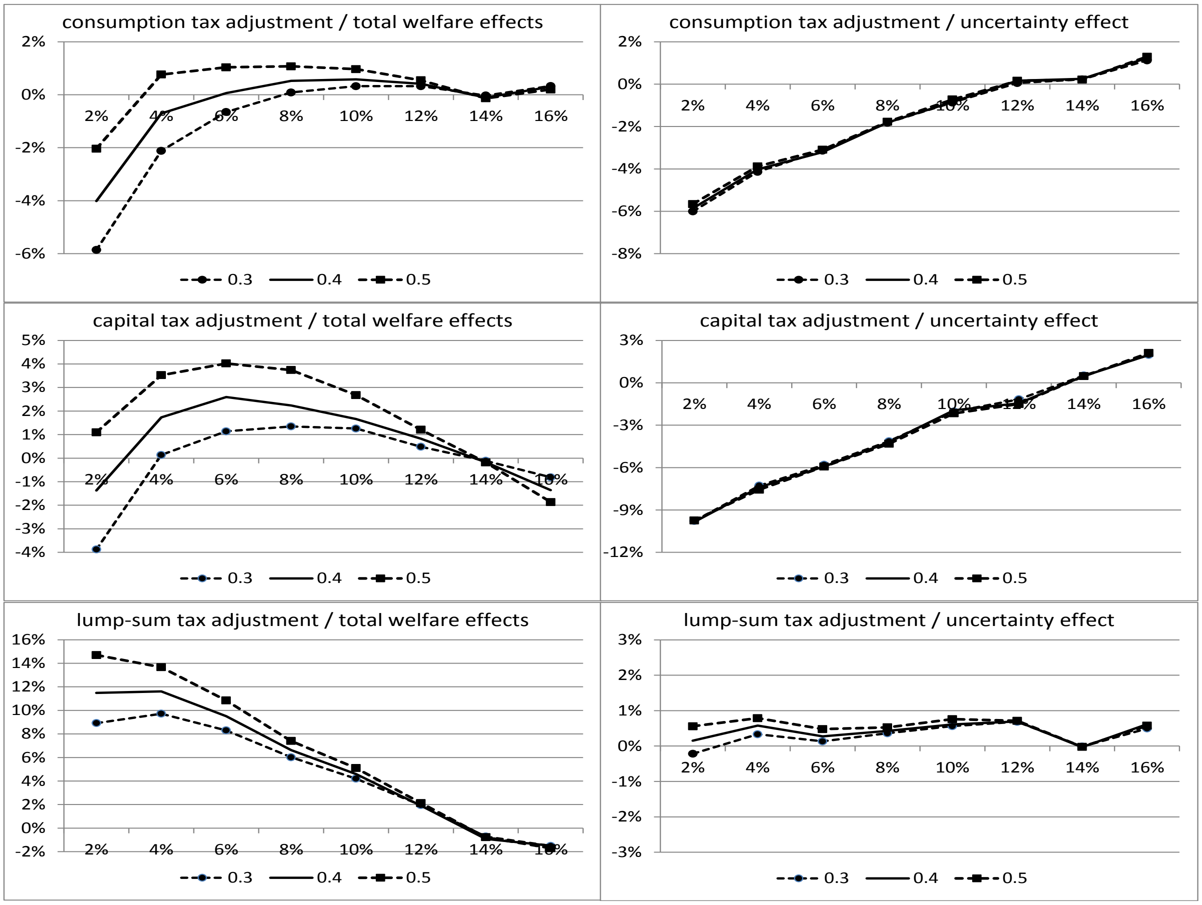

Figure 8: Welfare implications (x-axis: share of public workers; y-axis: welfare effects). Alternative values for η.

5 Conclusion

In this paper, we show that public employment is an important source of social insurance. In our preferred experiment, although optimal public employment is nearly flat, ranging from 8 to 12 percent, if public employment was reduced from 12 to 8 percent of the workforce, losses due to a decrease in the degree of insurance would be nearly 2 percent. Of course, this effect is counteracted by welfare gains due to inequality and level effects. Importantly, the welfare gains due to uncertainty are fairly robust to a missepecification of the production technology associated with the public sector. We also find that individuals with college education are the group most benefited by an increase in public employment. In particular, the public sector provides a more effective insurance scheme, through public wages, to this group.

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Appendix

A Equilibrium Definition

In order to define the equilibrium, we need a framework that accounts for the heterogeneity in the economy. At every point in time, the agents are heterogeneous with regard to their age t that evolves deterministically, a fixed level of human capital, and the individual state $x = (a, s, z)$ that evolve stochastically. Assume that a takes values in the compact $[0, \overline{a}]$. Let $X \equiv [0, \overline{a}] \times \{z_1, ..., z_n\} \times \{y, g\}$ be the state space and $\mathcal{B}(X)$ be the Borel σ-algebra on X. Moreover, let $(X, \mathcal{B}(X), \psi_{t,\theta})$ be a probability space, where $\psi_{t,\theta}$ is a probability measure that returns the fraction of agents with age t and human capital θ for each subset of X in $\mathcal{B}(X)$.

Since we assume agents are born with zero assets, it follows that the distribution of $t = 1$ agents at any level of human capital θ is given by the exogenous initial distribution of the productivity shock z. At subsequent ages, the distribution of agents in the state space is defined recursively by

$$\psi_{t+1,\theta}(\mathcal{X}) = \int_X p_{t,\theta}(x, \mathcal{X}) d\psi_{t,\theta}(x), \text{ for all } \mathcal{X} \in \mathcal{B}(X),$$

where the transition function $p_{t,\theta}(x, \mathcal{X})$ expresses the probability that an agent with age t, human capital θ and individual state x fall into the set $\mathcal{X} \in \mathcal{B}(X)$ in the next period.

We are ready to define the equilibrium concept. A stationary competitive recursive equilibrium consists of policy functions for the agents $c_t(x; \theta)$, $a'_t(x; \theta)$ and $s'_t(x; \theta)$; value functions $V_t(x; \theta)$ and $\tilde{V}_t(a)$; accidental bequests beq; policies for the firm K_y and H_y; prices w_y and r; government policies C_g, G and $\underline{z}(\theta)$, for all θ; and stationary distributions $\psi_{t,\theta}$, for all t, θ such that:

1. Given prices and government policies, the policy functions $c_t(x; \theta)$, $a'_t(x; \theta)$ and $s'_t(x; \theta)$ solve the agent problem defined in the text, with $V_t(x; \theta)$ and $\tilde{V}_t(a)$ being the associated value functions.

2. Given prices r and w_y, policies for the firm K_y and H_y maximize profits, i.e. $G^{\xi} K_y^{\alpha} H_y^{1-\alpha} - (r + \delta_y) K_y - w_y H_y$.

3. Accidental bequests, $beq = \sum_t \mu_t (1 - \pi_{t+1}) \sum_\theta \mu_\theta \int_X a'_t(x;\theta) d\psi_{t,\theta}(x)$, are distributed lump-sum to all agents.

4. Market clears:

$$\begin{aligned} \text{Capital market} \quad &: \quad \sum_t \mu_t \sum_\theta \mu_\theta \int_X a d\psi_{t,\theta}(x) = K_y + D \\ \text{Private labor market} \quad &: \quad \sum_{t<T_r} \mu_t \sum_\theta \mu_\theta \int_X \int \mathbf{I}_{\{s'_t(x;\theta)=y\}} q_y(t,\theta,z) d\psi_{t,\theta}(x) = H_y \end{aligned}$$

5. The government chooses C_g to balance its budget:

$$\tau_a r(K_y+D) + \tau_c C_y + (\tau_h + \tau_{ss})(w_y H_y + w_g H_g) + \tau_{beq} beq = C_g + I_g + \Upsilon + rD + w_g H_g + \sum_{t \geq T_r} \mu_t b,$$

where the other government policies – defined in the text – are treated as parameters in the computation of the benchmark economy.

6. The production of public goods is given by $G = A_g K_g^{\eta} H_g^{1-\eta}$, where $K_g = I_g / \delta_g$ and $H_g = \sum_{t<T_r} \mu_t \sum_\theta \mu_\theta \int \mathbf{I}_{\{s'_t(a,s,z;\theta)=g\}} q_g(t,\theta,z) d\psi_{t,\theta}(a,s,z)$.

7. For each θ, the government sets a minimum level of required productivity $\underline{z}(\theta)$ in order to hire $\lambda(\theta)$ workers, which is specified exogenously.

8. Stationary distributions are defined recursively by

$$\psi_{t+1,\theta}(\mathcal{X}) = \int_X p_{t,\theta}(x, \mathcal{X}) d\psi_{t,\theta}(x), \text{ for all } \mathcal{X} \in \mathcal{B}(X),$$

with $\psi_{1,\theta}$ being the invariant distribution of the productivity shock. Moreover, the transition probability function $p_{t,\theta}(x, \mathcal{X})$ is consistent with the policy functions for the agents and the stochastic process for the productivity shock.

B Welfare Decomposition

The methodology used to decompose the welfare gains is based on Flodén [2001]. In particular, we adapt it to an environment with overlapping generations in which social welfare weights only newborn agents under the veil of ignorance. For further discussion on this methodology we refer the aforementioned article.

First, note that the expected lifetime utility of a newborn agent, i.e. with age $t = 1$, with human capital θ at state (a, z, s) is given by

$$V_1(a, s, z; \theta) = \mathbb{E}\left[\sum_{t=1}^{T} \beta^{t-1} \left(\prod_{i=1}^{t} \pi_i \right) \frac{c_t^{1-\gamma}}{1-\gamma} \middle| (a, s, z) \right].$$

The ex-ante utilitarian social welfare is given by the expected lifetime utility of a newborn agent under the veil of ignorance, which reads

$$W = \sum_{\theta} \mu_\theta \int V_1(a, s, z; \theta) d\psi_{1,\theta}(a, s, z).$$

Define economy A as the benchmark economy and economy B as the new stationary equilibrium after the policy change. We define total welfare gains ω by how much lifetime consumption has to increase uniformly across newborn agents in the benchmark economy in order to equalize welfare measures across stationary equilibriums.

Definition 1. *The total welfare gains ω of a given policy change is defined implicitly by*

$$\sum_{\theta} \mu_\theta \int \mathbb{E}\left[\sum_{t=1}^{T} \beta^{t-1} \left(\prod_{i=1}^{t} \pi_i \right) \frac{[(1+\omega)c_t^A]^{1-\gamma}}{1-\gamma} \middle| (a, s, z) \right] d\psi_{1,\theta}(a, s, z) = W^B.$$

Notice we use superscripts A and B to denote equilibrium objects in their respective economies. The left hand side measures the social welfare under a hypothetical percentage change of ω in lifetime consumption, while the right hand side measures social welfare under the new policy. Finally, it can be shown that $\omega = (W^B/W^A)^{1/(1-\gamma)} - 1$.

The total welfare effect can be decomposed into three categories: (i) the level effect associated with changes in aggregate consumption; (ii) the inequality effect associated with changes in the distribution of consumption; and (iii) the uncertainty effect associated with changes in the degree of uncertainty in the economy.

Consider the level effect. Define the average consumption by

$$C = \sum_t \mu_t \sum_\theta \mu_\theta \int c_t(a,s,z;\theta) d\psi_{t,\theta}(a,s,z).$$

The level effect ω^{lev} is the percentage change in average consumption due to the new policy.

Definition 2. *The level effect ω^{lev} is given by*

$$\omega^{lev} = \frac{C^B}{C^A} - 1.$$

Consider the inequality and uncertainty effects. Let the certainty equivalent consumption bundle $\{\bar{c}(a,s,z;\theta)\}_{t=1}^T$ of a newborn agent at state (a,s,z) with human capital θ be defined implicitly by

$$V_1(a,s,z;\theta) = \sum_{t=1}^T \beta^{t-1} \left(\prod_{i=1}^t \pi_i \right) \frac{\bar{c}(a,s,z;\theta)^{1-\gamma}}{1-\gamma}.$$

Hence, the average certainty equivalent consumption is given by

$$\bar{C} = \sum_\theta \mu_\theta \int \bar{c}(a,s,z;\theta) d\psi_{1,\theta}(a,s,z).$$

Let p^{unc} and p^{ine} be the cost associated with uncertainty and inequality, respectively. In particular, p^{unc} is implicitly defined by

$$\sum_{t=1}^T \beta^{t-1} \left(\prod_{i=1}^t \pi_i \right) \frac{[(1-p^{unc})C]^{1-\gamma}}{1-\gamma} = \sum_{t=1}^T \beta^{t-1} \left(\prod_{i=1}^t \pi_i \right) \frac{\bar{C}^{1-\gamma}}{1-\gamma}.$$

In a stationary equilibrium, p^{unc} captures the cost of eliminating uncertainty in an equalitarian society, in which agents consume the same amount of goods. It can be shown that $p^{unc} = \bar{C}/C - 1$.

Definition 3. *The uncertainty effect* ω^{unc} *is given by*

$$\omega^{unc} = \frac{1 - p^{unc,B}}{1 - p^{unc,A}} - 1 = \frac{\bar{C}^B}{\bar{C}^A}\frac{C^A}{C^B} - 1.$$

Similarly, p^{ine} is implicitly defined by

$$\sum_{t=1}^{T} \beta^{t-1} \left(\prod_{i=1}^{t} \pi_i \right) \frac{[(1 - p^{ine})\bar{C}]^{1-\gamma}}{1 - \gamma} = W.$$

In a stationary equilibrium, p^{ine} captures the cost of eliminating inequality by giving the same average certainty equivalent consumption to all newborn agents. It can be shown that $p^{ine} = W^{1/(1-\gamma)}/\bar{C} \times constant - 1$.

Definition 4. *The inequality effect* ω^{ine} *is given by*

$$\omega^{ine} = \frac{1 - p^{ine,B}}{1 - p^{ine,A}} - 1 = \frac{\bar{C}^A}{\bar{C}^B} \left(\frac{W^B}{W^A} \right)^{\frac{1}{1-\gamma}} - 1.$$

Finally, we can apply the previous definitions to prove the following proposition adapted from Flodén [2001].

Proposition 1. *Total welfare effect* ω *is decomposable into a level effect* ω^{lev}*, a inequality effect* ω^{ine}*, and a uncertainty effect* ω^{unc} *according to the following equation:*

$$(1 + \omega) = (1 + \omega^{lev})(1 + \omega^{ine})(1 + \omega^{unc}).$$

C Wage Setting Rules

In order to calibrate the model and estimate the wage setting rules, we use data on workers from the PNAD. Following Braga et al. [2009], we restrict the sample to those workers who had worked between 20 and 70 hours and received positive earnings in the week of reference. As specified in the model, we only consider workers who are between 21 and 80 years old.

The variables of interest are experience t an individual has, which is proxied by the difference of the current age and the age at the first job,[28] The aim is to estimate

$$\ln(wage) = \text{constant}+\gamma_1^y\cdot(t-1)+\gamma_2^y\cdot(t-1)^2+\gamma_3^y(\theta)+z = \text{constant}+\gamma_1^y t+\gamma_2^y t^2+\gamma_3^y(\theta)+\rho z_{-1}+\varepsilon,$$

where $wage$ is the hourly wage paid in the private sector according to the wage setting rule defined in the main text. Notice that $\gamma_3^y(\theta_i)$ is the coefficient associated with the dummy variable for the i-th level of schooling.

We omit z_{-1}, which is non-observable, and estimate the equation above by ordinary least square. In principle, the estimated coefficients could be biased as selection in the public sector may induce a correlation between z_{-1} and the other variables of interest (levels of schooling and experience). Hence, in order to mitigate this concern, we also control for individual characteristics that might correlate with z_{-1}, such as tenure in the job, and dummies whether the individual is male, white, head of the household, has a farm job, and lives in an urban area.[29]

Moreover, by controlling for these individual characteristics, we claim that the variance of the residual, which is $z = \rho z_{-1} + \varepsilon$, captures the residual wage inequality. Notice that $\text{var}(z) = \sigma^2/(1-\rho^2)$, where ρ and σ are the parameters associated with the AR(1) process for z. Therefore, after specifying a value for ρ and estimating $\text{var}(z)$, we are able

[28] In order to make model and data compatible, we set $t = 1$ for individuals with experience between 0 and 4 years, $t = 2$ for those with experience between 5 and 9 years, and so on.

[29] For a small number of workers, at least one of these variables is misspecified. We exclude them from the sample. We end up with 19,873 public workers and 116,699 private workers. All descriptive statistics and estimations are weighted to make them representative of Brazil.

to calculate σ.

Finally, by relying on this same methodology, we also estimate the public wage setting rule and calculate $\hat{\sigma}$. Results are reported in Sections 4.1.2 and 4.1.3.

Departamento de Economia PUC-Rio
Pontifícia Universidade Católica do Rio de Janeiro
Rua Marques de Sâo Vicente 225 - Rio de Janeiro 22453-900, RJ
Tel.(21) 31141078 Fax (21) 31141084
www.econ.puc-rio.br
flavia@econ.puc-rio.br