TEXTO PARA DISCUSSÃO

No. 624

Economic gains of realized volatility in the Brazilian stock market

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Abstract

This paper evaluates the economic gains associated with following a volatility timing strategy based on a multivariate model of realized volatility. To study this issue we build a high frequency database with the most actively traded Brazilian stocks. Comparing with traditional volatility methods, we find that economic gains associated with realized measures perform well when estimation risk is controlled and increase proportionally to the target return. When expected returns are bootstrapped, however, performance fees are not significant, which is an indication that economic gains of realized volatility are offset by estimation risk.

Keywords: Realized volatility, utility, forecasting.

JEL Code: G11, G17.

1. Introduction

Given the growth of financial markets and the increasing complexity of its securities, volatility models play an essential role to help the task of risk management and investment decisions. Since realization of volatility returns based on daily data is not observable, the traditional approach is to invoke parametric assumptions regarding the evolution of the first and second moments of the returns, which is the idea behind ARCH and stochastic volatility models. Nevertheless, these models fail to capture some stylized facts such as autocorrelation persistence and fat tail of returns. The availability of intraday data opens up the possibility of approximating volatility directly from asset returns. The use of an observable variable, in turn, facilitates the task of dealing with problems that involves a significant number of assets. Indeed, traditional methods suffer from the curse of dimensionality, which is to say, the difficulty of these methods to handle with a wider range of assets.

The advantages of realized measures have been extensively analyzed in the recent period, when technological barriers have been gradually surmounted so as to provide the kind of data necessary to its calculation. Andersen et al (2003) compared it to traditional methods and confirmed its superiority in terms of forecasting performance. Identical conclusion has been reached by many concurrent studies, like Engle et al (2008). Previous findings relating microstructure parameters and volatility were revisited considering realized measures. Chan & Fong (2006), for instance, found that trading volume is the main factor driving the relationship between volume and realized volatility, as opposed to studies that pointed out order imbalance as the most important one. In Brazil, Carvalho et al (2006) found that returns displayed a normal distributional when standardized by realized measures, a useful property concerning risk management purposes, namely Value-at-Risk statistics. The authors based their conclusions on a sample of the five most liquid stocks traded at the domestic stock exchange, sampling at a 15-min frequency. In spite of the apparent consensus over the subject, there are many relevant issues that deserve attention, in particular the bias originated by microstructure noise and measurement errors. McAleer & Medeiros (2008) documented a review of the literature, stressing the future improvements that must be made in order to deal with such biases.

The objective of this paper is to evaluate the economic gains associated with the use of realized measures in the context of an investment decision where investors take conditional volatility forecasts as the main parameter in the portfolio optimization problem: the so-called volatility timing strategy proposed by Fleming et al (2001). We examine this issue by comparing forecasts of the covariance matrix obtained by means of a multivariate version of Corsi's Heterogeneous Auto-Regressive (HAR) model with the ones provided by traditional volatility models. Our database consists of high frequency transactions prices from the twenty most liquid stocks from the Brazilian Stock Exchange, BMF&Bovespa, and covers the period from February 2006 to January 2011.

This paper contributes to the attempt of applying models of realized volatility to a multivariate framework. Alone, this is not an innovation to the literature. However, we employ a greater than usual number of assets and it is not straightforward to infer that the results will continue to hold.

Moreover, the fact that Brazil is an emerging market raises the question whether adaptations to models originally designed to fit consolidated markets are required.

We find that economic gains associated with realized volatility increase proportionally to the target return. We also show that, when target returns are close to the risk-free, portfolios weights are heavily dependent on the risk-free asset. This finding allows us concluding that realized volatility performs better for increasing levels of risk. Using the unconditional mean as a reference for expected returns, an investor would be willing to pay substantial positive fees to switch from a portfolio based on forecasts taken from traditional volatility methods to one based on realized volatility forecasts, when target returns are superior to 15% per year. When expected returns are bootstrapped, although fees are still positive on average, high standard deviation values lead us to conclude that utility gains are equal at a statistical viewpoint. Actually, when estimation risk is significant, average portfolio returns are far apart from target returns irrespective of the volatility measure used, which is an indication that economic gains of realized volatility are offset by estimation risk.

We also perform robustness checks that confirm that, when estimation risk is negligible, economic gains are robust to changes in the parameters of the economic utility and of the optimization problem. Finally, we conclude that the inclusion of an external risk factor, aimed at adapting the model to an emerging economy, do not add in terms of utility gain.

Our results represent an important input to the applicability of realized volatility as a reference for risk estimates for the stock market in Brazil. It provides additional evidence to the literature that links positive economic values associated to realized volatility, as Christoffersen et al (2012), in their study of the benefits of realized volatility measures for option pricing, and Fleming et al (2003), which used it as an input for a volatility timing strategy based on four assets traded at the US futures market.

The text is organized as follows. In Section 2, we provide key realized volatility concepts. Next, we document the database sources and how we compute multivariate volatility. In Section 4, we present our version of the HAR model and describe its application to a multivariate setting.

Then, we present the methodology behind the evaluation of the economic gains and, in Section 6, discuss results and robustness checks. Finally, we offer our concluding remarks in Section 7.

2. Theoretical background: Realized Volatility

We will provide a brief review of the theoretical framework underlying the realized volatility (RV) measure. The theory of Quadratic variation is the baseline to understand how we obtain this direct measure of volatility.

We begin by assuming that logarithm prices (p_t) follow a continuous-time diffusion process given by:

$p_t = p_0 + \int_0^t \mu(s).ds + \int_0^t \sigma(s)dW(s)$, where W(t) is a standard Brownian motion, μ(t) is the mean process with finite variation and σ(t) is the instantaneous volatility which, by definition, is a positive process.

Over the time interval [t-k,t], the continuous compound return ($r_{t,k} = p_t - p_{t-k}$) is given by the following process:

$$r_t = \int_{t-k}^t \mu(s).ds + \int_{t-k}^t \sigma(s)dW(s) \qquad (2.1)$$

When we sum up the contribution of the mean component, μ(t), to the variation of returns we will find out it can be ignored. This is because $\mu(t).dt$ is of lower order of magnitude when compared to the second term $\sigma(t)dW(t)$ in terms of second order properties (see Andersen & Benzoni (2008)). Then, Quadratic Variation (QV) is defined as follows:

$$QV(t,k) = \int_{t-k}^t \sigma(s)^2 ds \qquad (2.2)$$

Suppose that one has all available information on intraday returns of an asset making it possible to calculate the sum of the squared returns sampled at a given frequency, over a trading day:

$$RV_t = \sum_{i=1}^T r_t^2 \qquad (2.3)$$

Where RV is the Realized Volatility measure.

With no microstructure noise, Andersen et al (2003) showed that QV converges in probability to RV. So, RV, defined as the sum of squared intraday returns, is the discrete version of the quadratic variation process. However, it does not come without a cost as it raises a set of issues related to microstructure of transaction that will be discussed in section 3, as we describe the construction of the database.

3. Database construction

We use a database that contains all intraday trading prices of the stocks traded at BM&FBovespa. The time series ranges from February/2006 to January/2011 and we select the twenty stocks listed in Table 3.1. There are two main reasons behind the outcome of this selection. First of all, we want to work at the highest possible frequency and minimize microstructure biases that arise when working with stocks with low liquidity. As we will see, all of the selected stocks meet this liquidity criterion. Besides, since we are doing out-of-sample forecasts that require a large number of days to work properly, we rule out stocks that belong to the database for less than 300 trading days. In fact, we also benefit from the longer time period of the database[1] by performing robustness checks with different estimation windows.

Stocks in Brazil are divided into preferred (PN) and common (ON) shares. The main difference is that the first type has the priority over dividend distributions, but does not give voting rights. As you can see in the following table, both types of shares are well represented in our database which consists of actively traded assets whose gaps between consecutive trades do not exceed 26.9 seconds. The range of sectors imposed by our stocks' selection just reflects the diversification of Brazilian industry. Thus, the concentration on the basic materials' industry is not a surprise, but other important industries such as financial and utilities are represented as well.

[1] The five-year time-period of the database is superior to other high frequency studies concerning Brazilian stocks (Mota & Fernando (2004), Carvalho et al (2006) and Wink Junior & Pereira (2012) Vicente et al (2014)).

Table 3.1: List of market indicators for each stock

This table provides a set of information regarding all the selected stocks. Stocks' codes and respective industry sectors were obtained from BMF&Bovespa. The total number of transactions is the sum of closed deals for each stock during the sample period. We also report the average gap between transactions which is the average time, measured in seconds, between consecutive trades. The sample period is February/2006 to January/2011.

Stock	Total number of transactions (millions)	Sector	Average gap between transactions (in seconds)
Ambev PN (AMBV4)	1.49	Consumer	26.9
Bradesco PN (BBDC4)	6.00	Financial	6.7
Bradespar PN (BRAP4)	2.12	Financial	18.9
Banco do Brasil ON (BBAS3)	4.70	Financial	8.5
Cemig PN (CMIG4)	3.04	Utilities	13.1
Cia Siderúrgica Nacional ON (CSNA3)	4.36	Basic Materials	9.2
Cyrella ON (CYRE3)	3.69	Real State	10.8
Eletrobras PN (ELET6)	2.03	Utilities	19.6
Gafisa ON (GFAS3)	3.14	Real State	12.7
Gerdau PN (GGBR4)	6.23	Basic Materials	6.4
Petrobras PN (PETR4)	17.53	Basic Materials	2.3
Usiminas PN (USIM5)	4.91	Basic Materials	8.1
Cia Vale do Rio Doce (VALE5)	15.26	Basic Materials	2.6
OGX Petróleo ON (OGXP3)	3.29	Basic Materials	6.4
Itausa Investimentos ON (ITSA4)	5.22	Financial	7.6
Itau Unibanco Holding S.A. ON (ITUB4)	3.67	Financial	3.7
PDG Realty S.A. ON (PDGR3)	2.54	Real State	12.6
Hypermarcas ON (HYPE3)	0.98	Consumer	22.7
BMFBOVESPA S.A. ON (BVMF3)	6.01	Financial	3.2
Redecard ON (RDCD3)	2.58	Financial	10.9

Note: In parenthesis, the stock code at BMF&Bovespa.
ON – common share PN – preferred share.
Source of Information: BMF&Bovespa.

Before constructing the realized volatility estimates, the first decision concerns the sampling frequency. The choice of the optimal frequency involves a trade-off between microstructure issues and loss of information as we will discuss below. If we increase the sampling frequency, we bring to light microstructure problems, such as the bid-ask bounce and error measurement due to price discreteness. In this respect, Aït-Sahalia (2005) showed that, in the presence of microstructure noise, it is optimal to sample less frequently than it would otherwise. On the other hand, over decreasing the sampling frequency does not make sense in an intraday analysis. The 5-min sampling frequency is our choice as is the common practice in the realized measure literature, (Fleming et al (2003), Andersen et al (2000), among others), and also in stock market applied studies (Chiriac & Voev (2011), Golosnoy et al (2012), Andersen et al (2003), among others).

Once having chosen the 5-min frequency, the next step was to interpolate transaction prices in order to obtain a regularly spaced time series starting at 10:00 AM, local time, and ending at 17:00 PM[2]. After cleaning the database for outliers and treating simultaneous observations[3], we

[2] Opening time varied through the sample, and calculations were modified according to these changes.
[3] Due to trading report approximations, some transactions occurred at the same time. In these cases, we took the average value as a solution.

identified the transaction prices nearest to the 5-min grid. As all the stocks have high liquidity parameters, it is fair to consider that this price remains valid until the end of a given 5-min grid. By first differencing the log prices for all grids, we obtain the 5-min returns. In table 3.2, we can see that, although autocorrelations are very close to zero in most cases, they remain mostly negative until lag 5 probably due to residual microstructure noise.

Table 3.2: Autocorrelation of intraday returns

This table presents the autocorrelation of intraday returns considering a sampling frequency of five minutes. Autocorrelations are computed for each stock and different lag lengths. The 5-min returns are based on intraday transaction prices nearest to each five-minute gridpoint. The sample period is February/2006 to January/2011.

Stock	Lag 1	Lag 2	Lag 5
Ambev PN (AMBV4)	-0.019	-0.025	-0.005
Bradesco PN (BBDC4)	-0.014	-0.010	0.006
Bradespar PN (BRAP4)	-0.013	-0.002	0.009
Banco do Brasil ON (BBAS3)	-0.018	-0.021	-0.004
Cemig PN (CMIG4)	-0.027	-0.027	0.007
Cia Siderúrgica Nacional ON (CSNA3)	-0.018	0.000	0.005
Cyrella ON (CYRE3)	-0.029	-0.006	0.001
Eletrobras PN (ELET6)	-0.015	-0.017	-0.004
Gafisa ON (GFAS3)	-0.026	-0.017	0.001
Gerdau PN (GGBR4)	-0.026	-0.002	0.007
Petrobras PN (PETR4)	0.000	0.002	0.005
Usiminas PN (USIM5)	-0.020	-0.016	0.012
Cia Vale do Rio Doce (VALE5)	-0.013	0.002	0.007
OGX Petróleo ON (OGXP3)	-0.046	-0.018	-0.004
Itausa Investimentos ON (ITSA4)	-0.013	-0.004	0.008
Itau Unibanco Holding S.A. ON (ITUB4)	-0.036	-0.010	-0.007
PDG Realty S.A. ON (PDGR3)	-0.047	-0.013	-0.016
Hypermarcas ON (HYPE3)	-0.026	-0.016	-0.021
BMFBOVESPA S.A. ON (BVMF3)	-0.019	-0.004	0.014
Redecard ON (RDCD3)	-0.031	-0.007	-0.007

Note: Own elaboration. Source of information: BMF&Bovespa.

To compute realized volatilities, we refer to Liu, Patton & Sheppard (2013) who compared a great variety of realized measures in terms of forecasting accuracy and concluded that it is difficult to beat a simple 5-min RV measure, the one without correction for microstructure or the use of tick-by-tick information. In this sense, RV is defined for each day t and stock i as:

$$RV_{i,t} = \sum_{i=1}^{T} r_{i,t}^2 \qquad (3.1)$$

Where $r_{i,t}$ is the 5-min return and T is the number of 5-min intervals.

The covariance measures are estimated as below, for each day t and pair of stocks (i,j):

$$RCOV_{ij,t} = \sum_{i=1}^{T} r_{i,t} . r_{j,t} \qquad (3.2)$$

Where $r_{i,t}$ and $r_{j,t}$ are the 5-min returns for stocks (i,j) and T is the number of 5-min intervals.

This measure is the covariance counterpart of the RV measure, without correction for microstructure and will be used in order to preserve the compatibility with the RV measure[4].

4. The model: Heterogeneous Auto-Regressive (HAR)

Before getting into the details of our model, it is interesting to discuss the desired properties of a volatility model. One stylized fact is that the distribution of returns departs from a normal distribution. As you can see in Table 4.1, this is especially true for all the stocks analyzed in the study in that all exhibit excess kurtosis and rejection of the null hypothesis of a Jarque-Bera test. A simulation exercise made by Corsi (2009) showed that we can recover this characteristic by applying a simple HAR model.

Table 4.1: Summary statistics for daily returns

This table shows summary statistics for daily returns between February/2006 and January/2011. Returns are the mean daily returns based on closing prices. Standard deviation and kurtosis provide a measure of dispersion and peakedness of the distribution of daily returns, respectively. We also report the Jarque-Bera statistical test of normality for a 5% level of significance.

Stock	Returns * 100	(Standard deviation) * 100	Kurtosis	Jarque-Bera
Ambev PN (AMBV4)	0.01	2.66	7,98	Rejection of null hypothesis of normality
Bradesco PN (BBDC4)	0.06	2.75	7,64	
Bradespar PN (BRAP4)	0.01	3.08	6,58	
Banco do Brasil ON (BBAS3)	-0.01	4.00	8,85	
Cemig PN (CMIG4)	0.02	3.00	6,71	
Cia Siderúrgica Nacional ON (CSNA3)	0.02	259	5,04	
Cyrella ON (CYRE3)	0.01	3.91	7,49	
Eletrobras PN (ELET6)	0.08	3.17	8,87	
Gafisa ON (GFAS3)	0.01	2.25	7,52	
Gerdau PN (GGBR4)	0.07	2.98	7,50	
Petrobras PN (PETR4)	0.00	2.56	9,61	
Usiminas PN (USIM5)	0.04	2.87	8,44	
Cia Vale do Rio Doce (VALE5)	0.07	2.10	7,97	
OGX Petróleo ON (OGXP3)	0.03	0.042	6,54	
Itausa Investimentos ON (ITSA4)	0.04	0.027	9,94	
Itau Unibanco Holding S.A. ON (ITUB4)	0.02	0.018	5,66	
PDG Realty S.A. ON (PDGR3)	0.09	0.038	7,71	
Hypermarcas ON (HYPE3)	0.07	0.030	5,70	
BMFBOVESPA S.A. ON (BVMF3)	0.00	0.040	7,86	
Redecard ON (RDCD3)	-0.03	0.030	5,40	

Note: Own elaboration. Source of information: BMF&Bovespa.

Another stylized fact is that financial volatility returns are usually long memory processes where large volatility shocks are not quickly forgotten. In ARCH and GARCH models, autocorrelation decreases exponentially when it should be a hyperbolic decay. In fact, our data suggests a very slow decay of autocorrelation in square daily returns, up to lag 100, or approximately 5 months.

[4] The literature has provided a number of possibilities to correct for microstructure. McAleer & Medeiros (2008) described some of these methods that vary according to the assumptions regarding the noise structure. More recently, Corsi & Audrino (2012) proposed a tick-by-tick approach.

Table 4.2: Autocorrelation of squared daily returns

This table reports the sample autocorrelation function for different lag lengths. Squared daily returns are computed using closing prices. We select lag lengths of short, medium and long term horizons in order to assess volatility persistence. The sample period is February/2006 to January/2011.

Stock	Lag 1	Lag 20	Lag 100
Ambev PN (AMBV4)	0.22	0.28	0.01
Bradesco PN (BBDC4)	0.13	0.10	0.03
Bradespar PN (BRAP4)	0.06	0.08	0.04
Banco do Brasil ON (BBAS3)	0.29	0.18	0.06
Cemig PN (CMIG4)	0.22	0.16	0.07
Cia Siderúrgica Nacional ON (CSNA3)	0.18	0.07	-0.03
Cyrella ON (CYRE3)	0.18	0.15	0.01
Eletrobras PN (ELET6)	0.28	0.16	0.04
Gafisa ON (GFAS3)	0.05	0.03	0.02
Gerdau PN (GGBR4)	0.11	0.10	0.05
Petrobras PN (PETR4)	0.16	0.08	0.04
Usiminas PN (USIM5)	0.18	0.06	0.02
Cia Vale do Rio Doce (VALE5)	0.10	0.09	0.03
OGX Petróleo ON (OGXP3)	0.33	0.24	0.01
Itausa Investimentos ON (ITSA4)	0.18	0.12	-0.01
Itau Unibanco Holding S.A. ON (ITUB4)	0.12	0.03	-0.01
PDG Realty S.A. ON (PDGR3)	0.19	0.22	-0.02
Hypermarcas ON (HYPE3)	0.08	0.12	-0.01
BMFBOVESPA S.A. ON (BVMF3)	0.36	0.31	0.03
Redecard ON (RDCD3)	0.17	0.10	0.03

Note: Own elaboration. Source of information: BMF&Bovespa.

In this regard, Corsi (2009) proposes an additive model that is simple to estimate and able to replicate the long memory characteristic of volatility processes: the Heterogeneous Auto-Regressive (HAR). Although, Fractional difference operators (FIGARCH and ARFIMA models) share the same long memory property of HAR processes, it lacks flexibility and economic interpretation and also requires a great amount of data to work properly.

The HAR model is supported by economic theory and, thus, adds the advantage of having economic interpretation. The Heterogeneous Market Hypothesis was first presented by Muller et al (1997). The idea behind the theory is that if all traders were the same, prices should converge immediately to its real price and the correlation between market presence and volatility should be negative. However, this is not what happens in practice and it is due to the heterogeneity of agents that execute transactions in different market situations and trading frequencies. Besides, they are motivated by a different set of factors such as endowment, degree of information, prior belief, as. Market makers and active investors, for instance, have a more immediate trading horizon and focus on short term results. On the other hand, there are portfolio managers (financial and non-financial) as well as investors which focus on medium and long term prospects, rebalancing their positions less frequently. These characteristics make room for a model with short, medium and long term components such as HAR.

4.1. The univariate case

We will provide the main features of the model that departs from the framework from Corsi (2009) and Corsi & Reno (2012) with two exogenous variable included and additional features to allow for the implementation of a multivariate setting. The basis for the construction of the HAR models is LeBaron (2001), which shows that the long memory property can be reproduced by a sum of three different linear processes. In both articles, agents' heterogeneity is represented by a few time scales (day, week and month). For each level of the cascade, we define an unobservable component, a partial volatility measure ($\tilde{\sigma}_t^{(d)}, \tilde{\sigma}_t^{(w)}, \tilde{\sigma}_t^{(m)}$), as a function of the past observation of the realized volatility and the expectation of the partial volatility of the next time scale. This last term accounts for the asymmetric propagation of volatility which incorporates a stylized fact that longer term volatility have stronger influence on short term ones than the inverse. In the framework of the Heterogeneous Market Hypothesis, it is very reasonable to say that short term traders are more interested in the longer term volatility than the other way round. Thus, for the longer time span (monthly, in our case), only the past observation remains. Accordingly, the model can be written as:

$$\tilde{\sigma}_{t+1d}^{(d)} = \alpha_d + \beta^{(d)} RV_t^{(d)} + \gamma^{(d)} E_t \tilde{\sigma}_{t+1w}^{(w)} + \omega_{t+1d}^{(d)} \quad (4.1.1)$$

$$\tilde{\sigma}_{t+1w}^{(w)} = \alpha_w + \beta^{(w)} RV_t^{(w)} + \gamma^{(w)} E_t \tilde{\sigma}_{t+1m}^{(m)} + \omega_{t+1w}^{(w)} \quad (4.1.2)$$

$$\tilde{\sigma}_{t+1m}^{(m)} = \alpha_m + \beta^{(m)} RV_t^{(m)} + \omega_{t+1m}^{(m)} \quad (4.1.3)$$

Where $RV_t^{(d)}$, $RV_t^{(w)}$ and $RV_t^{(m)}$ stands for daily, weekly and monthly realized measures, respectively.

Over longer time horizons, realized volatility is defined as an average of daily past realized volatilities over the time scale:

$$RV_t^{(x)} = \frac{1}{x} \left(RV_{t-1d}^{(d)} + RV_{t-2d}^{(d)} + \cdots RV_{t-xd}^{(d)} \right) \quad (4.1.4)$$

Where x is the number of days.

A single variable setting only requires an adequate errors' structure to ensure positive definiteness. Alternatively, one can use logarithms instead of the original variables. As we will see, extending this framework to a multivariate setting will require additional steps.

Besides, the return process is a function of the highest frequency component ($r_t = \sigma_t^{(d)}.\varepsilon_t$). By recursive substitution, we reach the following simple specification for the cascade model:

$$\tilde{\sigma}_{t+1d}^{(d)} = \alpha + \beta^{(d)} RV_t^{(d)} + \beta^{(w)} RV_t^{(w)} + \beta^{(m)} RV_t^{(m)} + \omega_{t+1d}^{(d)} \qquad (4.1.5)$$

By assuming that measurement errors ($\omega_{t+1d}^{(d)}$, $\omega_{t+1d}^{(w)}$, $\omega_{t+1d}^{(m)}$) are contemporaneously and serially independent zero mean variates, the partial volatility measure can be substituted by the realized volatility directly into equation (4.1.5):

$$RV_{t+1d}^{(d)} = \alpha + \beta^{(d)} RV_t^{(d)} + \beta^{(w)} RV_t^{(w)} + \beta^{(m)} RV_t^{(m)} + \epsilon_{t+1d}^{(d)} \qquad (4.1.6)$$

4.2. The extension to the multivariate case

The first concern when working in a multivariate setting is ensuring the positive definiteness of the covariance matrix. Hence, we need to decompose the time-varying covariance matrix in such a way to guarantee this property. Chiriac & Voev (2011) proposes a Choleski factorization that ensures positive definiteness. However, this option was rejected due to the fact that the results were dependent on the ordering of the assets, probably due to the larger number of assets we included in our work (twenty instead of six). Bauer & Vorkink (2011) offer a more suitable solution for the decomposition. The method takes advantage of some useful properties of matrix exponential and logarithmic functions.

First of all, taking the matrix logarithm of a real, positive definite matrix results in a real, symmetric matrix. Thus, consider the covariance matrix Σ of dimension 20x20, which is symmetric and positive definite and apply the logarithm function[5].

$$A_t = logm(\Sigma_t) \qquad (4.2.1)$$

[5] After computing the autocorrelation functions, we conclude that the long memory is preserved even after the logarithmic transformation.

Where the *logm* function computes the matrix logarithm using the algorithm proposed by Higham & Davies (2003).

Note that this transformation involves a rotation in the original elements, which are not represented one by one in the new space, the log-space. It yields a real, symmetric matrix which will be used for the purpose of forecasting. This is done by stacking the columns of the upper portion of matrix A_t one under another, into a single column.

$$a_t^{(i)} = vech\ (A_t^{(i)}) \qquad (4.2.2)$$

Where the vector a_t has 210 elements and *vech* is the function that creates a column vector whose elements are the stacked columns of the upper portion of a given matrix. The index (i) refers to daily (d), weekly (w) and monthly (m) covariance matrices.

To estimate the conditional variance we use different specifications for the multivariate HAR model. As we aim a good out-of-sample fitness, we need to control the degree of parameterization. If, for each equation, we included the information from all the other assets we would have at least 12 additional regressors at each forecast. So, we needed to wrap up cross-asset information into few variables. By applying a Principal Component Analysis, we can consolidate into as many variables as we want. The first component already accounts for more than 70% of the variance of the realized volatilities and it suffices for the purpose of forecasting. We will call this the market volatility (MV).[6]

As Brazilian market is notoriously affected by external factors, another additional feature is to include a proxy for the volatility of the US market. The VIX index is a measure of the implied volatility of S&P 500 traded at Chicago Board Options Exchange Market. For the sake of simplicity, we will use only its last observation, avoiding introducing another cascade variable. Besides, it is fair to say that distant external volatility horizons are already absorbed by the domestic realized measures.

[6] In the remainder of the paper, the volatility timing strategy will be evaluated under different estimation windows. In order to ensure that this exercise is strictly out-of-sample, the MV variable has been computed over the same rolling window as the regression estimates. Note, however, that some assets do not belong to the database since February, 2006. We progressively incorporate new assets in the variable MV when its number of trading days is greater than the estimation window.

In what follows, we present the final specification that will be tested in equation (4.2.3). To Corsi's HAR specification, we include the daily VIX index and market volatility (MV) as exogenous variables.

$$a_{t+1d}^{(d)} = \alpha + \beta^{(d)} a_t^{(d)} + \beta^{(w)} a_t^{(w)} + \beta^{(m)} a_t^{(m)} + \delta^{(d)} MV_t^{(d)} + \delta^{(w)} MV_t^{(w)} + \delta^{(m)} MV_t^{(m)} + \theta . \log(VIX_t) + \epsilon_{t+1d}^{(d)} \quad (4.2.3)$$

Finally, to obtain the forecasted value in the original parameter space, we apply the exponential matrix function (*expm*) to the covariance matrix in the log-space, a procedure that preserves its positive definiteness.

$$\Sigma_{t+1} = expm(A_{t+1}) \quad (4.2.4)$$

Where A_{t+1} is the symmetric matrix reconstructed by the elements of the forecasted column vector a_{t+1}.

The function *expm* in Matlab performs the following mathematical operation: $e^{A_t} = \sum_{k=0}^{\infty} \frac{1}{k!} {A_t}^k$, following the scaling and squaring method proposed by Higham (2005) that, according to Moler & Loan (2003), is one of the preferable methods.

Model (4.2.3) is our multivariate version of the HAR model using realized volatility and will be referred as MHAR-RV in the remainder of the paper.

4.3. Out-of-sample performance

Now we turn our attention to obtain forecasting estimates for the next-period realized volatility. The time horizon has been defined as one day. Ideally, we want that realizations of the forecasting error to be unpredictable. Andersen et al (2006) proposes a natural diagnostic of this property with a simple linear regression under a general loss function:

$$E_t \left[\frac{\partial L(y_{t+1}, \widehat{y_{t+1|t}})}{\partial \hat{y}} \right] = a + b' x_t + \varepsilon_{t+1} \quad (4.3.1)$$

Where the regressor x_t can be defined arbitrarily.

This regression can be estimated by OLS taking into account the heteroskedasticity of errors. A well calibrated forecasting method should result in *a=b=0*. If we take the loss function as quadratic and x_t as the forecasted volatility, we can rewrite equation (4.3.1) as:

$$\sigma_{t:t+1}^2 = a + (b+1)\widehat{\sigma_{t:t+1}^2} + \varepsilon_{t+1} \tag{4.3.2}$$

Realized volatility allows us to compute the *ex-post* measure ($\sigma_{t:t+1}^2$) and our multivariate HAR-RV models will generate different measures of the *ex-ant* one ($\widehat{\sigma_{t:t+1}^2}$). This is the so-called Mincer-Zarnowitz regressions which are applied to our realized variance/covariance measures.

As you can see in Table 4.3.1, *'b'* coefficients are significantly equal to zero for the majority of the stocks. The rejection of the null hypothesis of all *'a'* coefficients, in turn, implies the presence of systematic forecasting errors. Stocks with the lowest liquidity levels, that is, the ones with the most average gap between transactions as in Table 3.1, showed the poorest fits suggesting that it can be an important driver to the models' performance.

Table 4.3.1: Mincer-Zarnowitz regression for variances

The database covers the period from February 2006 to January 2011. In this table, we refer to the realized variances of the twenty stocks listed in Table 3.1. We compute Mincer-Zarnowitz regressions according to (4.3.2) for a level of significance of 5%. Model (4.2.3) provides the forecasted realized volatility measures according to an estimation window equal to 150 trading days.

Null a=0	Null b=0	$Mean\ R^2$
Rejected (20)	Not rejected (17)	0.55

Note: In parenthesis, the number of stocks in which the null hypothesis was not rejected.

With respect to covariances, we will focus on the forecasting regression relatively to PETR4, the most representative stock in the database. Once more, we identify the presence of systematic forecasting errors. Note also that variance fit is superior to the covariance one as implied by the comparison of the mean R^2 statistics. The difference can be attributed to the fact that our MHAR-RV model does not allow for potential divergences in terms of long memory properties between variance and covariances.

Table 4.3.2: Mincer-Zarnowitz regression for PETR4 covariances

The database covers the period from February 2006 to January 2011. In this table, we refer to the realized covariances between PETR4 and all the stocks listed in Table 3.1. We compute Mincer-Zarnowitz regressions according to (4.3.2) for a level of significance of 5%. Model (4.2.3) provides the forecasted covariance measures according to an estimation window equal to 150 trading days.

Null a=0	Null b=0	$Mean\ R^2$
Rejected (20)	Not rejected (18)	0.10

Note: In parenthesis, the number of stocks in which the null hypothesis was not rejected.

In Brazil, Wink Junior & Pereira (2012) analyze out-of-sample realized volatility forecasting performance of five Brazilian stocks. The authors conclude that there were no significant different between Corsi's HAR-RV and Mixed Data Sampling (MIDAS-RV), developed by Ghysels et al (2004).

5. The comparison of economic value

Besides forecasting, conditional covariance estimation will be valuable for the purpose of portfolio optimization. According to its investment horizons, agents rebalance its portfolios in the face of events or trends that redefine the perception of each stock parameter, especially mean and variance of the expected returns. A multivariate framework is particularly important as we are usually dealing with multiple assets whose comovements need to be taken into account. The set up of the optimization problem depends on what one wants to test.

Fleming et al (2001) defined an optimization problem based on a volatility timing strategy that is well suited to our subject of interest, i.e., the analysis of covariance measures and model´s estimation. The authors compared a daily rebalanced portfolio with a static one in terms of economic utility. This methodology seeks to estimate how many basic points a mean-variance investor would be willing to pay to switch strategies. Fleming et al (2003) did the same kind of analysis just switching from a GARCH estimation procedure to a model based on realized volatility.

We compare our specification of the MHAR-RV model with two traditional models, namely Multivariate GARCH (MVGARCH) and Exponential Weighted Moving Average (EWMA). EWMA is the most common approach to calculate time-varying covariance matrices. As defined by Riskmetrics, the daily volatility is calculated as follows:

$$\widehat{\Omega_t} = \lambda \sum_{i=1}^{N} (1-\lambda)^{i-1} Y_{t-i} Y_{t-i}{}' \qquad (5.1)$$

Where Y_t is the matrix of daily returns. Although the parameter λ is defined arbitrarily, Riskmetrics recommends using 0.06 for stocks. The idea is to assign higher values to most recent observations.

The MVGARCH calculation builds on the work of Engle (2002) to deal with large conditional covariance matrices. Decomposing the conditional covariance matrix, we obtain:

$$\Omega_{t|t-1} = \mathrm{D}_{t|t-1}\Gamma_{t|t-1}\mathrm{D}_{t|t-1} \qquad (5.2)$$

Bollerslev (1990) assumes that the temporal variation in the covariances is driven only by standard deviations, making $\Gamma_{t|t-1=}\Gamma$, for every t. Engle's DCC (dynamic conditional correlation) model assumes that the correlation process follows a GARCH (1,1) process, avoiding the oversimplification from Bollerslev's model.

Consider an investor that follows a volatility timing strategy, where he wants to minimize volatility subject to a target return (μ_p). Let Σ_t be the 20x20 conditional covariance matrix, R_{t+1} and $\mu=E(R_{t+1})$ be a 20 x1 vector of risk asset returns and its expectation. Let also be the risk-free asset $R_{f,t}$ and w_t a 20x1 matrix of the portfolio weights.

$$\min_{w_t} w_t'.\Sigma_t.w_t$$

$$s.t. w_t'.\mu + (1 - w_t'.\mathbf{1}).\boldsymbol{R_{f,t}} = \mu_p \qquad (5.3)$$

Concerning the risk-free asset, some remarks must be made about the Brazilian economy. First of all, the choice of the reference rate is not free of controversy. As we do not have a highly active secondary market for federal government bonds, it is necessary to choose a reference for the short-term rate. The 30-day swap, a futures contract traded at BVM&F, is a market reference for the evolution of short-term interest rates. The interbank deposit rate (CDI – Certificado de Depósito Interbancário) would be another option. However, this measure is more sensitive to market conditions that do not affect the fundamentals of a riskless asset. Besides, note that the variable was indexed by time due to the fact that the prime interest rate varied considerably over the sample period. This is in odds with to the constant value assumption, as of Fleming et al (2001,2003). Allowing for short selling, the solution for the minimization problem results in the following equation for portfolio weights.

$$w_t = \frac{(\mu_p - R_{f,t}).\Sigma_t^{-1}.(\mu - R_{f,t}.\mathbf{1})}{(\mu - R_{f,t}.\mathbf{1}).\Sigma_t^{-1}.(\mu - R_{f,t}.\mathbf{1})} \qquad (5.4)$$

The computation of portfolio returns generated by (5.4) take into account trading costs, which may be a relevant portion of high frequency strategies: $R_{p,t} = \sum_{i=1}^{20} w_{i,t}.R_{i,t} - (w_{i,t} - w_{i,t-1})Trading\ costs_t)$, where $w_{i,t}$ is the weight of stock i at time t and $R_{i,t}$ its daily return. In this respect, BMF&Bovespa provides information on trading and post-trading costs to day-trade operations. Since such costs vary according to trading volume, we will consider the highest-cost scenario (0.025% per trading), which corresponds to investments up to $ 4 million and $ 20 million Brazilian reais, respectively to individual and institutional investors.

According to Fleming et al (2001), volatility timing strategy benefits from smoother conditional covariance matrix values. In Appendix, Graphs 1 to 3 show the comparison between different volatility measures for the most liquid stocks of our database (Vale and Petrobras) and their covariance. Although volatility series move together for the most part of the sampling period, MHAR-RV diverges in the period surrounding the 2008's financial crisis and also around the second trimester of 2007.

Now consider the rationale for the evaluation of the economic value. The choice of the utility is arbitrary but should be consistent with the problem at hand. In this sense, in an optimization problem with first and second moments involved, a quadratic utility is a natural candidate. Moreover, it can be viewed as a second order approximation of any investor´s utility. From this point, we will follow the utility format proposed by Fleming et al (2001),where W_{t+1} is the investor wealth at t+1 and *a* is his absolute risk aversion.

$$U(W_{t+1}) = W_t.R_{p,t+1} - \frac{aW_t^2}{2}.R_{p,t+1}^2 \quad (5.5)$$

Where $R_{p,t+1}$ is the portfolio return, including the risk-free asset.

If we hold $a.W_t$ constant, this is equivalent to setting the relative risk aversion constant (γ). It allows us to calculate the average utility as follows.

$$\overline{U}(.) = W_0.(\sum_{t=0}^{T-1} R_{p,t+1} - \frac{\gamma}{2.(1+\gamma)}.R_{p,t+1}^2) \quad (5.6)$$

Where W_0 is the initial wealth.

The value of volatility timing is calculated by equating the average utility of two alternative portfolios. This equality is obtained by including an operator Δ in one side of the equation and, then, calculating the value that equates both sides (the result of a second order polynomial).

$$\sum_{t=0}^{T-1}(R_{1p,t+1} - \Delta) - \frac{\gamma}{2.(1+\gamma)}.(R_{1p,t+1} - \Delta)^2 = \sum_{t=0}^{T-1} R_{2p,t+1} - \frac{\gamma}{2.(1+\gamma)}.R_{2p,t+1}^2 \quad (5.7)$$

If asset 1 is the MHAR-RV portfolio, the value of Δ can be interpreted as its economic gain or performance fee associated to switching to an alternative portfolio based on a different measure of volatility (asset 2). It resembles the concept of certainty equivalent as the value of Δ can also be interpreted as the risk premium associated with the choice of a strategy based on realized measures.

6. Results

We propose a baseline scenario which will be explored in detail and we will additionally introduce changes one at a time, thus enabling us to isolate the effect of each choice and confirm if they are not neutral to the results. Regarding the volatility forecasting procedures, for instance, each econometric approach requires a minimum window length for the estimation in order to avoid small sample bias. In this sense, our baseline scenario considers an estimation window equal to 150 trading days, equivalent to approximately 7 months.

Back to equations (5.5) and (5.6), it becomes clear that large levels of risk aversion impose a penalty on large variations of the portfolio returns. Using different utility specification, Issler & Piqueira (2000) found that investors in Brazil are more risk averse than in US. However, their estimated results did not indicate an unambiguous value for this parameter. Consequently, concerning the investors´ utility, it is more realistic to start with an average risk-averse investor (risk aversion parameter equal to three) and we treat extreme investors (risk aversion parameter equal to one and ten) as a special case in section 6.4. For the same reason, we opt for a daily investment horizon to account for more active traders which rebalance their portfolio at higher frequencies.

In the minimization problem (5.3), you can see that there are no restrictions on short selling. Short selling is a key tool for hedging purposes and many financial economists believe that it is necessary to prevent prices from reflecting only the views of the most optimistic investors in the market. Hence, a decrease in return volatility is the expected effect of such a strategy with higher weights on the risk-free asset, while the excess return is obtained through the alternation of long and short positions. The size of the stock lending market in Brazil can be used as a proxy to infer the frequency of short selling operations. According to Chague et al (2014), stock lending experienced a substantial increase, from $ 1.56 billion in 2000 to $ 436.3 billion dollars in 2011. More importantly, almost 300 stocks were involved in at least one lending operation in 2011, endorsing the reference scenario where short selling is allowed. In this respect, we will also perform a robustness check with no short selling allowed, where we expect not only the weight on the risky assets to increase but return volatility as well.

In asset pricing, estimation risk refers to investor's uncertainty about the parameters of the return, playing a major role in our optimization problem. As estimation risk can offset or even overestimate possible economic gains associated with our reference portfolio, MHAR-RV, how should we deal with this issue? We will split economic gain analysis according to the level of control over estimation risk. First, we consider a situation of minor estimation risk that is controlled by using ex-post information, i.e., the unconditional expected returns are applied through the whole sample. As the riskless asset changes its price, risky assets tend to move accordingly. In other words, considering R_f conditional on time is inconsistent with an unconditional μ. In this sense, we considered a second level of estimation risk that takes short term expectations into account by calculating expected returns based on the conditional mean. We will also consider a third situation, where estimation risk is accounted for by bootstrapping each return series.

Before proceeding, we should remind that we do not want to find the best method to forecast returns as we know all the difficulties inherent in this task. However, we believe that changing this assumption and amplifying the scope of the study allows us to make more sound conclusions about it. The economic gains and all returns and interest rates are expressed in an annualized basis.

6.1. Controlling estimation risk

6.1.1. Unconditional mean

This is the so called “no estimation risk” situation described by Fleming et al (2001). From Table 6.1.1, we can see that MHAR-RV economic gains are positively correlated with target return levels. For a target level of 17.5% and when short selling is allowed, an investor would be willing to pay 30.9 and 109.3 basis points to switch from a portfolio based on EWMA and MVGARCH, respectively, to a portfolio based on MHAR-RV forecasts. Fleming et al (2003) made a similar comparison and found a performance fee of 21.9 basis points[7] between a rolling RV estimator and a EWMA approach. In spite of the fact that the results are not directly comparable[8], the fact that they are in the same order of magnitude even when a greater number of assets is included in the multivariate setting should be seen as an indication of the benefits of realized volatility.

For low target levels, realized volatility does not have a superior performance, probably due to the proximity between the target level and risk-free levels in the Brazilian economy (see Graph 4 in the Appendix) leading to portfolio weights that reduce the value of volatility timing. In fact, the optimization problem induces a self-financed portfolio, with near-unity risk-free portfolio weights and alternate long and short positions in the risky assets of low absolute value, depending on the expected returns.

Table 6.1.1: Economic gain (in basis points) between the reference and alternative portfolios

The database covers the period from February 2006 to January 2011 for the twenty stocks listed in Table 3.1. Portfolio weights were computed according to the volatility-timing strategy described in equation (5.4), allowing for short selling and equaling the expected return for each stock to its unconditional mean. The forecasted values of realized volatility are based on equation (4.2.3). Utility gains were then computed as in equation (5.7), with $\gamma=3$. The size of the estimation window is 150 trading days. Target returns are expressed on an annual basis.

Reference Portfolio	Alternative portfolio	Target return		
		μ_p=12.5%	μ_p=15.0%	μ_p=17.5%
MHAR-RV	EWMA	-68.6	18.8	30.9
	MVGARCH	-11.8	49.3	109.3

With Table 6.1.2, we are able to take a closer look at the results in term of weights and returns. As expected, increasing values for the target return leads to higher risk levels, as measured by the return´s standard deviation. Moreover, weights on the risk-free asset are near to 100% in all

[7] With no correction for microstructure and inclusion of overnight returns in the volatility measurement procedure and γ=1.

[8] There are differences in the RV forecasting model and in the level of risk of the assets involved. The target return is 10% while the risk-free level is 6% a year and assumed to be constant over the sample period.

instances. To account for risk reward, Sharpe Ratios show that MHAR-RV portfolio is superior for target levels of 17.5%, but this advantage is not strong at the 15.0% and 12.5% levels just as the economic gain analysis reveals. Note also that average portfolio returns are an increasing function of the target return confirming the efficacy of our strategy to control for estimation risk, but we will see that this superior performance is only promising as long as we use ex-post information, that is, if one has informational advantage.

Table 6.1.2: Descriptive statistics for portfolio returns

Statistics are derived from the daily portfolio returns, constructed with the weights generated by the volatility-timing strategy (5.4), in an annualized basis. Short selling is allowed and expected return for each stock is equal to its unconditional mean. The size of the estimation window is 150 trading days. The Sharpe Ratio measures the excess return relatively to the average risk-free rate (11.7 %) per unit of deviation. For each target return, we also report the average annualized daily return, annualized standard deviation and the average weight on the risk-free asset. Target returns are expressed on an annual basis.

Target return=12.5%			
	MHAR-RV	EWMA	MVGARCH
Average Return	12.19%	12.29%	12.08%
Standard Deviation	1.71%	1.88%	1.77%
Sharpe Ratio	0.29	0.31	0.21
Weight on risk-free asset	100.0%	99.8%	99.8%
Target return=15.0%			
	MHAR-RV	EWMA	MVGARCH
Average Return	13.87%	13.20%	13.86%
Standard Deviation	3.48%	3.76%	3.44%
Sharpe Ratio	0.62	0.40	0.63
Weight on risk-free asset	101.7%	99.7%	99.9%
Target return=17.5%			
	MHAR-RV	EWMA	MVGARCH
Average Return	16.87%	14.04%	15.49%
Standard Deviation	5.65%	6.11%	5.56%
Sharpe Ratio	0.92	0.38	0.68
Weight on risk-free asset	103.0%	99.4%	99.9%

6.2. Conditional mean

Using the conditional mean as the parameter for the estimation of expected returns, we aim to progressively increase the exposure to estimation risk. With that in mind, we calculated the conditional mean as an annualized average return of the past six-months, or 120 trading days approximately. Additionally, to avoid excess variability, we assume that the investors updated expected returns on a monthly basis, i.e., the conditional mean remained constant over the next 20 trading days.

Comparing to the “no estimation risk”, Table 6.2.1 shows that economic gains not only increased substantially but are positive in all comparisons. Performance fees remains positively correlated to the target return and gains are superior when EWMA is the alternative portfolio. Economic gains range from 13.0 to 152.1 basis points taking MVGARCH as the alternative portfolio.

Table 6.2.1: Economic gain (in basis points) between the reference and alternative portfolios

The database covers the period from February 2006 to January 2011 for the twenty stocks listed in Table 3.1. Portfolio weights were computed according to the volatility-timing strategy described in equation (5.4), allowing for short selling and equaling the expected return for each stock to its conditional mean. The forecasted values of realized volatility are based on equation (4.2.3). Utility gains were then computed as in equation (5.7), with γ=3. The size of the estimation window is 150 trading days. Target returns are expressed on an annual basis.

Reference Portfolio	Alternative Portfolio	Target return		
		μ_p=12.5%	μ_p=15.0%	μ_p=17.5%
MHAR-RV	EWMA	19.4	125.9	231.1
	MVGARCH	13.0	83.1	152.1

Turning to Table 6.2.2, we are able to evaluate the dramatic effect of estimation risk on portfolio returns given that volatility timing strategies are not able to deliver returns that are even close to the target, except when target returns are close to the risk-free rate. One direct consequence is the occurrence of negative Sharpe Ratios as long as the average returns are always lower than the average risk-free rate. The risk-free asset maintains a high share in portfolio composition and the poor results can be attributed to the failure of conditional mean as a viable return forecast. Moreover, average returns lose the positive association with target returns, producing additional evidence of the fundamental role of estimation risk in the outcome of the volatility-timing strategy.

Table 6.2.2: Descriptive statistics for portfolio returns

Statistics are derived from the daily portfolio returns, constructed with the weights generated by the volatility-timing strategy (5.4), in an annualized basis. Short selling is allowed and expected return for each stock is equal to its conditional mean. The size of the estimation window is 150 trading days. The Sharpe Ratio measures the excess return relatively to the average risk-free rate (11.7 %) per unit of deviation. For each target return, we also report the average annualized daily return, annualized standard deviation and the average weight on the risk-free asset. Target returns are expressed on an annual basis.

Target return=12.5%			
	MHAR-RV	EWMA	MVGARCH
Average Return	10.78%	10.57%	10.64%
Standard Deviation	0.62%	0.64%	0.57%
Sharpe Ratio	-1.48	-1.77	-1.86
Weight on Riskfree asset	99.5%	99.8%	99.8%
Target return=15.0%			
	MHAR-RV	EWMA	MVGARCH
Average Return	10.59%	9.20%	9.66%
Standard Deviation	1.23%	1.27%	1.13%
Sharpe Ratio	-0.90	-1.97	-1.81
Weight on Riskfree asset	98.5%	99.5%	99.4%
Target return=17.5%			
	MHAR-RV	EWMA	MVGARCH
Average Return	10.41%	7.89%	8.78%
Standard Deviation	2.03%	2.06%	1.84%
Sharpe Ratio	-0.64	-1.85	-1.59
Weight on Riskfree asset	97.4%	99.2%	98.9%

6.3. Bootstrap

In order to obtain comparable results, we performed a simulation approach similar to the one employed by Fleming et al (2001). For each asset, we first generated a bootstrapped series of 2000 observations with replacement. Then, we calculated the average return of the first 500 observations and executed the same steps for 1000 times. From Table 6.3.1, we conclude that estimation risk offsets economic gains associated with our realized volatility measure. Although performance fees are in general positive on average, standard deviations are too large at a statistical viewpoint. While economic gains increase with the target level, the same goes for the standard deviation figures.

Table 6.3.1: Average economic gain (in basis points) between the reference and alternative portfolios

The database covers the period from February 2006 to January 2011 for the twenty stocks listed in Table 3.1. The results in the table are based on 1000 simulation trials. Portfolio weights were computed according to the volatility-timing strategy described in equation (5.4), allowing for short selling and equaling the expected return for each stock to its bootstrapped mean. The forecasted values of realized volatility are based on equation (4.2.3). Utility gains were then computed as in equation (5.7), with γ=3. The size of the estimation window is 150 trading days. Standard deviations are reported in parenthesis. Target returns are expressed on an annual basis.

Reference Portfolio	Alternative portfolio	Short selling allowed		
		μ_p=12.5%	μ_p=15.0%	μ_p=17.5%
MHAR-RV	EWMA	-19.6 (66.9)	0,5 (61.5)	3.5 (66.5)
	MVGARCH	-100.4 (201.7)	9.2 (220.5)	52.4 (223.0)

A closer look at the behavior of returns and weights show that average returns are stable over different target levels and not too far from the average risk-free rate (11.7%). Thus, under different return scenarios provided by our bootstrap simulation, volatility timing strategy is only able to incorporate a small premium over the risk-free rate on average.

Table 6.3.2: Descriptive statistics for portfolio returns

Statistics are derived from the daily portfolio returns, constructed with the weights generated by the volatility-timing strategy (5.4), in an annualized basis. Short selling is allowed and expected return for each stock is equal to its bootstrapped mean. The size of the estimation window is 150 trading days. The Sharpe Ratio measures the excess return relatively to the average risk-free rate (11.7 %) per unit of deviation. For each target return, we also report the average annualized daily return, annualized standard deviation and the average weight on the risk-free asset. Target returns are expressed on an annual basis.

Target return=12.5%			
	MHAR-RV	EWMA	MVGARCH
Average Return	12.15%	11.83%	12.24%
Standard Deviation	1.23%	0.95%	1.02%
Sharpe Ratio	0.37	0.14	0.53
Weight on Riskfree asset	101.0%	100.1%	100.3%
Target return=15.0%			
	MHAR-RV	EWMA	MVGARCH
Average Return	11.95%	11.99%	11.86%
Standard Deviation	1.14%	0.98%	0.97%
Sharpe Ratio	0.22	0.30	0.16
Weight on Riskfree asset	100.7%	97.9%	97.5%
Target return=17.5%			
	MHAR-RV	EWMA	MVGARCH
Average Return	12.26%	12.27%	12.03%
Standard Deviation	1.28%	1.26%	1.05%
Sharpe Ratio	0.44	0.45	0.31
Weight on Riskfree asset	100.0%	100.0%	100.0%

6.4. Additional Robustness checks

In an attempt to isolate the effect of the different volatility measures in a volatility-timing strategy, all robustness checks will consider the “no estimation risk” case, setting aside the issue of the role of expected returns in the optimization problem. So far, out-of-sample forecasts have been obtained with an estimation window of 150 days. As you can see in Table 6.4.1, results are robust to increasing window sizes as long as our first insights did not change. Economic gains are still positively related to target returns. The only noticeable change is the improvement of MVGARCH-based portfolios relatively to the EWMA ones, suggesting that MVGARCH performs better for larger estimation windows.

Table 6.4.1: Economic gain (in basis points) between the reference and alternative portfolios

The database covers the period from February 2006 to January 2011 for the twenty stocks listed in Table 3.1. Portfolio weights were computed according to the volatility-timing strategy described in equation (5.4), allowing for short selling and equaling the expected return for each stock to its unconditional mean. The forecasted values of realized volatility are based on equation (4.2.3). Utility gains were then computed as in equation (5.7), with γ=3. Two different estimation windows are used (200 and 250 trading days). Target returns are expressed on an annual basis.

Reference Portfolio	Alternative Portfolio	Target return		
		μ_p=12.5%	μ_p=15.0%	μ_p=17.5%
Window Size= 200				
MHAR-RV	EWMA	-39.8	5.1	50.1
	MVGARCH	-18.0	10.1	37.5
Window Size= 250				
MHAR-RV	EWMA	-12.0	51.0	114.0
	MVGARCH	2.9	39.5	74.9

Recall that, to the basic structure of a HAR model, we included two exogenous variables: VIX and a proxy for the domestic market volatility. We wonder if economic gains are a consequence of a more complex MHAR-RV setting than EWMA and GARCH models. In Table 6.4.2, we compute the economic gain with a basic HAR without exogenous variables in the "no estimation risk" case and, again, the relation between target levels and economic gains is positive. Despite the fact that the performance at the lowest target level (12.5%) is inferior, when we compare all instances of Table 6.1.1, results are similar and sometimes mixed. Hence, we can state that the realized volatility gains cannot be attributed to our model specification provided that overall conclusions are robust to it. Remember that the inclusion of exogenous variables aimed at adapting the model to an emerging country environment. The results, thus, show that such adaptations do not improve the volatility-timing strategy based on realized measures.

Table 6.4.2: Economic gain (in basis points) between the reference and alternative portfolios

The database covers the period from February 2006 to January 2011 for the twenty stocks listed in Table 3.1. Portfolio weights were computed according to the volatility-timing strategy described in equation (5.4), allowing for short selling and equaling the expected return for each stock to its unconditional mean. The forecasted values of realized volatility are based on a basic HAR specification, with no exogenous variables. Utility gains were then computed as in equation (5.7), with γ=3. Estimation window is equal to 150 trading days. Target returns are expressed on an annual basis.

Reference Portfolio	Alternative portfolio	Short selling allowed		
		μ_p=12.5%	μ_p=15.0%	μ_p=17.5%
MHAR-RV	EWMA	-49.4	8.8	66.8
	MVGARCH	7.5	77.1	145.4

The introduction of a short selling restriction comes as a natural robustness check as we expect a huge change in portfolio composition, with substantial lower weights of the risk-free asset. It happens to be the case that, when the target return is 17.5%, the risk-free asset responds for less than 30% of portfolio composition irrespective of the volatility measure used. Besides reducing economic gains, utility gains and target returns lose the positive association verified in previous results, as you can see in Table 6.4.3..

Table 6.4.3: Economic gain (in basis points) between the reference and alternative portfolios
The database covers the period from February 2006 to January 2011 for the twenty stocks listed in Table 3.1. Portfolio weights were computed according to the volatility-timing strategy described in equation (5.4), with no short selling and equaling the expected return for each stock to its unconditional mean. The forecasted values of realized volatility are based on equation (4.2.3). Utility gains were then computed as in equation (5.7), with γ=3. Estimation window is equal to 150 trading days. Target returns are expressed on an annual basis.

Reference Portfolio	Alternative portfolio	Target return		
		μ_p=12.5%	μ_p=15.0%	μ_p=17.5%
MHAR-RV	EWMA	0.9	36.3	-47.6
	MVGARCH	-20.1	-30.4	-166.0

In the reference case, investors' relative risk aversion (γ) has been set to 3. When γ alternates between extreme cases (1 and 10), the investor will impose lower (γ=1) or higher (γ=10) penalties over large variations in volatility forecasts. As we can see in the following table, MHAR-RV economic gains are positively related to target levels and no marked changes are present, except those concerning the comparative performance between EWMA and MVGARCH.

Table 6.4.4: Economic gain (in basis points) between the reference and alternative portfolios
The database covers the period from February 2006 to January 2011 for the twenty stocks listed in Table 3.1. Portfolio weights were computed according to the volatility-timing strategy described in equation (5.4), allowing for short selling and equaling the expected return for each stock to its unconditional mean. The forecasted values of realized volatility are based on equation (4.2.3). Utility gains were then computed as in equation (5.7), with γ=1 and γ=10. Estimation window is equal to 150 trading days. Target returns are expressed on an annual basis.

Reference Portfolio	Alternative Portfolio	Target return		
γ=1				
		μ_p=12.5%	μ_p=15.0%	μ_p=17.5%
MHAR-RV	EWMA	-68.7	-19.1	30.2
	MVGARCH	-11.8	49.3	109.4
γ=10				
		μ_p=12.5%	μ_p=15.0%	μ_p=17.5%
MHAR-RV	EWMA	-68.5	-18.6	31.3
	MVGARCH	-11.7	49.2	109.2

As a final comment on the regression outcomes, it is pertinent to make it explicitly clear which factors may explain the appearance of negative economic gains, especially, but not exclusively, when target returns are closer to the risk-free rate. Although we provide evidence of impact of estimation risk in the computation of the true economic gains of realized volatility, other factors may account for it. First of all, it is true that Fleming et al (2003) only found positive economic gains, but note that they compared realized measures with a rolling estimators based on daily returns and a static portfolio, as opposed to our benchmark models that provide conditional measures of the covariance matrix based on traditional models extensively used in practice.

It is also noteworthy to compare the characteristics of the assets under study. There is a great deal of complexity associated with modeling the covariance matrix for twenty individual stocks. As a matter of fact, Chiriac & Voev (2011) selected six highly liquid stocks while the work of Fleming et al (2003) did not include individual stocks. This conjecture may be examined empirically by future works that consider alternative and more sophisticated models for the conditional covariance matrix in order to minimize specification errors.

7. Conclusion

We have characterized the economic gains associated with the use of multivariate realized measures of volatility applied to a comprehensive set of twenty Brazilian stocks between February 2006 and January 2011. The forecasting procedure has been based on Corsi's HAR-RV model applied to a multivariate setting, as proposed by Bauer & Vorkink (2011). Portfolio weights have been computed in the context of a volatility timing strategy and the resulting daily portfolio returns are the basis for the evaluation of the economic gains of a quadratic utility.

We find that economic gains associated with realized measures increase are substantial for higher levels of the target return when estimation risk is controlled with ex-post information. Using the unconditional mean as a reference for expected returns, an investor would be willing to pay 30.9 and 109.3 basis points to switch from a portfolio based on EWMA and MVGARCH, respectively, to a portfolio based on (MHARV-RV) forecasts, when subjected to a target level of 17.5% per year. and no restriction to short selling. Economic gains are also robust to changes in the parameters of the utility of the optimization problem. When estimation risk is significant, however, it tends to offset economic gains of realized volatility. Besides, restricting short selling eliminates the association between target returns and economic gains.

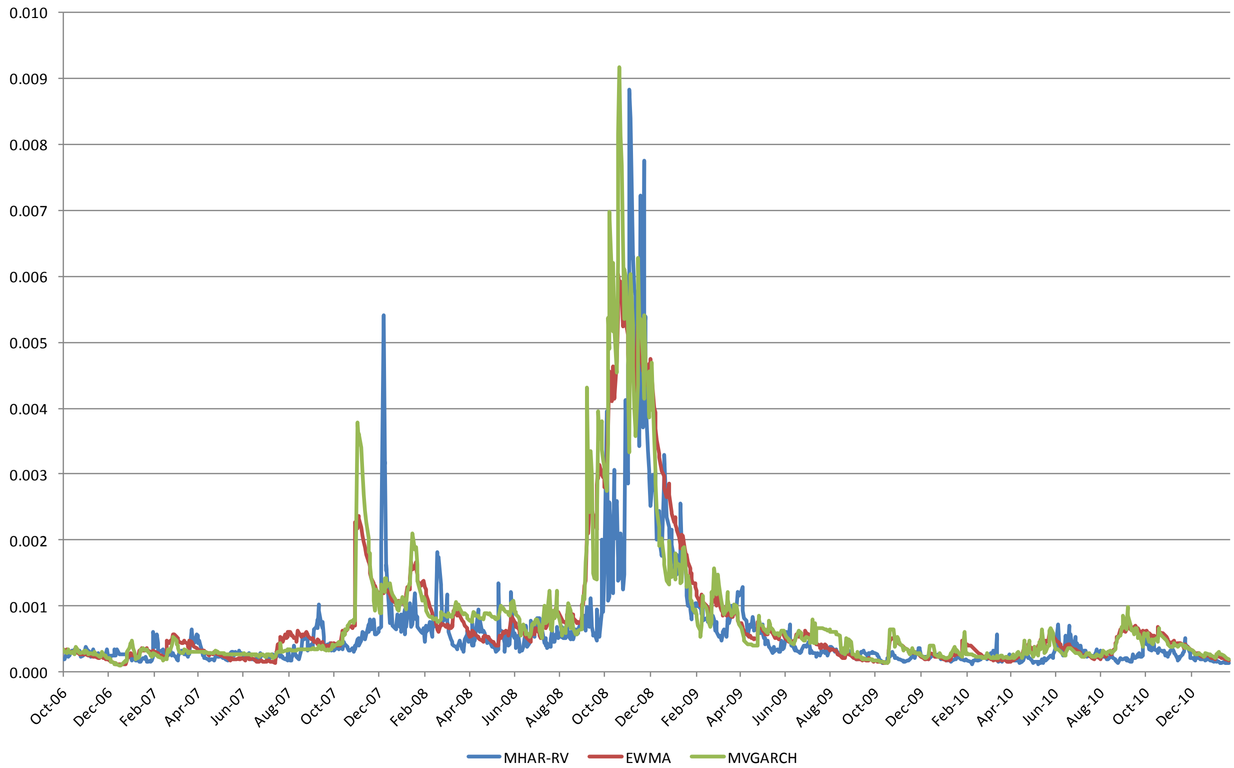
For lower levels of the target return, as we observe higher weights on the risk-free asset, economic gains are decreasing and we cannot attest for its superiority over the competing forecasting methods, EWMA and MVGARCH. It is also important to highlight that estimation risk plays a key role on the estimation procedure. When we depart from the “no estimation risk” case, economic gains associated with bootstrapped expected returns display positive values, but high standard deviations.

Our results contribute to the literature as we provide evidence of the benefits of realized measures even when are dealing with a great number of assets, all of them with high estimation risk and volatility shifts. Although Fleming et al (2001, 2003) already offered clear indication on such benefits; their work considered a lower number of assets and it is not straightforward to imply that results hold whatever assets’ dimension. We should point out, however, that utility gains are only significant when we control for estimation risk with ex-post information, suggesting that poor forecasts of expected returns offset utility gains associated with realized volatility.

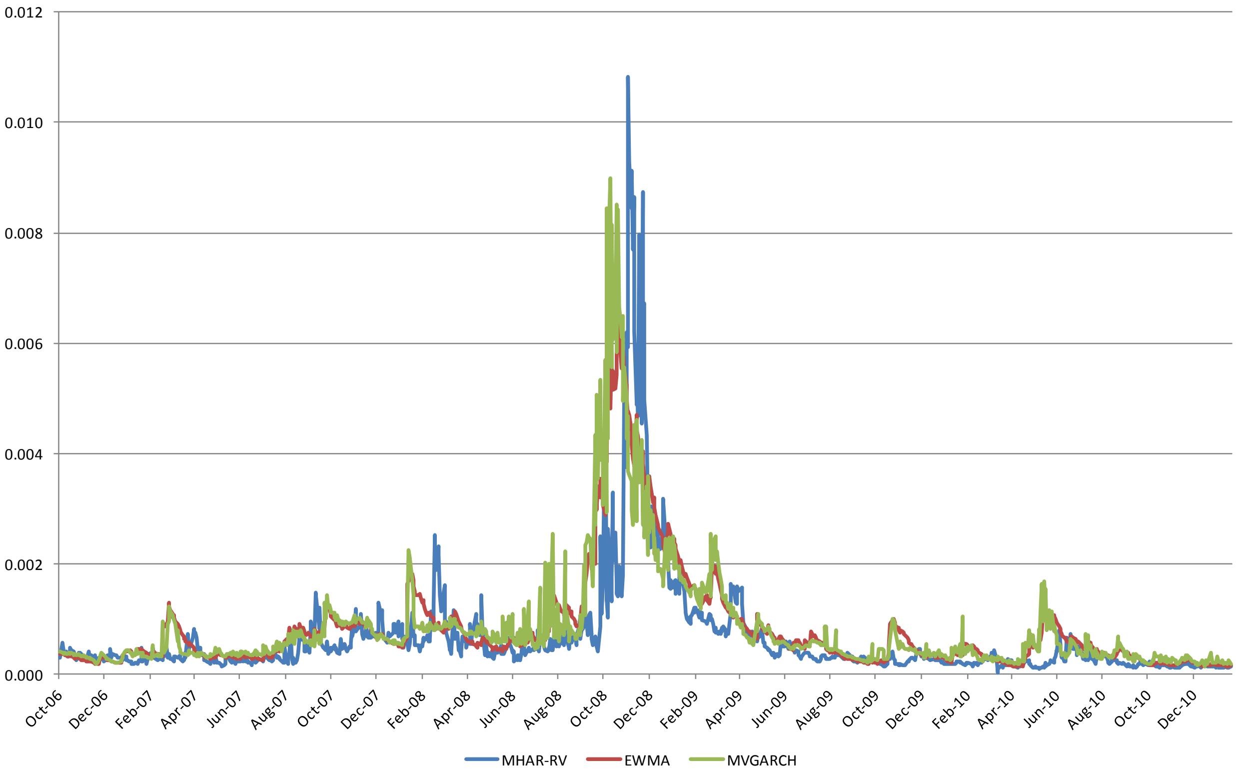
Taylor (2013) investigated the economic value related to volatility forecasts of portfolios based US bond and stock futures. The author notes that the gains associated with the knowledge of volatility dynamics are timely and state-dependent. Hence, future research should consider the use of sub-samples in order to explore dynamic features of the results. Since our choice for the economic utility is arbitrary, another possibility is to assess economic performance by the use of alternative function forms. Finally, alternative models for expected returns and for the conditional covariance matrix should provide results more independent from estimation risk and specification error considerations.

Appendix

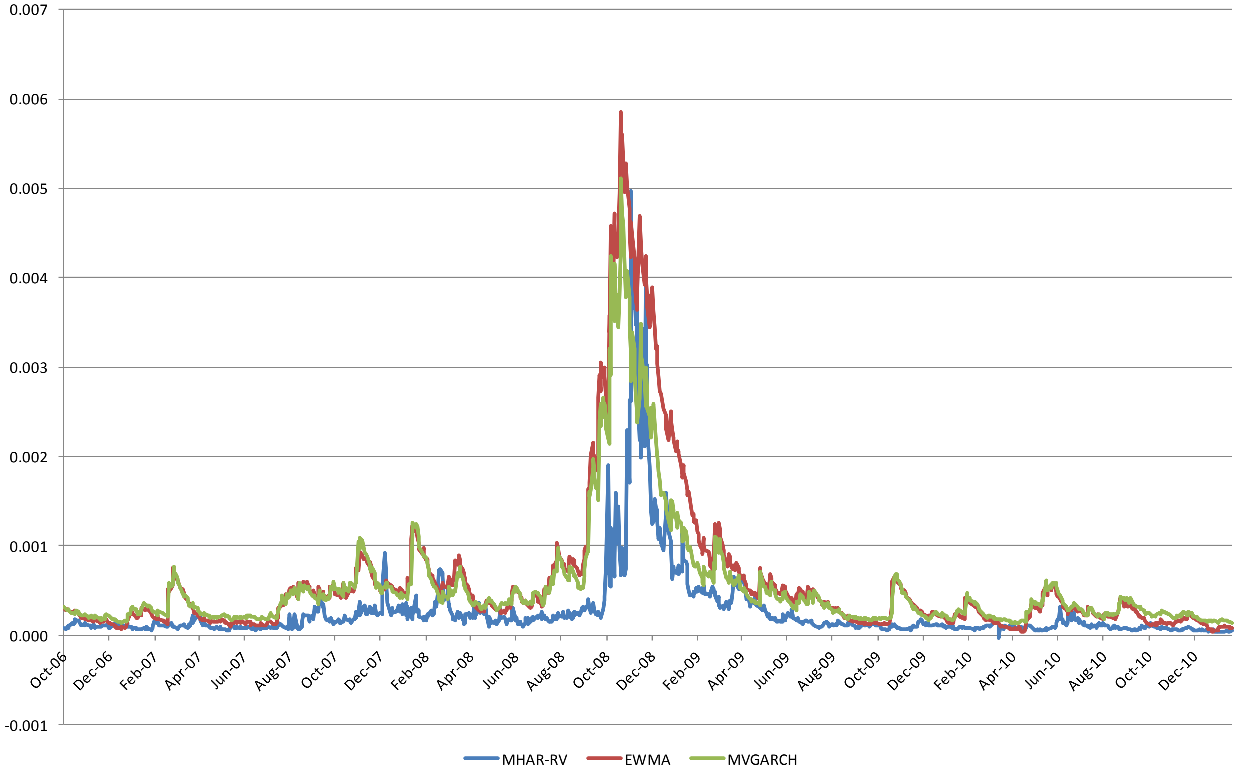
Graph 1: Daily volatility of Petrobras (PETR4) according to different forecasting models (EWMA, MVGARCH and MHAR-RV)



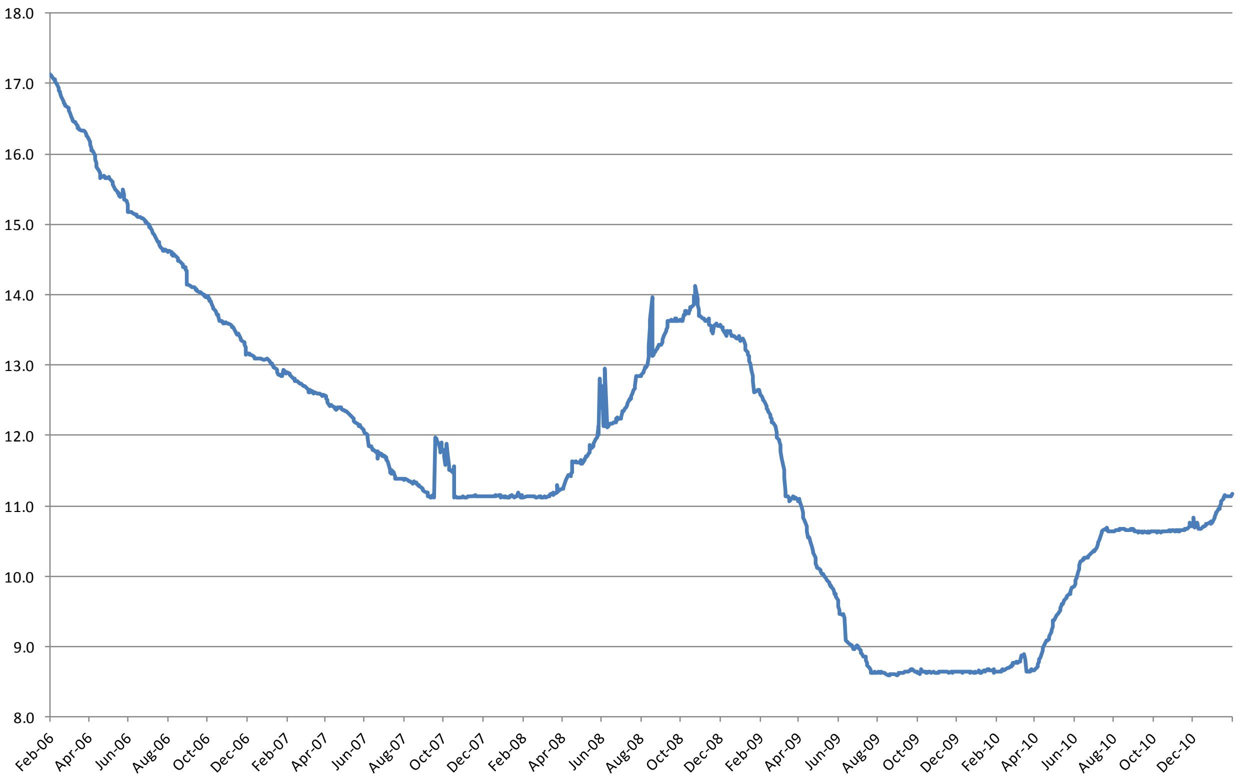
Graph 2: Daily volatility of Vale (VALE5) according to different forecasting models (EWMA, MVGARCH and MHAR-RV)



Graph 3: Daily covariance of Petrobras (PETR4) and Vale (VALE5) according to different forecasting models (EWMA, MVGARCH and MHAR-RV)



Graph 4: Evolution of the risk-free asset between February 2006 and January 2011 (expressed in percent a year)



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