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Abstract

For a given frequency of price adjustment, monetary non-neutrality is smaller if older prices are disproportionately more likely to change. This type of selection for the age of prices provides a complete characterization of price-setting frictions in time-dependent sticky-price models. Selection for older prices is weaker if: 1) the hazard function of price adjustment is less strongly increasing; 2) there is sectoral heterogeneity in price stickiness; 3) durations of price spells are more variable. Weaker selection for old prices implies larger monetary non-neutralities. In particular, the Taylor (1979) model exhibits maximal selection for older prices, whereas the Calvo (1983) model exhibits no selection.

JEL classification codes: E10, E30

Keywords: time-dependent pricing, selection effect, monetary non-neutrality, general hazard function, heterogeneity.

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1 Introduction

Infrequent price changes at the microeconomic level do not necessarily imply that monetary disturbances have large real macroeconomic effects. For the same frequency of price changes, the real effects of a monetary shock are small if the firms adjusting their prices are also the ones most likely to change prices by a large amount. The importance of this *selection effect* has been well understood at least since Caplin and Spulber (1987). In their model, large price adjustments by a small fraction of firms completely offset monetary shocks and induce money neutrality. This is because in Caplin and Spulber (1987), as in menu-cost models more generally, there is *self-selection*: firms always have the option of incurring a menu cost to adjust their prices, so that adjusting firms are also the ones which would like to adjust their prices by the greatest amount.[1]

In this paper we argue that selection effects do not necessarily hinge on self-selection. In fact, we show that selection is relevant in time-dependent sticky-price models, where the probability of a price change depends only on the time elapsed since the price was last reset. This is because the real effects of a monetary shock differ depending on whether adjusting firms are more or less likely to have prices that pre-date the shock. More fundamentally, we show that in such an economy, for a given average frequency of price changes, the real effects of a monetary shock depend *solely* on this type of selection. In particular, the real effects of nominal shocks are larger if older prices are relatively less likely to be adjusted.

A proper understanding of the fundamental role of selection in determining the real effects of monetary shocks in time-dependent models is important, since such models are prevalent in the sticky-price literature. While originally used for tractability, subsequent literature has shown that time-dependent pricing rules emerge optimally in the presence of information costs as in Caballero (1989), Bonomo and Carvalho (2004), and Reis (2006).

We tie selection to features of the distribution of price spells, some of which have been singled out in previous literature as important in determining the extent of monetary non-neutrality in time-dependent price-setting models.[2] In particular, we show that:

1) Calvo (1983) pricing implies no selection, as the probability of price changes does

[1]While Caplin and Spulber (1987) do not consider menu costs explicitly, the state-dependent pricing rule that they postulate can be rationalized by the presence of such costs. For seminal analyses of selection effects in menu-cost models, see Danziger (1999) and Golosov and Lucas (2007).

[2]While all results that explicitly tie selection to non-neutrality are original to this paper, some of the other mathematical results presented here were first derived in Carvalho and Schwartzman (2008) – a retired working paper that we never submitted for publication. Whenever appropriate, we point out which results were derived in our 2008 paper.

not depend on the age of the price. In contrast, Taylor (1979) pricing implies maximum selection, since changing prices are always the ones that have been in place for longest. This explains why, for a given frequency of price changes, Taylor pricing generates a lower degree of monetary non-neutrality than Calvo pricing (Kiley, 2002).

2) If the hazard of price adjustment is increasing, then selection for older prices is relatively stronger, and the real effects of a nominal shock are smaller than under Calvo pricing. This conforms with discussions by Dotsey, King, and Wolman (1997) and Wolman (1999). Moreover, the more increasing the hazard of price adjustment is, the larger selection effects are.

3) Under certain conditions, we can show that cross-sectoral heterogeneity in price stickiness is associated with lower (and possibly negative) selection, as sectors with low frequency of price changes have both a larger proportion of old prices and a lower probability of price changes. This clarifies and generalizes the finding in Carvalho (2006) that heterogeneity in price setting can lead to larger real effects of monetary shocks.

4) When comparing two economies, one in which the distribution of the duration of price spells is a mean preserving spread of the other, selection for older prices is weaker – and the real effects of the shock are larger – in the economy with more variable price spells. In particular, for a commonly used specification for monetary shocks, the mean and the variance of the duration of price spells are sufficient statistics for the real effects of nominal disturbances.[3]

Our framework encompasses a great degree of generality. As in Dotsey, King, and Wolman (1997, section 3), price changes arrive according to a generic function of the time elapsed since the last price adjustment. We are able to analyze the impact of quite general monetary shocks thanks to an equivalence between the real effects of monetary shocks in sticky-price models and in sticky-information models, as in Mankiw and Reis (2002).[4] For most of the paper, we focus on an environment in which the optimal price for a given firm is neither a strategic substitute nor a strategic complement to the prices set by other firms – what we refer to as *strategic neutrality in price setting.* As a robustness check, we investigate the role of selection in settings with strategic complementarity or substitutability through numerical

[3]Without linking it to selection, we first proved this result in Carvalho and Schwartzman (2008). Subsequently, Vavra (2010) and Alvarez et al. (2012) provided alternative proofs of the same result. The result is also complementary to the one in Alvarez, Le Bihan, and Lippi (2014), who, in a different setup, connect the real effects of monetary shocks to a moment of the size distribution of price changes rather than moments of the distribution of price spells.

[4]We first proved this equivalence result in Carvalho and Schwartzman (2008).

simulations. The results suggest that the relationship that we uncover between selection and real effects of monetary shocks is robust to those strategic interactions. Finally, we also analyze the implications of selection for private and social efficiency in face of monetary and other types of shocks.

Our paper is not the first one to identify a role for selection in time-dependent pricing models. Using a recursive formulation in a discrete-time setting, Sheedy (2010) shows that selection for older prices is associated with higher inflation persistence – an issue that we do not examine. He does not, however, examine the implications of selection for the real effects of monetary shocks. Subsequent work by Alvarez, Le Bihan, and Lippi (2014) elaborates on the link that we uncover between time-dependent selection and monetary non-neutrality in a different setting. The main difference is that, in their model, time-dependence takes the form of a combination of Taylor and Calvo pricing.[5] Otherwise, their main focus is on other types of selection effects under state-dependent pricing. In contrast, we cover the whole space of time-dependent pricing models. Another related paper is Yao (2015), who uses numerical examples to show how differences in the distribution of price durations affect the dynamics of the economy in response to shocks. Finally, Vavra (2010) explores the empirical distribution of price durations estimated from micro-data for the U.S. to study monetary non-neutrality.

We proceed as follows: In Section 2 we lay out the model, which is a continuous time, perfect foresight version of the baseline New Keynesian model, with general distribution of price durations. Section 3 introduces our concept of selection in time-dependent price-setting models, and states the key propositions linking selection to the real effects of a monetary shock. Section 4 shows how selection relates to various ways of summarizing the distribution of price spells, that is, it states and discusses results 1) to 4) listed above. In Section 5 we present the numerical results for cases allowing for strategic interactions in price setting. Section 6 analyzes the implications of selection for social and private efficiency, and the last section concludes.

[5] In their model, Taylor pricing arises as a limiting case in which firms have infinitely many products, and the payment of a menu cost allows them to change all prices at once. In this context, a law of large numbers leads firms to reprice at fixed intervals (for small monetary shocks). They expand the space of models they consider to include Calvo pricing, by allowing for random opportunities to change prices at no cost.

2 Model

There is a representative household that derives utility from a continuum of differentiated consumption goods aggregated in a Dixit-Stiglitz composite and supplies a continuum of firm-specific varieties of labor. Labor is hired by monopolistically competitive firms that produce the goods. The household owns these firms, so it receives back whatever profits they generate. Firms hire labor in competitive markets. We assume a cashless economy with a risk-free nominal bond in zero net supply as in Woodford (2003) and abstract from fiscal policy.

In our analysis, we rely on a first-order approximation of the model around a zero inflation steady state. This allows us to resort to the certainty equivalence principle and focus on the dynamic response of the economy to one-time, zero probability shocks in a world of otherwise perfect foresight. We use a continuous-time formulation since it yields tractable closed-form solutions, although none of the key results or intuitions rely on the continuous-time assumption. The representative household maximizes:

$$E_t \left[\int_0^\infty e^{-\rho s} \left(\frac{C(t+s)^{1-\sigma} - 1}{1-\sigma} - \int_0^1 \lambda \frac{L_j(t+s)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} dj \right) ds \right]$$

$$s.t.\ \dot{B}(t+s) = i(t+s) B(t+s) + \int_0^1 W_j(t+s) L_j(t+s) dj - P(t+s) C(t+s) + T(t+s),$$

and subject to a no-Ponzi condition. Here ρ is the discount rate, σ is the inverse of the elasticity of intertemporal substitution, ψ is the Frisch elasticity of labor supply, $C(s)$ is consumption of the composite good, $L_j(s)$ is the quantity of labor supplied for the production of variety j, $W_j(s)$ is the nominal wage for labor of variety j, $T(s)$ are firms' flow profits received by the consumer, $B(s)$ denotes bond holdings that accrue a nominal interest at rate $i(s)$, and $P(s)$ is a price index to be defined below. E_t is the expectations operator with respect to information available at time t. Given the assumption of perfect foresight except for a one-time, zero probability shock, we can ignore the expectations operator for the solution of the household problem.

The composite consumption good is given by:

$$C(t) \equiv \left[\int_0^1 C_j(t)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $C_j(t)$ is consumption of the variety of the good produced by firm j. The elasticity of substitution between varieties is $\varepsilon > 1$. Denoting by $P_j(t)$ the price charged by firm j at time t, the corresponding consumption price index is:

$$P(t) = \left[\int_0^1 P_j(t)^{1-\varepsilon} \, dj \right]^{\frac{1}{1-\varepsilon}}.$$

The first-order conditions for the representative consumer's optimization problem are:

$$\frac{W_j(t)}{P(t)} = \lambda C(t)^{\sigma} L_j(t)^{\frac{1}{\psi}}, \; j \in [0,1], \tag{1}$$

$$\frac{\dot{C}(t)}{C(t)} = \sigma^{-1} \left[i(t) - \frac{\dot{P}(t)}{P(t)} - \rho \right],$$

$$C_j(t) = C(t) \left(\frac{P_j(t)}{P(t)} \right)^{-\varepsilon}, \; j \in [0,1]. \tag{2}$$

Firms transform labor into output one for one. They sell their products at a nominal price that they only change infrequently. In the meantime, they commit to producing as much as necessary to satisfy the demand for their output given their chosen price. The timing of those occasional price changes depends probabilistically on the time elapsed since the firm's last price change – i.e., price setting is *time dependent*. Particular examples of time-dependent models include Taylor (1979) and Calvo (1983). We follow Section 3 in Dotsey, King, and Wolman (1997) and consider a general time-dependent setting. We denote the probability of a *new* price surviving for a period of length less than s by a generic cumulative distribution function $G(s)$. At this point, the only restriction we impose is that $G(s)$ depends only on the time elapsed since the price was last reset but not on the particular date in which it was reset. Note that G being a c.d.f. implies that $\lim_{s\to\infty} G(s) = 1$, so that all price spells come to an end with probability one. Certain results require additional restrictions on G that we will introduce as needed.

A firm that resets its price at time t chooses the price $X_j(t)$ to solve:

$$\begin{aligned}
&\max_{X_j(t)} E_t\left[\int_0^\infty e^{-\rho s}\left(1-G(s)\right)\left[X_j(t)\,Y_j(t+s)-W_j(t+s)\,N_j(t+s)\right]ds\right]\\
&s.t.\ Y_j(t+s)=N_j(t+s)\,,\\
&Y_j(t+s)=\left(\frac{X_j(t)}{P(t+s)}\right)^{-\varepsilon}Y(t+s)\,,
\end{aligned} \tag{3}$$

where $N_j(t+s)$ is the amount of labor demanded by the firm, and where the demand function already takes into account that goods market clearing implies $C_j(t)=Y_j(t)$. The first-order condition yields:

$$X_j(t)=E_t\left[\frac{\varepsilon}{\varepsilon-1}\frac{\int_0^\infty e^{-\rho s}\left(1-G(s)\right)P(t+s)^{\varepsilon}\,Y(t+s)\,W_j(t+s)\,ds}{\int_0^\infty e^{-\rho s}\left(1-G(s)\right)P(t+s)^{\varepsilon}\,Y(t+s)\,ds}\right].$$

As is usual in the literature, we focus on the symmetric equilibrium in which all adjusting firms choose the same nominal price. This allows us to drop the j subscripts and denote the price set by any firm at time t as $X(t)$. Moreover, we assume *uniform staggering* of pricing decisions, meaning that: 1) conditional on the time since the last price adjustment, the event that one firm changes price is independent of any other firm changing price, and 2) the cross-sectional distribution of survival times is stationary. Thus, the aggregate price index satisfies:

$$P(t)=\left[\int_{-\infty}^{t}\Lambda\left(1-G(t-v)\right)X(v)^{1-\varepsilon}\,dv\right]^{\frac{1}{1-\varepsilon}},$$

where $\Lambda dt\equiv\left[\int_0^\infty\left(1-G(s)\right)ds\right]^{-1}dt$ is the constant "fraction" of prices that are changed over an infinitesimally small interval dt. We refer to Λ as the average frequency of price changes in the economy. Using integration by parts, it is straightforward to show that:

$$\Lambda^{-1}=\int_0^\infty s\,dG(s)\,,$$

which is the average duration of price spells.

The model is closed by a monetary policy specification that ensures existence and uniqueness of a rational expectations equilibrium. Following standard practice in the price-setting literature (e.g. Mankiw and Reis, 2002), we assume an exogenous path for nominal aggregate demand, $M(t) = P(t) Y(t)$, which we leave unspecified at this point.

We log-linearize the model around a zero-inflation steady state. In this log-linear environment, firms that change prices at time t set (lowercase variables denote log-deviations from the steady state):

$$x(t) = x_j(t) = E_t \left[\frac{\int_0^\infty e^{-\rho s} (1 - G(s)) \, w_j(t+s) \, ds}{\int_0^\infty e^{-\rho s} (1 - G(s)) \, ds} \right]. \tag{4}$$

Log-linearizing the labor supply condition in equation (1), and combining the log-linear versions of the production function (equation 3), the household's demand for varieties (equation 2), and the market clearing condition $C_j(t) = Y_j(t)$ yields the following equilibrium expression for nominal wages:

$$w_j(t+s) = p(t+s) + \left(\sigma + \psi^{-1}\right) y(t+s) - \varepsilon \psi^{-1} \left(x(t) - p(t+s)\right).$$

Note that $w_j(t+s)$ is the same for all firms j that change price at the same time, so that, consistent with the symmetry assumption above, $x_j(t)$ is also the same for all such firms. We can also use $m(t+s) = p(t+s) + y(t+s)$ to substitute out $y(t+s)$, rearrange slightly, and obtain:

$$w_j(t+s) = \left(1 + \varepsilon \psi^{-1} - \sigma - \psi^{-1}\right) p(t+s) + \left(\sigma + \psi^{-1}\right) m(t+s) - \varepsilon \psi^{-1} x(t).$$

Substituting the expression above in the first-order condition for the firm's problem (equation 4) and rearranging yields:

$$x(t) = E_t \left[\frac{\int_0^\infty e^{-\rho s} (1 - G(s)) \left[\alpha m(t+s) + (1 - \alpha) p(t+s)\right] ds}{\int_0^\infty e^{-\rho s} (1 - G(s)) \, ds} \right], \tag{5}$$

where $\alpha = \frac{\sigma + \psi^{-1}}{1 + \varepsilon \psi^{-1}}$.

According to equation (5), the model implies *strategic neutrality* in price setting if $\alpha = 1$. This means that the nominal marginal cost for a given firm and, therefore, its desired price, only depends on the exogenous process $m(t+s)$ – and not on decisions made by other firms. Strategic neutrality arises under specific constellations of primitive parameters such as, for example, $\sigma = 1$ and $\psi \to \infty$ (log utility in consumption and linear disutility of labor). Given our framework, a necessary and sufficient condition for strategic neutrality is $\sigma + 1/\psi = 1 + \varepsilon/\psi$. More generally, pricing decisions will be either strategic substitutes or strategic complements. If $\alpha < 1$, there is strategic complementarity in price setting, meaning that firms will choose prices close to what they expect the aggregate price level to be. With $\alpha > 1$ pricing decisions are strategic substitutes.

Finally, the aggregate price level is given by:

$$p(t) = \int_{-\infty}^{t} \Lambda \left(1 - G(t - v)\right) x(v)\, dv. \tag{6}$$

2.1 Monetary shocks

The economy starts with a constant level of nominal aggregate demand M^{old}, with associated pricing decisions X^{old}, the aggregate price level P^{old}, and constant output Y^{old}. We analyze the impact of a one-time, unforeseen shock to nominal aggregate demand. The shock hits the economy at $t = t_0$, yielding thereafter a new path for nominal aggregate demand $M^{new}(t)$, and associated paths for pricing decisions, aggregate price level, and output – respectively, $X^{new}(t)$, $P^{new}(t)$, and $Y^{new}(t)$. The assumptions that price setting is purely time dependent and that price changes are uniformly staggered over time allow us to set, for notational convenience, $t_0 = 0$ without loss of generality.

In log-linear terms, the ex-post path of nominal income is:

$$m(t) = \begin{cases} m^{old}, \text{ if } t < 0, \\ m^{new}(t), \text{ if } t \geq 0. \end{cases} \tag{7}$$

The assumption of a one-time unforeseen shock implies that $E_t[M(t+s)] = M^{old}$ if $t < 0$ and $E_t[M(t+s)] = M^{new}(t)$ if $t \geq 0$, and analogously for $X(t)$, $P(t)$, and $Y(t)$. Thus, from this point onward, we drop the expectations operator and use the superscripts "new" and "old" instead.

3 Selection and monetary non-neutrality

In this section we introduce a concept of selection appropriate for time-dependent models. We show that, for a given frequency of price changes, this type of selection contains the same information as the distribution of price durations, so that it provides a complete description of nominal rigidities in the model. Finally, we show how selection affects monetary non-neutrality, with lower selection for prices set before the shock being associated with higher real effects of nominal shocks.

3.1 Selection

In statistics, there is a selection bias if a sample is not a random draw from the population. In that case, sample moments provide biased estimates of population moments. By analogy, the prices being reset at a given point in time are a sample of the population encompassing all existing prices. As a measure of selection, we focus on the fraction of prices set before the shock ("old prices") being reset at t, as compared to the corresponding fraction of old prices in the population still in place at t.

Because the distribution of the duration of price spells, G, is time-invariant, at any time $t \geq 0$, the fraction of old prices among changing prices is equal to $1 - G(t)$ – which is the probability that a price survives for t or longer. In turn, $1 - \omega(t) \equiv 1 - \int_0^t \Lambda(1 - G(s))\, ds$ is the fraction of old prices in the population at time t. In this context, we say there is *positive* selection for old prices if $1 - G(t) > 1 - \omega(t)$ and *negative* selection otherwise. This suggests a natural measure of selection for old prices at each point in time after a shock.

Definition 1. *For all t such that $\omega(t) < 1$, selection (at t), denoted by $\mu(t)$, is defined as*

$$\mu(t) \equiv \frac{1 - G(t)}{1 - \omega(t)} - 1,$$

and for t such that $\omega(t) = 1$,

$$\mu(t) = 0.$$

The extension of the definition for the cases in which $\omega(t) = 1$ is natural, since with $\omega(t) = 1$ all adjusting prices as well as all prices in the population have been set after the shock (that is, they are all "new"). Hence, the "sample" of prices that can adjust at any point in time has the same composition as the population and there is no selection.

The state of the economy at any $t \geq 0$ is a function of the history of selection for old prices starting at the time of the shock. To capture this history, we also employ a related measure, which emphasizes not selection at a given point in time, but cumulative selection since the shock hit:

Definition 2. *Cumulative selection (at t), denoted by $\Xi(t)$, is defined as*

$$\Xi(t) \equiv \int_0^t \mu(s)\, ds.$$

We refer loosely to selection for old prices in economy A being stronger than in economy B if either $\mu_A(t) > \mu_B(t)\ \forall t$ and/or $\Xi_A(t) > \Xi_B(t)\ \forall t$. It is easy to see that the first ordering implies the second, but that the converse is not necessarily true.

We now proceed to show how the population of old prices at any point in time is determined by the history of selection up to that point. After the monetary shock hits, the pool of new prices $\omega(t)$ increases as firms *a)* have the opportunity to change prices (this is given by the frequency of price changes, Λ) and *b)* are doing so for the first time after the shock (this applies to a fraction $1 - G(t)$ of price changers). Therefore:

$$\frac{\partial \omega(t)}{\partial t} = \Lambda(1 - G(t)). \tag{8}$$

Solving the differential equation (8) with $\omega(0) = 0$ as a boundary condition and using the definitions above yields the following:[6]

$$1 - \omega(t) = e^{-\Lambda t - \Lambda \int_0^t \mu(v) dv} = e^{-\Lambda t - \Lambda \Xi(t)}. \tag{9}$$

Equation (9) suggests that, given Λ, G can be obtained from μ (and vice versa). As the following Lemma shows, this is indeed the case:[7]

Lemma 1. *Let $\mathcal{M}$ be the set of all functions $\mu : [0, \infty) \to \mathbb{R}$ that can be constructed using **Definition 1**. Let $\mathcal{G}^{\Lambda}$ be the set of all functions $G : [0, \infty) \to [0, 1]$ satisfying*

[6] See Lemma A.1 in Online Appendix for a formal statement and proof.

[7] The proofs to all Lemmas and Propositions are in Section 7 of the Online Appendix.

$\int_0^\infty (1 - G(s))\, ds = \Lambda^{-1}$. *Then, there is a mapping* $f : \mathcal{M} \to \mathcal{G}^\Lambda$ *that allows us to recover* G *from* μ.

The last result implies that, given the frequency of price changes Λ, μ and G are equally valid primitives for the general class of time-dependent pricing models that we consider.

3.2 Monetary non-neutrality

We now examine the link between selection and monetary non-neutrality. We focus on the case with strategic neutrality in price setting, since it allows us to isolate the role of selection from the well-known effects of interactions between firms' pricing decisions. Using numerical simulations, in Section 5 we examine whether our main results survive the presence of pricing interactions.

The effects of the shock on real output are given by:

$$y^{new}(t) - y^{old}. \tag{10}$$

We measure the degree of monetary non-neutrality by the discounted cumulative effect of the shock on output. More specifically, our measure of non-neutrality is given by:

$$\Gamma = \int_0^\infty e^{-\rho t} \left[y^{new}(t) - y^{old} \right] dt.$$

In Section 6.2 we show that, up to a first-order approximation, this measure is proportional to the ex-post utility impact generated by the monetary shock. More broadly, it is a useful summary statistic of the (positive) effects of monetary shocks on the real economy, which we then use to assess the implications of selection.[8] We refer to Γ generically as *the real effects of the monetary shock.*

3.2.1 Level shocks

We start by analyzing the commonly used case where, following the shock, the level of nominal income changes once and for all, that is, $m^{new}(t) = m^{new} = m^{old} + \Delta m$ for some

[8] This view is shared by other papers in the literature, as evidenced by the widespread usage of that measure to this effect (see, for example, Alvarez, F. and F. Lippi 2014, Alvarez, F., F. Lippi, and L. Paciello 2012, Vavra 2010, among others).

constant Δm. Apart from being a common benchmark, this case is interesting because the link between selection and the real effects of monetary shocks is particularly transparent.

From (5) it follows immediately that:

$$x^{old} = m^{old},\, x^{new} = m^{new}.$$

Taking into account the different price-setting decisions made before and after $t = 0$, we can then write the evolution of the aggregate price level for $t \geq 0$ as:

$$p^{new}(t) = p(t) = \omega(t)\, m^{new} + (1 - \omega(t))\, m^{old}, \tag{11}$$

where $\omega(t)$ is the fraction of firms with new prices in the population (i.e., who last set their prices after the shock).

The effects of the shock on real output are thus given by:

$$y^{new}(t) - y^{old} = m^{new} - p^{new}(t) - \left(m^{old} - p^{old}\right) = \Delta m\,(1 - \omega(t)).$$

In words, the output effect at t is proportional to the size of the shock Δm and to the fraction of firms with old prices at t, $1 - \omega(t)$. Thus, for a given sized shock, the real effects at t are larger if the pool of old prices is larger.

Using (10), we can write the real effects of the shock as:

$$\Gamma = \Delta m \int_0^\infty e^{-\rho t}(1 - \omega(t))\, dt. \tag{12}$$

The real effects are increasing in the integral over time of the fraction of old prices in the population. The longer the fraction of old prices in the population takes to shrink to zero after the shock, the larger are its real effects. It follows from (9) that:

$$\frac{\Gamma}{\Delta m} = \int_0^\infty e^{-\rho t}(1 - \omega(t))\, dt = \int_0^\infty e^{-(\rho+\Lambda)t - \Lambda \int_0^t \mu(v)dv} dt = \int_0^\infty e^{-(\rho+\Lambda)t - \Lambda \Xi(t)} dt. \tag{13}$$

We can thus derive the following immediate implications, summarized in the lemma below:

Lemma 2. *Given Λ and strategic neutrality ($\alpha = 1$), the effects of a shock to the level of* *nominal income ($m_1 = m_0 + \Delta m$) are larger if either*

1) selection, $\mu(t)$, is smaller for all t, or

2) cumulative selection, $\Xi(t)$, is smaller for all t.

3.2.2 General shocks

One difficulty in establishing analytical results for general shocks is that, differently from the simple case with a level shock, the cross-sectional distribution of prices is, in general, not concentrated on only two values – one for old prices and one for new prices. In spite of that, in the benchmark case of strategic neutrality, we are able to handle more general shocks thanks to the following proposition:[9]

Proposition 1. *Consider an economy characterized by a distribution of price spells G and strategic neutrality ($\alpha = 1$). The real effects of a monetary shock of the general form considered in equation (7) are*

$$\Gamma = \int_0^\infty e^{-\rho t} (1 - \omega(t)) \left(m^{new}(t) - m^{old}\right) dt.$$

This proposition holds in spite of the fact that, in general, $y^{new}(t) - y^{old}$ is not equal to $(1 - \omega(t)) \left(m^{new}(t) - m^{old}\right)$. The fact that it holds is a consequence of optimality of firms' price-setting decisions. Given strategic neutrality in price-setting, a firm j choosing its price after the shock would like to set $x_j(t) = m^{new}(t+s)$ for all s, but this is impossible if $m^{new}(t+s)$ varies over time. As a "compromise", it optimally sets $x_j(t)$ to be equal to a weighted average of $m^{new}(t+s)$, with weights given by the probability with which it expects the price to remain in place at each date $t+s$. For some period of time, $x_j(t)$ will remain below $m^{new}(t+s)$, and for some other period it will remain above. Over time these differences, as weighted by the probabilities of the price remaining in place, cancel out exactly, so that overall the real effects are the same as if the firm was able to set $p_j(t+s) = m^{new}(t+s)$ for all $s \geq 0$.

In the Online Appendix, we show that Proposition 1 can be alternatively formulated as stating that the real effects of a monetary shock in a sticky-price model are identical to those effects in a sticky-information model, so long as the distribution of price spells in the former

[9] Since we rely on a log-linear approximation to the model around a zero inflation steady state, these more general shocks should be such that the economy remains near that steady state.

is identical to the distribution of price plans in the latter. Thus all the analytical results in this paper translate to an equally large class of models with sticky information, as in Mankiw and Reis (2002).[10]

As in Section 3, we can write the result in Proposition 1 in terms of cumulative selection:

$$\Gamma = \int_0^\infty e^{-(\rho+\Lambda)t-\Lambda\Xi(t)} \left(m^{new}(t) - m^{old}\right) dt. \tag{14}$$

From equation (14), it is evident that, given Λ, so long as $m^{new}(t) \geq m^{old}$ for all t (or vice-versa), monetary shocks have smaller real effects in economies with larger cumulative selection $\Xi(t)$ everywhere. This last result implies a generalization of Lemma 2:

Proposition 2. *Consider a shock to nominal aggregate demand characterized by $m^{new}(t) \geq m^{old}$ for all t.*

Consider the impact of the shock in two economies, A and B, characterized by distributions of price durations $G_A(t)$ and $G_B(t)$, with $\int_0^\infty (1 - G_A(t))\, dt = \int_0^\infty (1 - G_B(t))\, dt = \Lambda^{-1}$. Then, $\Gamma_A < \Gamma_B$ if either

1) $\mu_A(t) \geq \mu_B(t)\ \forall t$ or,

2) $\Xi_A(t) \geq \Xi_B(t)\ \forall t$.

The restriction that $m^{new}(t) > m^{old}$ ("monotonic shocks") is important. In particular, if shocks are non-monotonic (for example, if they lead nominal income to oscillate around its previous level), lower selection can lead to lower non-neutrality.[11]

Equation (14) also provides some insight into how the dynamic properties of monetary shocks may interact with selection to give rise to smaller or larger real effects. In particular, if we restrict our attention to mean-reverting shocks for which $\int_0^\infty \left(m^{new}(t) - m^{old}\right) dt < \infty$, we can write

$$\begin{aligned} \Gamma &= \int_0^\infty e^{-(\rho+\Lambda)t-\Lambda\Xi(t)} dt \int_0^\infty \left(m^{new}(t) - m^{old}\right) dt \\ &\quad + cov\left(e^{-(\rho+\Lambda)t-\Lambda\Xi(t)}, m^{new}(t) - m^{old}\right), \end{aligned}$$

where $cov\left(e^{-(\rho+\Lambda)t-\Lambda\Xi(t)}, m^{new}(t) - m^{old}\right)$ denotes the covariance over time between the

[10] We first proved this "equivalence" between sticky-price and sticky-information models in Carvalho and Schwartzman (2008), Proposition 1.

[11] We thank an anonymous referee for calling our attention to this case.

shock $m^{new}(t) - m^{old}$ and the selection term $e^{-(\rho+\Lambda)t-\Lambda\Xi(t)}$. The first term contains the original intuition: The real effects of shocks are larger if selection is smaller (lower $\Xi(t)$) and if the shock itself is larger on average (larger $\int_0^\infty \left(m^{new}(t) - m^{old}\right) dt$). The covariance term states that, additionally, the real effects of a nominal shock are larger if $m^{new}(t) - m^{old}$ is relatively large when cumulative selection, $\Xi(t)$, is small. Since $(\rho + \Lambda)t + \Lambda\Xi(t)$ increases with t, this implies that, for a given average size $\int_0^\infty \left(m^{new}(t) - m^{old}\right) dt$, shocks that mean revert faster (i.e., that are more "front-loaded") will feature larger real effects than more persistent shocks (which are relatively more "back loaded").

4 Selection and the distribution of price durations

We now turn to results concerning how selection is related to different properties of the distribution of price spells. We start by discussing the benchmark cases of Taylor and Calvo pricing. We then revisit two topics that have been the subject of previous work: The slope of hazard functions and ex-ante heterogeneity in price setting. Finally, we show that there is a link between selection and the variance of price durations, and we explore conditions under which this result allows us to derive a simple sufficient statistic for the real effects of monetary policy shocks.

4.1 Benchmark cases: Taylor and Calvo pricing

We start with a discussion of the two most widely used time-dependent models, which are the ones proposed by Taylor (1979) and Calvo (1983). We show that these cases are polar opposites insofar as selection is concerned. In particular, Taylor pricing implies maximal selection and Calvo pricing implies no selection. Thus, the real effects of monetary shocks will be minimal under Taylor and larger in Calvo than in any model with non-negative selection.

4.1.1 Taylor pricing

Firms set prices for a fixed period of time (given by Λ^{-1}). Thus, the distribution of price durations is degenerate at Λ^{-1}. This specification has been influential in the sticky-price literature, and, apart from Taylor (1979), it has been used in prominent papers such as

Chari, Kehoe, and McGrattan (2000).

For a given frequency of price changes Λ, we can define Taylor pricing in terms of our notation as:

$$G^{Taylor}(t) = \begin{cases} 0 \text{ if } t < \Lambda^{-1}, \\ 1 \text{ otherwise.} \end{cases}$$

Under Taylor pricing, selection at time t is:

$$\mu^{Taylor}(t) = \begin{cases} \frac{1}{1-\Lambda t} - 1 \text{ if } t < \Lambda^{-1}, \\ 0 \text{ otherwise.} \end{cases} \tag{15}$$

Selection is equal to zero for $t \geq \Lambda^{-1}$ since from that point onward the pool of old prices is thoroughly depleted, so that $1 - \omega^{Taylor}(t) = 0$. Selection is positive elsewhere. Within the range where selection is positive, it is also maximal, since all changing prices were set before the shock. We formalize the point in the following Lemma:

Lemma 3. *Consider an arbitrary time-dependent economy with distribution of price durations characterized by $G(t)$ and with average frequency of price changes Λ. Let $\mu(t)$ and $\Xi(t)$ be, respectively, the corresponding selection and cumulative selection functions. Let $\mu^{Taylor}(t)$ and $\Xi^{Taylor}(t)$ be, respectively, the selection and cumulative selection functions for a Taylor economy with average frequency of price changes Λ. Then $\mu^{Taylor}(t) \geq \mu(t)$ for all $t < \Lambda^{-1}$ and $\Xi^{Taylor}(t) \geq \Xi(t)$ for all t.*

Given Proposition 2, it follows immediately that, for a given Λ, Taylor pricing implies the smallest real effects among all time-dependent pricing models.[12]

4.1.2 Calvo pricing

A further leading example of time-dependent pricing used in the literature is the one proposed by Calvo (1983), which is the key building block of the canonical New Keynesian model. In this setting, the probability of a given firm changing its price over any given period of time does not depend on the time elapsed since it last adjusted. This implies an exponential decay of the survival probability of a price.

[12] Vavra (2010) provides a different proof of the fact that the real effects under Taylor are minimal. Without linking it to selection, we first proved that result in Carvalho and Schwartzman (2008).

In terms of our notation, we can denote the cumulative distribution of price durations under Calvo as:

$$G^{Calvo}(t) = 1 - e^{-\Lambda t}.$$

It is easy to verify that:

$$\omega^{Calvo}(t) = 1 - e^{-\Lambda t},$$

so that selection is given by:

$$\mu^{Calvo}(t) = \frac{e^{-\Lambda t}}{e^{-\Lambda t}} - 1 = 0.$$

Thus, under Calvo pricing there is no selection. In other words, price changing firms are a representative draw from the population.

4.2 Hazard functions

The empirical literature on price-setting has devoted substantial effort to estimating the shape of the hazard function of price adjustment.[13] The motivation is that, at least since the work of Dotsey, King, and Wolman (1997, 1999) and Wolman (1999), it has been clear that the shape of the hazard function matters for the real effects of monetary shocks.

Assuming G is differentiable, the hazard function can be defined as:

$$h(s) = \frac{\frac{\partial G(s)}{\partial t}}{1 - G(s)}.$$

We start by showing that the concept of selection and hazard functions are closely related. Specifically, the following holds:

Lemma 4. *Let μ and h be, respectively, the selection function and hazard function associated with a differentiable c.d.f. G. Let $t_1 \in [0, \infty)$ be the smallest value of t such that $\omega(t_1) = 1$. Then, for $t < t_1$,*

$$\mu(t) = \int_t^{t_1} \frac{h(s)}{\Lambda} \Psi_t(s)\, ds - 1, \tag{16}$$

where

$$\Psi_t(s) \equiv \frac{1 - G(s)}{\int_t^{t_1} (1 - G(v))\, dv}$$

[13] For a recent review of this literature, see Klenow and Malin (2010, section 5.3).

is the density of prices of age s among all prices older than t.

Thus, up to a constant, selection at t is proportional to a weighted average of the hazard function evaluated at t and later. The intuition is as follows: Selection at t is tied to the probability of prices set *before* time 0 changing at t. Given stationarity, this is equivalent to the probability of prices changing at age t or *afterwards*. Since the hazard function is the continuous-time analogue of the probability of a price changing at a given age, conditional on it having survived up to that age, selection at date t can be obtained from integrating the hazard function from t onwards using $\Psi_t(s)$ as weights. The normalization by the average frequency of price changes Λ reflects the fact that, unlike the hazard function, selection does not depend on the average frequency of price changes.

From (16), we can show that if a hazard function is strictly increasing, then there is positive selection at all t:

Lemma 5. *For a given distribution of price durations $G(t)$, consider the corresponding hazard $h(t) = \frac{\partial G(t)}{\partial t} / (1 - G(t))$ and selection $\mu(t) = \frac{1-G(t)}{1-\omega(t)} - 1$ functions. If $h(t') > h(t)$ for all $t' > t$, then $\mu(t) > 0$ for all $t > 0$.*

The result is intuitive. An increasing hazard function implies positive selection, since the probability of a price change increases with the age of the price. An immediate implication is that any economy featuring an increasing hazard of price adjustment will feature higher selection than an economy featuring Calvo pricing. The Lemma thus verifies the intuition spelled out by Wolman (1999) for the reason why, as compared to Calvo pricing, increasing hazard functions are associated with smaller real effects of monetary shocks.

The general intuition behind Lemma 5 extends to the comparison of two hazard functions. In this case, the c.d.f.'s can be ranked in terms of the associated cumulative selection. Given two economies, one with a more increasing hazard function than the other, the economy with the more increasing hazard function features higher cumulative selection and lower monetary non-neutrality:[14]

Proposition 3. *For two economies A and B with the same average frequency of price changes ($\Lambda_A = \Lambda_B$) and for which the relevant moments and derivatives are defined, $\Xi_A(t) < \Xi_B(t)$ $\forall t$ if either*

[14] The result is actually stronger than this, as all that is required is a single-crossing condition on the two hazards (see the proof in the Online Appendix).

1) there is a single crossing at some t^* so that $h_A(t) \geq h_B(t)$ for $t \leq t^*$ and $h_A(t) < h_B(t)$ for $t > t^*$, or

2) $\frac{\partial h_A(t)}{\partial t} < \frac{\partial h_B(t)}{\partial t}$ $\forall t$.

Given Proposition 3, it follows immediately from Proposition 2 that monetary shocks are associated with smaller real effects in economies in which the hazard function increases more quickly.

4.3 Heterogeneity in price stickiness

In this section we show that selection effects also shed light on, and allow us to generalize the findings in Carvalho (2006), that a one-sector model calibrated to the average frequency of price changes is likely to understate the real effects of nominal shocks relative to a model with cross-sectoral heterogeneity in price stickiness. These findings are of particular importance because, as documented by Bils and Klenow (2004) and others, there is substantial heterogeneity in the frequency of price changes.

Consider a heterogeneous sticky-price economy with K sectors indexed by k, each with a measure Φ_k of firms and sector-specific distribution of price-durations $G_k(t)$.[15] For notational convenience, we use $E[\cdot]$ to denote cross-sectoral weighted averages:

$$E[x_k] \equiv \sum_{k=1}^{K} \Phi_k x_k. \tag{17}$$

The price level in the heterogeneous economy is:

$$p(t) = E[p_k(t)],$$

where $p_k(t)$ is the price level in sector k. These sectoral price levels are aggregates of past pricing decisions:

$$p_k(t) = \int_{-\infty}^{t} \Lambda_k [1 - G_k(t-s)] x_k(s) ds,$$

where $\Lambda_k \equiv \left[\int_0^\infty (1 - G_k(s)) ds\right]^{-1}$ is the average frequency of price changes in sector k.

[15]For brevity, we do not specify the whole multisector model here and borrow the required log-linear equations directly from Carvalho and Schwartzman (2008).

Definition 1 for selection does not apply to the heterogeneous economy, but it is possible to construct a natural extension. First, if the fraction of new prices in sector k at time t is $\omega_k(t)$, it follows that the fraction of new prices in the economy as a whole (which we denote by $\omega^{het}(t)$) is just the average of the fraction of new prices across sectors:

$$\omega^{het}(t) = E[\omega_k(t)].$$

Calculating the fraction of new prices among changing prices is slightly more involved. Here, we have to take into account that the mass of prices changing in a given sector at any given interval dt is given by $\Phi_k \Lambda_k dt$ – the mass of firms in the sector, Φ_k, multiplied by the frequency of price changes Λ_k and by the length of time dt. If we denote the economy-wide fraction of new prices among changing prices at time t by $G^{het}(t)$, then:

$$G^{het}(t) = E\left[\frac{\Lambda_k}{E[\Lambda_k]} G_k(t)\right].$$

We can now generalize Definition 1 to the heterogeneous economy:

Definition 3. *For all t such that $E[\omega_k(t)] < 1$, selection (at t), denoted by $\mu^{het}(t)$, is defined as*

$$\mu^{het}(t) \equiv \frac{1 - G^{het}(t)}{1 - \omega^{het}(t)} - 1. \tag{18}$$

For t such that $\omega^{het}(t) = 1$,

$$\mu^{het}(t) = 0.$$

Definition 2 also generalizes to the heterogeneous economy in the natural way, so that $\Xi^{het}(t) = \int_0^t \mu^{het}(s)\,ds$. Given those definitions, it is possible to extend Proposition 2 to heterogeneous economies:

Proposition 2'. *Consider the real effects of a shock to nominal aggregate demand given by $m^{new}(t) \geq m^{old}$ for all t in two economies, A and B, characterized by sector specific distribution of price durations $\left\{G_k^A(t)\right\}_{k=1}^{KA}$ and $\left\{G_k^B(t)\right\}_{k=1}^{KB}$ and by sectoral weights $\left\{\Phi_k^A\right\}_{k=1}^{KA}$ and $\left\{\Phi_k^B\right\}_{k=1}^{K}$. Suppose, moreover, that the cross-sectoral average of the frequencies of price changes in both economies is the same, that is, $E\left[\Lambda_k^A\right] = E\left[\Lambda_k^B\right]$. Then, $\Gamma_A < \Gamma_B$ if either*

1) $\mu_A^{het}(t) \geq \mu_B^{het}(t)\ \forall t$, or

2) $\Xi_A^{het}(t) \geq \Xi_B^{het}(t)\ \forall t$.

We are now ready to show the role of selection in generating the results in Carvalho (2006). In that paper, all sectors feature Calvo pricing, with different hazards of price adjustment. This is a particular example of economies where the relevant source of heterogeneity across sectors in the distribution of price durations is summarized by a single sector-specific scaling parameter. We now generalize that setting.

Specifically, let the c.d.f. of price durations in sector k be given by $G_k(t) = \bar{G}(\Lambda_k t)$, with $\int_0^\infty 1 - \bar{G}(t)\, dt = 1$. Note that $\bar{G}$ is a generic c.d.f. common to all sectors, but that the average frequency of price change in sector k is equal to Λ_k.

Given this parameterization of the heterogeneous economy, we can compare it to a counterfactual one-sector economy with c.d.f. of price durations $\bar{G}(E[\Lambda_k]\, t)$, defined below:

Definition 4. *Consider a multisector economy characterized by sector-specific distribution of price durations* $\left\{\bar{G}(\Lambda_k t)\right\}_{k=1}^{K}$, *where* $\int_0^\infty 1 - \bar{G}(t)\, dt = 1$, *and sectoral weights* Φ_k. *The counterfactual one-sector economy is an economy with one sector and c.d.f. of price durations given by* $\bar{G}(E[\Lambda_k]\, t)$.

The following proposition compares the cumulative selection function in both economies:

Proposition 4. *Let* $\Xi^{het}(t)$ *denote cumulative selection of a multisector economy characterized by the sectoral c.d.f.'s of price durations* $\left\{\bar{G}(\Lambda_k t)\right\}_{k=1}^{K}$, *where* $\int_0^\infty 1 - \bar{G}(t)\, dt = 1$, *and sectoral weights* Φ_k, *and let* $\Xi^{count}(t)$ *denote cumulative selection of its counterfactual one-sector economy. Then,*

$$\Xi^{het}(t) < \Xi^{count}(t) \;\; \forall t. \tag{19}$$

Thus, cumulative selection in the multisector economy is always smaller than in the counterfactual one-sector economy. It follows immediately from Proposition 2' that a shock to nominal aggregate demand in the multisector economy has larger real effects than in the counterfactual one-sector economy.

The intuition for Proposition 4 is easiest to understand in the case considered by Carvalho (2006), where the hazard of price adjustment is constant within each sector, as in Calvo (1983). In this economy, $\bar{G}(t) = 1 - e^{-t}$, so that:

$$G_k(t) = 1 - e^{-\Lambda_k t}, \;\; \omega_k(t) = 1 - e^{-\Lambda_k t}. \tag{20}$$

Each sector features Calvo pricing so that within each sector there is no selection. This, however, is not true in the aggregate. We can check that selection in the heterogeneous economy is negative:

$$\mu^{het}(t) = \frac{\frac{E\left[\Lambda_k e^{-\Lambda_k t}\right]}{E[\Lambda_k]}}{E\left[e^{-\Lambda_k t}\right]} - 1 = \frac{cov\left(\Lambda_k, 1-\omega_k(t)\right)}{E\left[\Lambda_k\right] E\left[1-\omega_k(t)\right]} < 0, \tag{21}$$

where cov denotes the cross-sectional covariance given sectoral weights Φ_k.

The covariance term in equation (21) neatly summarizes the intuition behind the main result in this section. In the heterogeneous economy, price changes are disproportionately selected from sectors with high frequency of price changes (high Λ_k). However, exactly because these sectors have a high frequency of price changes, they have a smaller fraction of old prices (low $1-\omega_k(t)$). Therefore, $cov\left(\Lambda_k, 1-\omega_k(t)\right) < 0$ and selection is negative. In contrast, the counterfactual one-sector economy is just a Calvo economy, so that selection is zero. Thus, the heterogeneous economy features lower selection than its counterfactual one-sector counterpart, and higher real effects of monetary shocks.

4.4 The variance of price durations

For economists trying to calibrate time-dependent sticky-price models, the results presented so far may seem a bit discouraging. They imply that the average frequency of price changes is far from a being a sufficient statistic for the real effects of nominal shocks. Rather, they suggest that one cannot do without the whole distribution of price durations, since it is the shape of that distribution that determines selection.[16]

Our next results show that it may not be necessary to account for the entire distribution of price durations. They make the case that, for a given frequency of price changes, the variance of price durations may be a good scalar metric of selection effects and, in some particular cases, a sufficient statistic.

As a first step, we compare selection among two distributions of price durations where one is a mean preserving spread of the other. Proposition 5 states the result:

Proposition 5. *Consider two economies, A and B, characterized by the distributions of price spells G_A and G_B, where G_A is obtained from a mean preserving spread of G_B. Then,*

[16] For a model calibrated with microeconomic estimates of the full distribution of the duration of price spells, see Vavra (2010). As shown in that paper, an alternative to our approach is to consider the distribution of remaining durations of prices in place. Given stationarity, one is just a transformation of the other.

$\Xi_A(t) \leq \Xi_B(t)\ \forall t$. *Conversely, if* $\Xi_A(t) \leq \Xi_B(t)$ *and* $\Lambda_A = \Lambda_B$, *then* G_A *is a mean preserving spread of* G_B.

Thus, if we restrict ourselves to comparing economies that can be ordered in terms of cumulative selection, the variance of price durations is a sufficient statistic for that ordering. Given that restriction, in the context of Proposition 5, if the variance of price durations in economy A is higher than in economy B, then, selection is lower in A than in B.

For a given frequency of price changes, the variance of price durations is, furthermore, a sufficient statistic for the real effects of nominal disturbances in the case of shocks to the level of nominal income discussed in Section 3.2.1:[17]

Proposition 6. *Suppose an economy is characterized by a distribution of price spells* G *with finite mean and variance given by* Λ^{-1} *and* σ^2. *The real effects of a permanent level shock to nominal aggregate demand of size* Δm *satisfy*

$$\lim_{\rho \to 0} \frac{\Gamma}{\Delta m} = \frac{1}{2}\left(\Lambda^{-1} + \Lambda \sigma^2\right). \tag{22}$$

Note that, unlike Proposition 5, Proposition 6 does not require the economies under comparison to be ordered by degree of selection. In that sense, it applies more broadly than previous results that hinged on an ordering by cumulative selection.[18]

Furthermore, the Proposition presents a closed-form expression for the real effects of the monetary shock. As an example, it allows us to easily calculate the real effects of a level shock under Taylor and Calvo pricing. They are given by $\frac{\Lambda^{-1}}{2}\Delta m$ for Taylor and $\Lambda^{-1}\Delta m$ for Calvo. Hence, the real effects of a level shock are twice as large under Calvo pricing than under Taylor pricing.

The variance is not a sufficient statistic for more complicated shocks. Proposition 6 is a special case of Proposition 6', which applies to any shock whose impulse response function can be well approximated by a polynomial function.

Proposition 6'. *Suppose an economy is characterized by a distribution of price spells* G *with finite moments of order between* 1 *and* $K+1$. *Let the random variable* τ *be the realized duration of price spells. The real effects of a monetary shock characterized by* $m^{new}(t) -$

[17] This is not the case more generally because orderings by variance do not imply an ordering by mean preserving spreads.

[18] We first derived this result in Carvalho and Schwartzman (2008), Proposition 3.

$m^{old} = \sum_{k=1}^{K} a_k t^{k-1}$ *are:*

$$\lim_{\rho\to 0}\Gamma = \sum_{k=1}^{K} \frac{a_k}{k(k+1)} \frac{E\left[\tau^{k+1}\right]}{E[\tau]}.$$

Proposition 6' states that the number of (uncentered) moments necessary to characterize the real effects of a monetary shock increases with the number of polynomial terms necessary to approximate the new trajectory of nominal income.[19]

For example, consider the effects of permanent shocks to the growth rate of nominal aggregate demand. Such shocks are relevant for periods of disinflation such as the early 80s in the United States. The growth rate shock is:[20]

$$m(t) = \begin{cases} m^{old}, \ t < 0, \\ m^{old} + bt, \ t \geq 0. \end{cases} \tag{23}$$

In that case, the real effects of the shock are given by:

$$\lim_{\rho\to 0}\Gamma = \frac{b}{6}\left(\Lambda^{-2} + 3\sigma^2 + \Lambda\eta\sigma^3\right),$$

where σ^2 is the variance of price durations and η is the skewness. It is, again, straightforward to calculate the real effects of this kind of shock under Taylor and Calvo pricing. They are, respectively, $\frac{\Lambda^{-1}}{6}b$ and $\Lambda^{-1}b$, so that the real effects of the shock are six times as large under Calvo than under Taylor.

5 Interactions in pricing decisions

The analytical results presented above hold under strategic neutrality in price setting. In this section we perform some numerical exercises to assess whether our main results extend

[19]Shocks approximated by polynomials of order $K > 2$ may seem unrealistic, since they would imply a divergent inflation path. However, the polynomial only needs to be a good approximation to the shock up to some distant enough time T for Proposition 6' to be useful as a means of obtaining a good approximation. See Section 2 in the Online Appendix for a formal discussion.

[20]We first derived this result in Carvalho and Schwartzman (2008), Proposition 5. While this shock involves a change in steady-state inflation, Carvalho (2008, Appendix A.6) shows numerically that, as long as the discount rate (ρ) is not strictly equal to zero, this result is a good approximation for temporary but highly persistent shocks to the growth rate of nominal income, so that inflation converges slowly back to zero.

to more general cases.

We consider the real effects of a shock across different sticky-price economies indexed by T and characterized by the following family of survival functions:

$$1 - G^{T}(t) = \begin{cases} 1 - e^{-\theta t} \text{ if } t < T, \\ 0 \text{ if } t \geq T, \end{cases} \tag{24}$$

with θ such that:

$$\int_0^\infty \left(1 - G^{T}(t)\right) dt = D \text{ for all } T. \tag{25}$$

That is, for different values of T, we adjust θ to ensure that the average duration of price spells equals D. We take the unit of time to be a quarter and set $D = 2$, so that the average price-spell lasts 2 quarters in all economies.

This family includes the two leading cases of constant duration (Taylor, 1979) and constant hazard (Calvo, 1983). The first obtains if $T = 2$ and $\theta = 0$. Calvo pricing obtains with $T \to \infty$ and $\theta \to \frac{1}{2}$.

For any $T'' > T'$, it is easy to check that the survival function parameterized by T'' is a mean preserving spread of the survival function parameterized by T'.[21] Thus, from Proposition 5 it follows that, as T increases, cumulative selection decreases.

To perform the simulations, we consider the discrete-time analogue of the model in Section 2. The discrete-time analogue of the family of survival functions described in equations (24) and (25) is:

$$1 - G_t^{T} = \begin{cases} \theta^{t} \text{ if } t < T, \\ 0 \text{ if } t \geq T. \end{cases}$$

with θ such that:

$$\sum_{t=0}^{\infty} (1 - G_t)\, dt = 2 \text{ for all } T.$$

The reset price chosen by all firms adjusting in period t is:

$$x_t = \sum (1 - G_t)\left[\alpha m_t + (1 - \alpha) p_t\right], \tag{26}$$

[21] In particular, it is straightforward to verify that for any T'' and T' with $T'' > T'$, $\int_0^t G^{T''}(s)\, ds \geq \int_0^t G^{T'}(s)\, ds$ for all t.

where α determines whether pricing decisions are strategic complements or strategic substitutes. In our experiments we compare results with $\alpha = 1$ (strategic neutrality), $\alpha = 1/3$ (strategic complementarity), and $\alpha = 3$ (strategic substitutability).[22]

In order to perform the numerical exercises, we also need to parameterize the shock process. We follow Mankiw and Reis (2002) and consider a process that is mean reverting in the growth rate of nominal aggregate demand:

$$\Delta m_t = 0.5\Delta m_{t-1} + \epsilon_t. \tag{27}$$

For given T, equations (26), (27), and the discrete-time analogue of the aggregate price-level equation (6) define a standard linear rational-expectations model in $\{p_t, m_t, x_t\}$, which we solve using Dynare.

Figure 1 shows the log cumulative real effects of monetary shocks described in equation (27) for different levels of strategic interactions and different T's. As expected, the real effects are, for a given T, largest under strategic complementarity ($\alpha = \frac{1}{3}$) and smallest under strategic substitutability ($\alpha = 3$). Furthermore, for given α, they also increase noticeably as selection decreases (T increases).

Lastly, note that moving from a model with Taylor pricing ($T = 2$ and $\theta = 0$) to one approaching a constant hazard of price adjustment ($T = 20$ and $\theta = 0.4998$) implies an increase in the real effects by a factor of approximately two under strategic neutrality ($\alpha = 1$). This is close to what is implied by the analytical result in Proposition 6, even though the nominal income process in equation (27) does not imply instantaneous level shifts as assumed in the proposition. With strategic complementarities, real effects increase by a factor greater than two, whereas with strategic substitutability, they increase by a factor smaller than two.

6 Selection and efficiency

Economies with highest selection present the lowest cumulative effect of monetary shocks. To what extent does this translate into higher private or social efficiency? In order to be able

[22] Recall that $\alpha = \frac{\sigma+\psi^{-1}}{1+\theta\psi^{-1}}$. The parametrization with strategic complementarities obtains if, for example, $\sigma = 1$, $\psi = 1$, and $\theta = 5$. The parametrization with strategic substitutability obtains, for example, if $\sigma = 3$, $\psi \to \infty$, and any θ.

to analyze the impact of selection on efficiency from an ex-ante perspective, we need to allow for the possibility that before $t = 0$, firms expect shocks with some probability. We focus on cases where, before the shock, firms expect nominal income to remain constant, on average. In particular, we assume that, conditional on a shock occurring, $m^{new}(t) - m^{old} = \Delta m(t)$ with probability $\frac{1}{2}$ and $m^{new}(t) - m^{old} = -\Delta m(t)$ otherwise, so that $E\left[m^{new}(t) - m^{old}\right] = 0$ and $E\left[\left(m^{new}(t) - m^{old}\right)^2\right] = (\Delta m(t))^2$. In addition, in order to obtain analytical results, we revert to an environment with strategic neutrality in price-setting.

6.1 Private efficiency

Up to a second-order approximation, we can write the deviation of profit losses from frictionless level for a given firm j at time t as

$$\hat{\pi}_j(t) = -\chi (p_j(t) - m(t))^2 + t.i.s., \tag{28}$$

where χ is a constant function of parameters and $t.i.s.$ stands for "terms independent of selection" (i.e., those that are out of the control of the firm).[23] This follows from the usual result that, up to second order, profit losses are well approximated by the square of the deviation between the firm's price and the target price that it would choose in the absence of nominal rigidities. Due to strategic neutrality, the latter is equal to nominal income $m(t)$.

At any time t, the fraction of firms with price $x(s)$, $s \in [-\infty, t]$ is $\Lambda(1 - G(t-s))$. Hence, we can write the average deviation of profits from the steady state at a given time t, $\hat{\pi}(t)$, as (ignoring terms independent of selection):[24]

$$\hat{\pi}(t) = -\chi \int_{-\infty}^{t} \Lambda(1 - G(t-s))(x(s) - m(t))^2 \, ds. \tag{29}$$

We first focus on level shocks, as specified in Section 3.2.1. Using the fact that (see equations (8) and (9)):

$$\int_{-\infty}^{0} \Lambda(1 - G(t-s)) \, ds = e^{-\Lambda t - \Lambda \Xi(t)}, \tag{30}$$

[23] See the Online Appendix, Section 5.1 for details.

[24] Although each firm only cares about its own profits, our focus on average profits is justified given our ex-ante perspective. In this context, we take firms as not knowing at which point in the life of their price spell the shock will hit.

and that, for such a shock, $x(t) = m^{new}\ \forall t > 0$, we find that the expected profit losses from monetary shocks are decreasing in selection:

$$\hat{\pi}(t) = -\chi e^{-\Lambda t - \Lambda \Xi(t)} \Delta m^2. \tag{31}$$

For more general shocks, the expression for average profit losses, (29), can be written as:

$$\hat{\pi}(t) = -\chi e^{-\Lambda t - \Lambda \Xi(t)} \Delta m(t)^2 - \chi \left(1 - e^{-\Lambda t - \Lambda \Xi(t)}\right) E\left[(x(s) - m(t))^2 | s \geq 0\right]. \tag{32}$$

There are now two components. The first component captures the profit losses stemming from firms being surprised by the nominal shock and not being able to adjust prices immediately in response. The second term is the average loss for firms that have changed their prices since the shock. It represents the profit losses stemming from the fact that firms need to keep prices constant between adjustments, and hence cannot track nominal income perfectly. In the previous case of a permanent shock to the level of nominal income, the second term drops out, since constant prices set after the shock are perfectly capable of tracking nominal income – giving rise to equation (31).

Whether selection will increase or decrease profit losses under general shocks is thus ambiguous, since it depends on the relative size of the two components. Furthermore, selection influences the relationship between $x(s)$ and $m(t)$, since firms set $x(s)$ optimally. In numerical exercises we find that, for a wide range of simple AR(1)-type shocks, maximal selection (i.e., Taylor pricing) yields smaller profit losses than no selection (i.e., Calvo pricing). This conforms with the intuition that, by allowing firms to better track their preferred prices, higher selection increases private efficiency.[25] However, we can also provide an example in which profit losses under Calvo pricing are smaller than under Taylor pricing (see the Online Appendix for details).

6.2 Social efficiency

We now turn to the relationship between selection and social efficiency. Following Benigno and Woodford (2005), we can substitute equilibrium conditions into the household's utility

[25]We thank an anonymous referee for suggesting that intuition.

function so as to write the utility of the representative household as:

$$U_0 = \int_0^\infty e^{-\rho t} \left(\frac{Y(t)^{1-\sigma} - 1}{1-\sigma} - \frac{Y(t)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} \int_0^1 \left(\frac{P_j(t)}{P} \right)^{-\left(1+\frac{1}{\psi}\right)\varepsilon} dj \right) dt. \tag{33}$$

We can then obtain a second-order approximation of the utility function of households at $t = 0$:

$$U_0 = Y^{1-\sigma} \int_0^\infty e^{-\rho t} \left[a_y y(t) - a_{yy} y(t)^2 - a_\Delta \int_0^1 (p_j(t) - p(t))^2 dj \right] dt,$$

where a_y, a_{yy}, and a_Δ are constant functions of parameters, with positive sign for reasonable parametrizations.[26] To a first-order approximation, the utility function is proportional to the cumulative real effect of the monetary shock, $\int_0^\infty e^{-\rho t} y(t)\, dt$. This term is the dominant one if we are mostly concerned with the ex-post welfare impact of (small) monetary shocks. From an ex-ante perspective, however, the first-order term is zero in expectation.[27] Hence, we turn to an analysis of the two remaining terms: $y(t)^2$, capturing the effect of monetary shocks on consumption (and leisure) volatility, and $\int_0^1 (p_j(t) - p(t))^2 dj$, capturing the effect of relative price distortions.

First, let us restrict our attention to level shocks. It is then straightforward to verify that:

$$E_0 \Delta y(t)^2 = (1 - \omega(t))^2 \Delta m^2 = e^{-2\Lambda t - 2\Lambda \Xi(t)} \Delta m^2.$$

Therefore, in that case, the ex-ante cost of monetary shocks due to consumption (and leisure) volatility decreases with selection.

Turning to the second term, which captures price dispersion, we can show that:

$$\begin{aligned}
\int_0^1 (p_j(t) - p(t))^2 dj &= \int_{-\infty}^t \Lambda (1 - G(t-s)) (x(s) - p(t))^2 dj \\
&= \left(1 - e^{-\Lambda t - \Lambda \Xi(t)}\right) e^{-\Lambda t - \Lambda \Xi(t)} (\Delta m - \Delta p(t))^2 + e^{-\Lambda t - \Lambda \Xi(t)} \Delta p(t)^2 \\
&= \left(1 - e^{-\Lambda t - \Lambda \Xi(t)}\right) e^{-\Lambda t - \Lambda \Xi(t)} \Delta m^2, \qquad (34)
\end{aligned}$$

[26] In particular, $a_y \equiv \frac{1}{\varepsilon}$, $a_{yy} \equiv \frac{1}{2}\left(\sigma + \left(1 + \psi^{-1}\right)\frac{\varepsilon - 1}{\varepsilon} - 1\right)$ and $a_\Delta \equiv \frac{1}{2}\left(1 + \frac{1}{\psi}\right)\varepsilon(\varepsilon - 1)$. Note that $a_{yy} > 0$ if $\sigma \geq 1$, as commonly assumed.

[27] This follows from the assumption that positive and negative nominal income shocks average out.

where the last equality uses the fact that $\Delta p(t) = e^{-\Lambda t - \Lambda \Xi(t)} \Delta m$. The impact of selection on price distortions is thus ambiguous. On the one hand, higher selection means that prices converge to m^{new} faster, so that at any point in time there are fewer "misaligned" prices. On the other hand, since under higher selection average prices converge more rapidly, it also means that prices that have not changed are more misaligned.

The bottom line is that the relationship between selection and welfare, which also depends on the relative sizes of the a_{yy} and a_{Δ} coefficients in (33), is ambiguous. This result is not unique to permanent level shocks. In particular, in the Online Appendix we consider the welfare impact of mean-reverting shocks under Taylor and Calvo pricing. We find that welfare losses due to the price dispersion component are higher under Calvo than under Taylor pricing when shocks are relatively persistent, but those same losses become lower for Calvo pricing for shocks that mean revert faster.

6.3 Alternative sources of fluctuations

So far we have only analyzed the implications of selection for the efficiency costs of monetary shocks. One may alternatively be interested in knowing the implications of selection given other kinds of shocks. This is especially interesting in the present context, since it is well known from the optimal monetary policy literature that different shocks have different welfare implications (see, for example, Woodford, 2003; and Adam, 2007). To highlight the role of selection, we analyze cases in which monetary policy keeps nominal income constant in the face of those shocks.

Insofar as firm profitability is concerned, the analysis remains unchanged with the only difference being that we need to substitute $m(t)$ in equation (28) for the target price implied by the shock.[28] It is for household welfare that the source of shocks becomes most interesting. We consider the impact of two shocks: to productivity and to the desired markup of firms.

The aggregate productivity shock changes the marginal product of labor from 1 to $Z(t)$. The desired markup shock changes the elasticity of substitution between varieties of goods ε, leading to a new desired markup. The first-order approximation to the optimality condition

[28]For results concerning private efficiency, as well as the derivation of results presented in this section, see the Online Appendix, Sections 4 and 5.

for price-setting becomes:

$$x_j(t) = x(t) = E_t \left[\frac{\int_0^\infty e^{-\rho s} (1 - G(s)) [w_j(t+s) + \zeta(t+s) - z(t+s)] \, ds}{\int_0^\infty e^{-\rho s} (1 - G(s)) \, ds} \right] \ \forall j,$$

where $\zeta(t+s)$ is the log-deviation from the steady state of the desired markup $\frac{\varepsilon(t)}{\varepsilon(t)-1}$, and $z(t+s)$ is log labor productivity. Also, $y_j(t) = l_j(t) + z(t)$. Finally, to second order, the expected utility function can be approximated as

$$EU_0 = -Y^{1-\sigma} \int_0^\infty e^{-\rho t} \left[a_{yy} (y(t) - by^*(t))^2 + a_\Delta \int_0^1 (p_j(t) - p(t))^2 \, dj \right] dt + t.i.s., \quad (35)$$

where $t.i.s.$ are terms independent of selection, $y^*(t) = \frac{1+\psi^{-1}}{\sigma+\psi^{-1}} z(t)$ is the deviation from steady-state of the efficient level of output (i.e., the level that would prevail under flexible prices and perfect competition), and $b = (\varepsilon - 1) / \left(\varepsilon - \frac{1+\psi^{-1}}{\sigma+\psi^{-1}} \right)$.[29]

Consider first a level shock to productivity, so that $z^{new}(t) - z^{old} = \Delta z$ for $t > 0$ with probability $\frac{1}{2}$, and it equals $-\Delta z$ otherwise. Given strategic neutrality, it follows that $x(t) = -\frac{1+\psi^{-1}}{1+\psi^{-1}\varepsilon} z^{new} \ \forall t$ and $y(t) = -p(t) = -\left(1 - e^{-\Lambda t - \Lambda \Xi(t)}\right) x^{new} = \left(1 - e^{-\Lambda t - \Lambda \Xi(t)}\right) \frac{1+\psi^{-1}}{1+\psi^{-1}\varepsilon} z^{new}$. The price dispersion term $\int_0^1 (p_j(t) - p(t))^2 \, dj$ is proportional to $\left(1 - e^{-\Lambda t - \Lambda \Xi(t)}\right) e^{-\Lambda t - \Lambda \Xi(t)}$, so that, as before, selection has an ambiguous welfare effect.

The term in $y(t)$ can be written as

$$E_0 \left[(y(t) - by^*(t))^2 \right] = \left(\frac{1+\psi^{-1}}{1+\psi^{-1}\varepsilon} \right)^2 \left(b - 1 + e^{-\Lambda t - \Lambda \Xi(t)} \right)^2 (\Delta z)^2 .$$

Insofar as output effects are concerned, higher selection is unambiguously welfare-improving so long as $b - 1 + e^{-\Lambda t - \Lambda \Xi(t)} > 0$ for all t. Since $e^{-\Lambda t - \Lambda \Xi(t)} > 0$, this will necessarily be the case if $b > 1$. In that case, selection is beneficial because economies with higher selection approach efficient output more quickly. If, however, $b < 1$, the effect of selection is ambiguous. Intuitively, $b < 1$ introduces a motive for the planner to stabilize output relative to efficient output. By reducing the responsiveness of output to productivity shocks, low selection helps the planner achieve this objective.

Consider now the impact of a level shock to desired markups, so that $\xi^{new}(t) - \xi^{old} = \Delta \xi$ for $t > 0$ with probability $\frac{1}{2}$ and equals $-\Delta \xi$ otherwise. It follows that $x^{new}(t) = \frac{1}{1+\varepsilon\psi^{-1}} \xi^{new}$

[29] As before, first-order terms disappear because of the assumption that shocks average out in expectation.

for $t > 0$ and $y(t) = -p(t) = -\left(1 - e^{-\Lambda t - \Lambda \Xi(t)}\right) x^{new} = -\left(1 - e^{-\Lambda t - \Lambda \Xi(t)}\right) \frac{1}{1+\varepsilon \psi^{-1}} \xi^{new}$. The price dispersion term $\int_0^1 (p_j(t) - p(t))^2 \, dj$ is proportional to $\left(1 - e^{-\Lambda t - \Lambda \Xi(t)}\right) e^{-\Lambda t - \Lambda \Xi(t)}$. Since efficient output does not change (i.e., $y^*(t) = 0$), the output term $(y(t) - by^*(t))^2$ is now

$$E_0\left[(y(t) - by^*(t))^2\right] = \left(1 - e^{-\Lambda t - \Lambda \Xi(t)}\right)^2 \left(\frac{1}{1+\varepsilon \psi^{-1}}\right)^2 (\Delta \xi)^2 .$$

Thus, in terms of the welfare losses that are due to output fluctuations, higher selection is unambiguously detrimental to welfare. Intuitively, if markup shocks are prevalent, weaker selection can be socially beneficial, since it prevents actual markups from fluctuating as much.

6.4 Summary

In this section we analyzed the relationship between selection and efficiency (both private and social) in the face of different kinds of shocks. Differently from the analysis of non-neutrality in previous sections, this requires the use of a second-order approximation of objective functions. Up to second order, the effects of selection on private and social efficiency are ambiguous. Furthermore, because firms and society face different objective functions, the implications of selection for private efficiency may be opposite from those for social efficiency. For example, in face of markup shocks, high selection may be optimal for firms but detrimental for social welfare.

While we focus our exposition on aggregate shocks, our analysis of the implications of selection for private efficiency applies equally to cases in which shocks to firms' frictionless optimal prices are idiosyncratic. In this context, it is natural to entertain the possibility that the pattern of selection – rather than being arbitrary – should be the outcome of some optimization on the part of firms, with idiosyncratic shocks figuring prominently.[30]

The fact that firms and society face different objective functions, together with the fact that aggregate and idiosyncratic shocks may have different dynamic properties, can lead to a tension between private and social efficiency. Therefore, firms may choose a pattern of selection that is suboptimal from a social perspective.

[30] While we take the pattern of selection to be a primitive in our model, time-dependent pricing may be derived as stemming from firms' optimal decision-making in face of different kinds of frictions. For example, Bonomo and Carvalho (2004) and Alvarez and Lippi (2014) examine cases in which Taylor pricing is optimal. In contrast, Woodford (2009) develops a model in which the optimal policy can be well approximated by Calvo (1983) pricing.

7 Conclusion

We investigate the different ways in which the shape of the distribution of duration of price spells affects the real effects of nominal aggregate demand shocks. We highlight a mechanism that so far has barely been given attention in the literature: a selection for the time since prices were last adjusted. In fact, we show that selection provides a complete characterization of the distribution of price durations in time-dependent sticky-price models. We also analyze the implications of selection for private and social efficiency in the presence of monetary and other kinds of shocks.

The results in the paper suggest that a careful characterization of the distribution of price durations is of crucial importance for the proper evaluation of the aggregate implications of nominal price stickiness. While the results are derived for the case of time-dependent pricing, there is no reason why the selection effect identified here should not hold some relevance more broadly, whenever the timing of price changes is not entirely up to the discretion of the firms. This suggests that further research on price setting would do well to focus on models that are able to fully account for the distribution of price spells and investigate the extent to which the mechanisms emphasized here continue to matter.

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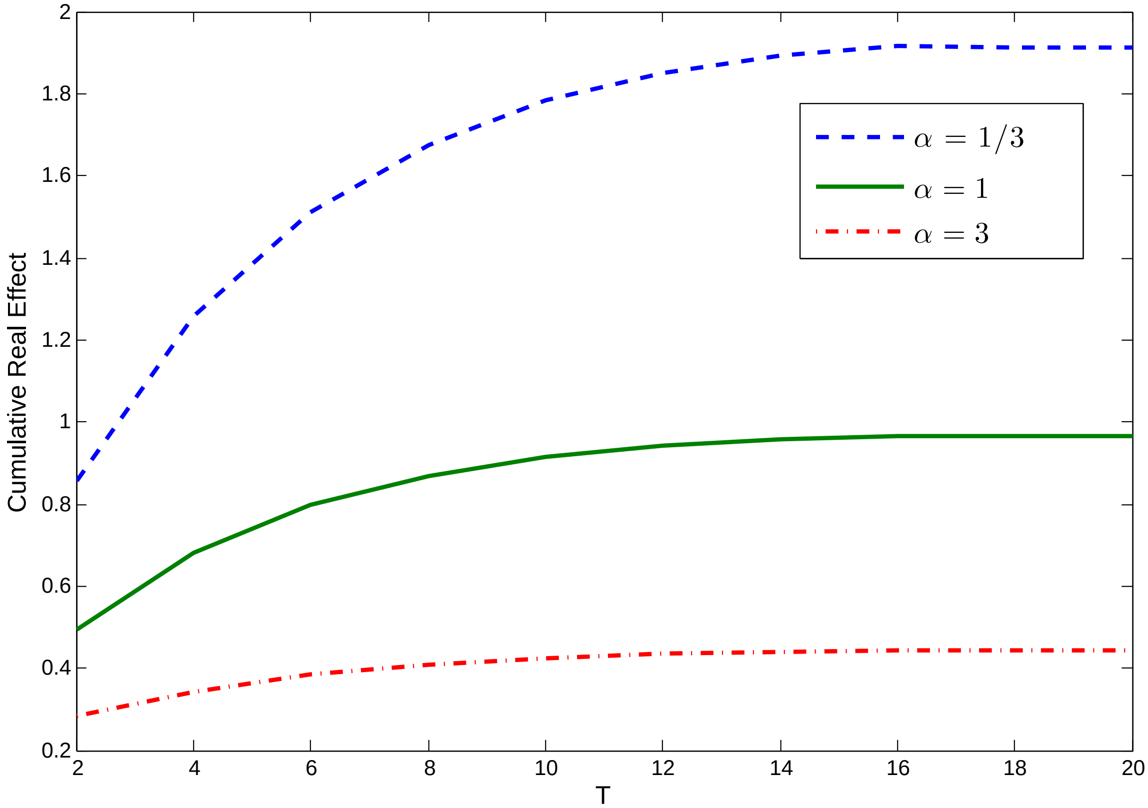

Figure 1: Cumulative real effect a shock to the level of nominal income: Same average duration, different variances.

Note: Average price duration is equal to 2. Distribution of price durations follow equations (24) and (25), see Online Appendix for details.

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