TEXTO PARA DISCUSSÃO

No. 628

Monetary Policy and Real Exchange Rate Dynamics in Sticky-Price Models_

Carlos Carvalho
Fernanda Nechio

DEPARTAMENTO DE ECONOMIA
www.econ.puc-rio.br

Monetary Policy and Real Exchange Rate Dynamics in Sticky-Price Models*

Carlos Carvalho
PUC-Rio

Fernanda Nechio
Federal Reserve Bank of San Francisco

March, 2015

Abstract

We study how real exchange rate dynamics are affected by monetary policy in dynamic, stochastic, general equilibrium, sticky-price models. Our analytical and quantitative results show that the source of interest rate persistence – policy inertia or persistent policy shocks – is key. When the monetary policy rule has a strong interest rate smoothing component, these models fail to generate high real exchange rate persistence in response to monetary shocks, as policy inertia hampers their ability to generate a hump-shaped response to such shocks. Moreover, in the presence of persistent monetary shocks, increasing policy inertia may *decrease* real exchange rate persistence.

JEL classification codes: F3, F41, E0

Keywords: real exchange rates, monetary policy, interest rate smoothing, PPP puzzle, persistence.

*For comments and suggestions we thank seminar participants at SF Fed, UT at Austin and EEA 2014. The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of San Francisco or the Federal Reserve System. E-mails: cvianac@econ.puc-rio.br, fernanda.nechio@sf.frb.org.

1 Introduction

The open economy macroeconomics literature has struggled to develop models that can replicate the empirical evidence of high persistence and volatility of real exchange rate (RER) fluctuations in response to shocks – in particular, to monetary shocks. This difficulty came to be known as the Purchasing Power Parity (PPP) puzzle, as put forth by Rogoff (1996). Within this literature, Engel (2012) and Benigno (2004) highlight the interaction between monetary policy and price stickiness. In a standard one-sector sticky-price model, they find that price stickiness only matters for real exchange rate persistence when monetary policy features policy inertia.[1] In that context, if the degree of policy inertia is strong enough (i.e., the smoothing component of the interest rate rule followed by the monetary authority is high enough), the model can generate some persistence in the dynamic response of the real exchange rate to monetary policy shocks that are serially uncorrelated.

However, in the empirical macroeconomics literature there is an ongoing debate about the source of interest rate persistence observed in the data. For example, Rudebusch (2002) provides evidence that it arises mainly from persistent monetary shocks, whereas Coibion and Gorodnichenko (2012) point to policy inertia as the main source of interest rate persistence. Both papers provide some evidence that monetary policy rules likely feature both sources of persistence.

In this paper we study the extent to which the modeling choice for the source interest rate persistence matters for real exchange rate dynamics in sticky-price DSGE models. To that end, we study versions of the two-country multisector model of Carvalho and Nechio (2011) with different specifications of the monetary policy rule. That model produces empirically plausible real exchange rate dynamics in response to nominal aggregate demand disturbances. We drop the assumption of an exogenous nominal aggregate demand process in favor of explicit monetary policy rules. In particular, we use a standard Taylor (1993) rule, and allow for persistent policy shocks and/or policy inertia (interest rate smoothing) as possible sources of interest rate persistence. Whereas the model of Carvalho and Nechio (2011) features heterogeneity in the degree of price stickiness across sectors, to relate our findings to Engel (2012) and Benigno (2004) we also entertain one-sector versions of the model, in which the degree of price rigidity is the same for all firms.

We find that the source of interest rate persistence – policy inertia or persistent policy shocks – matters a great deal. Quantitatively, persistent policy shocks go a long way in generating real exchange rate persistence – although much more so in the multisector model with heterogeneity in price stickiness. In contrast, policy inertia only manages to produce low levels of real exchange rate

[1] Benigno (2004) studies a one-sector model in which he allows the frequency of price changes for exporting goods to differ from the frequency for domestic price setting. He also allows for an asymmetry in the frequency of price changes across countries. He shows that when this heterogeneity leads to different frequencies of price changes within a same country (due to differences in frequencies for varieties produced by local versus foreign firms), the real exchange rate becomes more persistent.

persistence. In fact, if the policy rule followed by the monetary authorities has too strong an interest rate smoothing component, even the multisector sticky-price model fails to generate meaningful real exchange rate persistence in response to monetary shocks.

Our finding that policy inertia hampers the ability of the model to generate RER persistence may seem to contradict the results in Engel (2012) and Benigno (2004). However, this is not the case. Consistent with those papers, in response to i.i.d. policy shocks, we find that real exchange rate persistence is increasing in the degree of policy inertia. Likewise, absent policy inertia, RER persistence increases with the degree of (positive) autocorrelation of monetary shocks. These results hold for both multisector and one-sector versions of our model.

When both policy inertia and persistent monetary shocks are present, however, the comparative statics results become richer. If the degree of (positive) serial autocorrelation of monetary shocks is not too high, RER persistence continues to increase with the degree of policy inertia. So, the findings of Engel (2012) and Benigno (2004) extend to the case of not-too-persistent monetary policy shocks. However, this ceases to be true when monetary shocks are persistent enough. In that case, introducing policy inertia *decreases* RER persistence. We show these results analytically in a simplified one-sector model, and confirm that they continue to hold in our calibrated multisector and one-sector economies.

The reason why monetary policy matters so much and why policy inertia can hamper the ability of the models to generate meaningful degrees of RER persistence can be traced back to Steinsson (2008). He argues that the ability of a model to produce hump-shaped real exchange rate dynamics is critical to matching the degree of persistence seen in the data. Our results show that, depending on the nature of policy, monetary shocks can induce hump-shaped RER dynamics. This happens in our multisector model when the policy rule resembles the original Taylor (1993) rule, without interest rate smoothing. However, with a large enough degree of policy inertia, monetary shocks fail to generate hump-shaped responses. Echoing the findings in Steinsson (2008), in that case the model fails to generate large enough RER persistence.

In our baseline analysis, we follow the literature and compare model predictions to *unconditional* moments of real exchange data estimated by Steinsson (2008). While this is the common practice in the literature, one may question the comparison of unconditional data moments with model-implied moments generated by monetary shocks only. Hence, we redo our analysis using RER moments conditional on identified monetary policy shocks. To that end, we revisit the results of Eichenbaum and Evans (1995), extending their sample from 1990 to 2007, and estimating the response of the RER of various countries to identified shocks to the Federal Funds Rate. Conditional RER moments do differ from unconditional moments that the literature has focused on – in particular, the dynamic response of RERs to monetary policy shocks is even more persistent than its unconditional dynamics. However, the lessons from our analysis of the different versions of the model are unchanged, irrespective

of whether we discipline the reference model using unconditional or conditional RER moments.

Although we focus our analysis on open economy models, our lessons are not limited to this context. As highlighted by Engel (2012), the lessons apply to some closed-economy models that are isomorphic to their open-economy versions. Moreover, lessons from open economy models such as the ones we analyze are likely to hold approximately in their closed economy versions whenever the degree of home bias in consumption is large enough. In those cases, the effects of different monetary policy rules should show up in the dynamics of real variables such as consumption and output. Overall, our results on the different effects of alternative choices for the source of interest rate persistence in the model highlight the importance of the empirical debate on this issue (e.g. Rudebusch 2002, Coibion and Gorodnichenko 2012).

Besides the connection with Benigno (2004) and Engel (2012), our paper is related more broadly to the literature that uses dynamic sticky-price models to study the persistence of real exchange rates, such as Bergin and Feenstra (2001), Kollman (2001), Chari et al. (2002), Steinsson (2008), Johri and Lahiri (2008), Martinez-Garcia and Søndergaard (2008).

Section 2 presents the model. Section 3 provides details of the calibration, followed by a quantitative analysis of the effects of different monetary policy rules on RER dynamics. In section 4 we derive analytical results for a simplified version of the model, which show that RER persistence may decrease in the presence of policy inertia. We follow with a discussion of the mechanisms behind our findings. In Section 5 we revisit the seminal contribution of Eichenbaum and Evans (1995), an redo our analysis comparing model predictions to moments of RERs conditional on identified monetary shocks. The last section concludes.

2 The model

This section reproduces the main features of the model in Carvalho and Nechio (2011) and introduces an explicit monetary policy rule in place of an exogenous specification for nominal demand. The world economy consists of two symmetric countries, *Home* and *Foreign*. In each country, identical consumers supply labor to intermediate firms that they own, invest in a complete set of state-contingent financial claims, and consume a nontraded final good. Nontraded final goods are produced by competitive firms that combine intermediate varieties produced in the two countries. These varieties are produced by monopolistically competitive firms that are divided into sectors that differ in their frequency of price changes. Intermediate firms can price-discriminate across countries and set prices in local currency.

The Home representative consumer chooses consumption of the final good, C_t, and total labor supply N_t, to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} \right),$$

subject to the flow budget constraint:

$$P_t C_t + E_t \left[\Theta_{t,t+1} B_{t+1}\right] \leq W_t N_t + B_t + T_t,$$

and a standard "no-Ponzi" condition:

$$B_{t+1} \geq -\sum_{l=t+1}^{\infty} E_{t+1} \left[\Theta_{t+1,l} \left(W_l N_l + T_l\right)\right] \geq -\infty,$$

where $\Theta_{t,l} \equiv \prod_{l'=t+1}^{l} \Theta_{l'-1,l'}$, E_t denotes the time-t expectations operator, W_t is the corresponding nominal wage rate, and T_t stands for net transfers from the government plus profits from Home intermediate firms. The final good sells at the nominal price P_t, and B_{t+1} accounts for the state-contingent value of the portfolio of financial securities held by the consumer at the beginning of $t+1$. Under complete financial markets, agents can choose the value of B_{t+1} for each possible state of the world at all times. A no-arbitrage condition requires the existence of a nominal stochastic discount factor $\Theta_{t,t+1}$ that prices in period t any financial asset portfolio with state-contingent payoff B_{t+1} at the beginning of period $t+1$.[2] Finally, β is the time-discount factor, σ^{-1} denotes the intertemporal elasticity of substitution, and γ^{-1} is the Frisch elasticity of labor supply.

The maximization problem yields as first-order conditions for consumption and labor:

$$\begin{aligned} \frac{C_t^{-\sigma}}{C_{t+l}^{-\sigma}} &= \frac{\beta^l}{\Theta_{t,l}} \frac{P_t}{P_{t+l}}, \\ \frac{W_t}{P_t} &= N_t^{\gamma} C_t^{\sigma}. \end{aligned} \tag{1}$$

Under complete markets, one can price a one-period riskless nominal bond as:

$$\frac{1}{I_t} = \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}}\right], \tag{2}$$

where I_t is the short-term nominal interest rate.

Finally, the solution must also satisfy a transversality condition:

$$\lim_{l \to \infty} E_t \left[\Theta_{t,t+l} B_{t+l}\right] = 0.$$

The Foreign consumer solves an analogous problem and maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{*1-\sigma} - 1}{1-\sigma} - \frac{N_t^{*1+\gamma}}{1+\gamma}\right),$$

[2] To avoid cluttering the notation, we omit explicit reference to the different states of nature.

subject to the flow budget constraint:

$$P_t^* C_t^* + E_t \left[\Theta_{t,t+1}^* \frac{B_{t+1}^*}{\mathcal{E}_t} \right] \leq W_t^* N_t^* + \frac{B_t^*}{\mathcal{E}_t} + T_t^*, \tag{3}$$

and an analogous "no-Ponzi" condition. A superscript "$*$" denotes the Foreign counterpart of the corresponding Home variable. Without loss of generality, we assume that the complete set of state-contingent assets are denominated in the Home currency. As a result, in the budget constraint (3), B_t^* appears divided by the nominal exchange rate, $\mathcal{E}_t$, to convert the value of the portfolio into Foreign currency. $\mathcal{E}_t$ is defined as the price of the Foreign currency in terms of the Home currency, hence, it is quoted in units of Home currency per unit of the Foreign currency.

The Foreign consumer's optimality conditions for consumption and labor, and transversality condition are:

$$\begin{aligned} \frac{C_t^{*-\sigma}}{C_{t+l}^{*-\sigma}} &= \frac{\beta^s}{\Theta_{t,t+l}^*} \frac{\mathcal{E}_t P_t^*}{\mathcal{E}_{t+l} P_{t+l}^*}, \\ \frac{W_t^*}{P_t^*} &= C_t^{*\sigma} N_t^{*\gamma}, \\ \lim_{l \to \infty} E_t \left[\Theta_{t,t+l}^* B_{t+l}^* \right] &= 0. \end{aligned} \tag{4}$$

Since there are no arbitrage opportunities and assets are freely traded, the stochastic discount factor has to be the same for both countries. Defining $Q_t \equiv \mathcal{E}_t \frac{P_t^*}{P_t}$ as the real exchange rate, from equations (1) and (4):

$$Q_{t+l} = Q_t \frac{C_t^{-\sigma}}{C_{t+l}^{-\sigma}} \frac{C_{t+l}^{*-\sigma}}{C_t^{*-\sigma}}. \tag{5}$$

Iterating equation (5) backwards and assuming $Q_0 \frac{C_0^{-\sigma}}{C_0^{*-\sigma}} = 1$ yields:

$$Q_t = \frac{C_t^{*-\sigma}}{C_t^{-\sigma}}. \tag{6}$$

A representative competitive firm produces the final good, which is a composite of varieties of intermediate goods from both countries. Monopolistically competitive firms produce each variety of intermediate goods. The latter firms are divided into sectors indexed by $s \in \{1, ..., S\}$, each featuring a continuum of firms. Sectors differ in the degree of price rigidity, as we detail below. Overall, firms are indexed by the country where they produce, by their sector, and are further indexed by $j \in [0, 1]$. The distribution of firms across sectors is given by sectoral weights $f_s > 0$, with $\sum_{s=1}^{S} f_s = 1$.

The final good is produced by combining the intermediate varieties according to the technology:

$$Y_t = \left(\sum_{s=1}^{S} f_s^{\frac{1}{\eta}} Y_{s,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}, \tag{7}$$

$$Y_{s,t} = \left(\omega^{\frac{1}{\rho}} Y_{H,s,t}^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} Y_{F,s,t}^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}, \tag{8}$$

$$Y_{H,s,t} = \left(f_s^{\frac{\theta-1}{\theta}} \int_0^1 Y_{H,s,j,t}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}, \tag{9}$$

$$Y_{F,s,t} = \left(f_s^{\frac{\theta-1}{\theta}} \int_0^1 Y_{F,s,j,t}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}, \tag{10}$$

where Y_t is the Home final good, $Y_{s,t}$ is the aggregation of sector-s Home and Foreign intermediate goods sold in Home, $Y_{H,s,t}$ and $Y_{F,s,t}$ are the composites of intermediate varieties produced by firms in sector s in Home and Foreign, respectively, to be sold in Home, and $Y_{H,s,j,t}$ and $Y_{F,s,j,t}$ are the varieties produced by firm j in sector s in Home and Foreign to be sold in Home. The parameters $\eta \geq 0$, $\rho \geq 0$, and $\theta > 1$ are, respectively, the elasticity of substitution across sectors, the elasticity of substitution between Home and Foreign goods, and the elasticity of substitution within sectors. Finally, $\omega \in [0,1]$ is the steady-state share of domestic inputs.

A representative Home final-good-producing firm solves:

$$\begin{aligned} &\max \; P_t Y_t - \sum_{s=1}^{S} f_s \int_0^1 \left(P_{H,s,j,t} Y_{H,s,j,t} + P_{F,s,j,t} Y_{F,s,j,t}\right) dj \\ &s.t. \;\; (7)\text{-}(10), \end{aligned}$$

which yields as first-order conditions, for $j \in [0,1]$ and $s = 1, ..., S$:

$$\begin{aligned} Y_{H,s,j,t} &= \omega \left(\frac{P_{H,s,j,t}}{P_{H,s,t}}\right)^{-\theta} \left(\frac{P_{H,s,t}}{P_{s,t}}\right)^{-\rho} \left(\frac{P_{s,t}}{P_t}\right)^{-\eta} Y_t, \\ Y_{F,s,j,t} &= (1-\omega) \left(\frac{P_{F,s,j,t}}{P_{F,s,t}}\right)^{-\theta} \left(\frac{P_{F,s,t}}{P_{s,t}}\right)^{-\rho} \left(\frac{P_{s,t}}{P_t}\right)^{-\eta} Y_t. \end{aligned}$$

The price indices are given by:

$$P_t = \left(\sum_{s=1}^{S} f_s P_{s,t}^{1-\eta}\right)^{\frac{1}{1-\eta}}, \tag{11}$$

$$P_{s,t} = \left(\omega P_{H,s,t}^{1-\rho} + (1-\omega) P_{F,s,t}^{1-\rho}\right)^{\frac{1}{1-\rho}}, \tag{12}$$

$$P_{H,s,t} = \left(\int_0^1 P_{H,s,j,t}^{1-\theta} dj\right)^{\frac{1}{1-\theta}}, \tag{13}$$

$$P_{F,s,t} = \left(\int_0^1 P_{F,s,j,t}^{1-\theta} dj\right)^{\frac{1}{1-\theta}}, \tag{14}$$

where P_t is the price of the Home final good, $P_{s,t}$ is the price index of sector-s intermediate goods sold in Home, $P_{H,s,t}$ is the price index for sector-s Home-produced intermediate goods sold in Home, and $P_{H,s,j,t}$ is the price charged in the Home market by Home firm j from sector s. $P_{F,s,t}$ is the price index for sector-s Foreign-produced intermediate goods sold in Home, and $P_{F,s,j,t}$ is the price charged in the Home market by Foreign firm j from sector s. Both $P_{H,s,j,t}$ and $P_{F,s,j,t}$ are set in the Home currency.

The Foreign final firm solves an analogous maximization problem and its demands for intermediate inputs from Foreign ($Y^*_{F,s,j,t}$) and Home ($Y^*_{H,s,j,t}$) producers are:

$$Y^*_{F,s,j,t} = \omega \left(\frac{P^*_{F,s,j,t}}{P^*_{F,s,t}}\right)^{-\theta} \left(\frac{P^*_{F,s,t}}{P^*_{F,t}}\right)^{-\rho} \left(\frac{P^*_{s,t}}{P^*_t}\right)^{-\eta} Y^*_t,$$

$$Y^*_{H,s,j,t} = (1-\omega) \left(\frac{P^*_{H,s,j,t}}{P^*_{H,s,t}}\right)^{-\theta} \left(\frac{P^*_{H,s,t}}{P^*_{s,t}}\right)^{-\rho} \left(\frac{P^*_{s,t}}{P^*_t}\right)^{-\eta} Y^*_t.$$

In analogy to equations (11) to (14), Foreign price indices are given by:

$$\begin{aligned}
P^*_t &= \left(\sum\nolimits_{s=1}^{S} f_s P^{*1-\eta}_{s,t}\right)^{\frac{1}{1-\eta}}, \\
P^*_{s,t} &= \left(\omega P^{*1-\rho}_{F,s,t} + (1-\omega) P^{*1-\rho}_{H,s,t}\right)^{\frac{1}{1-\rho}}, \\
P^*_{H,s,t} &= \left(\int_0^1 P^{*1-\theta}_{H,s,j,t} dj\right)^{\frac{1}{1-\theta}}, \\
P^*_{F,s,t} &= \left(\int_0^1 P^{*1-\theta}_{F,s,j,t} dj\right)^{\frac{1}{1-\theta}},
\end{aligned}$$

where P^*_t is the price of the Foreign final good, $P^*_{s,t}$ is the price index of sector-s intermediate goods sold in Foreign, $P^*_{F,s,t}$ is the price index for sector-s Foreign-produced intermediate goods sold in Foreign, and $P^*_{F,s,j,t}$ is the price charged in the Foreign market by Foreign firm j from sector s. $P^*_{H,s,t}$ is the price index for sector-s Home-produced intermediate goods sold in Foreign, and $P^*_{H,s,j,t}$ is the price charged in the Foreign market by Home firm j from sector s. Both $P^*_{F,s,j,t}$ and $P^*_{H,s,j,t}$ are set in the Foreign currency.

Monopolistically competitive firms produce varieties of the intermediate good by employing labor. As in Carvalho and Nechio (2011), these firms set prices as in Calvo (1983). The frequency of price changes varies across sectors, and in each period, each firm j in sector s changes its price independently with probability α_s. This is the only source of (ex-ante) heterogeneity.

At each time a Home-firm j from sector s adjusts its price, it chooses prices $X_{H,s,j,t}$, $X^*_{H,s,j,t}$ to be charged in the Home and Foreign markets, respectively, with each price being set in the corresponding

local currency. The maximization problem is:

$$\begin{aligned}
\max\ & E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \left[\begin{array}{c} X_{H,s,j,t} Y_{H,s,j,t+l} + \mathcal{E}_{t+l} X^*_{H,s,j,t} Y^*_{H,s,j,t+l} \\ -W_{t+l} N_{s,j,t+l} \end{array} \right] \\
st \qquad & Y_{H,s,j,t} = \omega \left(\frac{P_{H,s,j,t}}{P_{H,s,t}} \right)^{-\theta} \left(\frac{P_{H,s,t}}{P_{s,t}} \right)^{-\rho} \left(\frac{P_{s,t}}{P_t} \right)^{-\eta} Y_t \\
& Y^*_{H,s,j,t} = (1-\omega) \left(\frac{P^*_{H,s,j,t}}{P^*_{H,s,t}} \right)^{-\theta} \left(\frac{P^*_{H,s,t}}{P^*_{s,t}} \right)^{-\rho} \left(\frac{P^*_{s,t}}{P^*_t} \right)^{-\eta} Y^*_t \\
& Y_{H,s,j,t} + Y^*_{H,s,j,t} = F(N_{s,j,t}) = N^{\chi}_{s,j,t},
\end{aligned}$$

where χ determines returns to labor.

The first-order conditions for optimal price setting lead to:

$$\begin{aligned}
X_{H,s,j,t} &= \frac{\theta}{\theta-1} \frac{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \Lambda_{H,s,t+l} \left(\chi N^{\chi-1}_{s,j,t+l} \right)^{-1} W_{t+l}}{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \Lambda_{H,s,t+l}}, \\
X^*_{H,s,j,t} &= \frac{\theta}{\theta-1} \frac{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \Lambda^*_{H,s,t+l} \left(\chi N^{\chi-1}_{s,j,t+l} \right)^{-1} W_{t+l}}{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \mathcal{E}_{t+l} \Lambda^*_{H,s,t+l}},
\end{aligned}$$

where:

$$\begin{aligned}
\Lambda_{H,s,t} &= \omega \left(\frac{1}{P_{H,s,t}} \right)^{-\theta} \left(\frac{P_{H,s,t}}{P_{s,t}} \right)^{-\rho} \left(\frac{P_{s,t}}{P_t} \right)^{-\eta} Y_t, \\
\Lambda^*_{H,s,t} &= (1-\omega) \left(\frac{1}{P^*_{H,s,t}} \right)^{-\theta} \left(\frac{P^*_{H,s,t}}{P^*_{s,t}} \right)^{-\rho} \left(\frac{P^*_{s,t}}{P^*_t} \right)^{-\eta} Y^*_t.
\end{aligned}$$

By analogy, the Foreign firm problem yields:

$$\begin{aligned}
X^*_{F,s,j,t} &= \frac{\theta}{\theta-1} \frac{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \Lambda^*_{F,s,t+l} \left(\chi \left(N^*_{s,j,t+l} \right)^{\chi-1} \right)^{-1} W^*_{t+l}}{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \Lambda^*_{F,s,t+l}}, \\
X_{F,s,j,t} &= \frac{\theta}{\theta-1} \frac{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \Lambda_{F,s,t+l} \left(\chi \left(N^*_{s,j,t+l} \right)^{\chi-1} \right)^{-1} W^*_{t+l}}{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1-\alpha_s)^l \Lambda_{F,s,t+l} / \mathcal{E}_{t+l}},
\end{aligned}$$

where

$$\begin{aligned}
\Lambda^*_{F,s,t} &= \omega \left(\frac{1}{P^*_{F,s,t}} \right)^{-\theta} \left(\frac{P^*_{F,s,t}}{P^*_{s,t}} \right)^{-\rho} \left(\frac{P^*_{s,t}}{P^*_t} \right)^{-\eta} Y^*_t, \\
\Lambda_{F,s,t} &= (1-\omega) \left(\frac{1}{P_{F,s,t}} \right)^{-\theta} \left(\frac{P_{F,s,t}}{P_{s,t}} \right)^{-\rho} \left(\frac{P_{s,t}}{P_t} \right)^{-\eta} Y_t.
\end{aligned}$$

Finally, the market clearing conditions for Home include:

$$N_t = \sum_{s=1}^{S} f_s \int_0^1 N_{s,j,t} dj,$$

and likewise for Foreign.

Note that when all sectors have the same frequency of price adjustment, the multisector economy becomes a standard one-sector sticky-price model. In our quantitative analysis, we consider a one-sector economy with frequency of price changes equal to the average frequency of adjustments in the multisector world economy: $\overline{\alpha} = \sum_{s=1}^{S} f_s \alpha_s$.

2.1 Monetary policy

To close the model we specify a monetary policy rule. We consider a Taylor-type interest rate rule of the form:

$$I_t = \beta^{-1} \left(IR_{t-1}\right)^{\rho_i} \left(\frac{P_t}{P_{t-1}}\right)^{(1-\rho_i)\phi_\pi} \left(\frac{GDP_t}{GDP}\right)^{(1-\rho_i)\phi_Y} e^{v_t}, \tag{15}$$

where I_t, as previously defined, is the nominal interest rate on one-period riskless bonds at time t, $GDP_t \equiv Y_t + \sum_{s=1}^{S} f(s) \int_0^1 Y^*_{H,s,j,t} dj - \sum_{s=1}^{S} f(s) \int_0^1 Y_{F,s,j,t} dj$ is gross domestic product,[3] GDP denotes gross domestic product in steady state, ρ_i, ϕ_π, and ϕ_Y are the parameters associated with the interest rate rule, and v_t is a persistent shock with process $v_t = \rho_v v_{t-1} + \sigma_{\varepsilon_v} \varepsilon_{v,t}$, where $\varepsilon_{v,t}$ is a zero mean, unit variance *i.i.d.* shock, and $\rho_v \in [0, 1)$. We assume throughout that monetary policy in Foreign follows an analogous rule as in Home, and that monetary shocks are uncorrelated across the two countries.

We solve the model by log-linearizing around a zero-inflation steady state. For more details about the model and solution, we refer the reader to Carvalho and Nechio (2011).

3 Quantitative results

The parameterization of the cross-sectional distribution of price stickiness follows Carvalho and Nechio (2011). The 271 categories of goods and services reported by Nakamura and Steinsson (2008) are aggregated into 67 expenditure classes. The frequency of price changes for each expenditure class is obtained as a weighted average of the frequencies for the underlying categories, using the expenditure weights provided by those authors. The resulting average monthly frequency of price changes is $\bar{\alpha} = \Sigma_{s=1}^{S} f_s \alpha_s = 0.211$ – this is used to calibrate the one-sector economy.

The following parameters are also set as in Carvalho and Nechio (2011). We set σ^{-1} to 1/3, the (Frisch) labor supply elasticity is set to unit ($\gamma = 1$), labor share χ is set to 2/3, and the consumer's

[3] This GDP definition follows from Chari et al. (2002).

time preference rate is set to 2% per year. The elasticity of substitution between varieties within sectors is set to $\theta = 10$, the elasticity of substitution between Home and Foreign goods is set to $\rho = 1.5$, the elasticity of substitution between varieties of different sectors is set to unit ($\eta = 1$), and the share of domestic goods is set to $\omega = 0.9$.[4]

Turning to the specifications for monetary policy, we set $\phi_\pi = 1.5$, $\phi_Y = 0.5/12$, which are standard values for Taylor rules. We consider cases with persistent shocks only ($\rho_v > 0$, $\rho_i = 0$), policy inertia only ($\rho_v = 0$, $\rho_i > 0$), and with both persistent shocks and policy inertia (ρ_v, $\rho_i > 0$). We use the model with policy inertia only as a reference point, and calibrate ρ_v to generate plausible RER dynamics. In particular, we roughly match both the level of persistence and additional, more nuanced features of the underlying impulse response function (see Table 1 below). Given the value for ρ_v, the other two cases, which feature interest rate smoothing, are meant to illustrate the effects of policy inertia. In Section 4 we extend the comparative statics and consider different parameter values for ρ_i and ρ_v.

Following standard practice in the literature (e.g., Chari et a. 2002, Steinsson 2008), our calibration targets unconditional RER moments. In Section 5 we redo our analysis with a reference model calibrated to roughly match the dynamic properties of RERs conditional on monetary shocks identified as in Eichenbaum and Evans (1995). Our results are robust to this alternative approach.

Solving and simulating the multisector model with 67 sectors is computationally costly. To sidestep this problem we work with a 3-sector approximation to the underlying 67-sector economy. We choose the frequencies of price changes and sectoral weights in the approximating model to match a suitably chosen set of moments of the cross-sectional distribution of price stickiness of the original 67-sector economy. In the Appendix, we show that this delivers a very good approximation to the 67-sector model.

Table 1: Alternative monetary policy rules

Persistence measures:	Data:	Multisector economy			One-sector economy		
		$\rho_v = 0.975$ $\rho_i = 0$	$\rho_v = 0$ $\rho_i = 0.975$	$\rho_v = 0.975$ $\rho_i = 0.975$	$\rho_v = 0.975$ $\rho_i = 0$	$\rho_v = 0$ $\rho_i = 0.975$	$\rho_v = 0.975$ $\rho_i = 0.975$
$\mathcal{HL}$	54	53	6	8	28	4	4
$\mathcal{QL}$	76	83	17	29	55	8	8
$\mathcal{UL}$	28	22	0	0	0	0	0
$\mathcal{CIR}$	80	71	14	24	40	6	7
ρ_1	0.8	0.9	0.7	0.7	0.8	0.6	0.6
Volatility: $\frac{\sigma_q}{\sigma_c}$	5.5	3.5	4.0	4.0	3.1	3.9	3.9

[4] Carvalho and Nechio (2011) provide more details on the choices for these parameter values.

Table 1 reports the results for different versions of the model. In terms of persistence, it shows results for the half-life ($\mathcal{HL}$) of RER deviations from parity, the quarter-life ($\mathcal{QL}$), the up-life ($\mathcal{UL}$) of those deviations, and the cumulative impulse response ($\mathcal{CIR}$). Results for all four measures are reported in months. The quarter-lives and the up-lives are meant to provide a better picture of the shape of the impulse response function. They correspond to, respectively, the time it takes for the impulse response function to drop below 1/4 of the initial impulse, and the time it takes for the real exchange rate to peak after the initial response. Hence, a hump-shaped impulse response function yields a nonzero up-life ($\mathcal{UL}$). The cumulative impulse response ($\mathcal{CIR}$) measures the area under the impulse response function. As an additional measure of persistence, we also report the first-order autocorrelation of the real exchange rate (ρ_1). Finally, to capture real exchange rate volatility, we report the standard deviation of real exchange rate relative to consumption.[5,6]

The first column of Table 1 shows empirical measures of persistence taken from Steinsson (2008), obtained by fitting simple time-series processes to RER data. His estimates imply that RER deviations from parity are long-lasting – within the 3-5 year "consensus" (Rogoff 1996) – and yield a hump-shaped impulse response function. The second, third and fourth columns present the results for the baseline multisector model for the three cases of interest, respectively: persistent shocks only ($\rho_v = 0.975$, $\rho_i = 0$); policy inertia only ($\rho_v = 0$ and $\rho_i = 0.975$); and both ($\rho_v = 0.975$ and $\rho_i = 0.975$). The last three columns present analogous results for one-sector economies. Recall that the value $\rho_v = 0.975$ is chosen so that the model with persistent shocks only roughly matches the moments in the data.

The results for multisector and one-sector models in Table 1 echo the findings of Carvalho and Nechio (2011). Multisector models significantly increase real exchange rate persistence when compared to their one-sector counterparts. In addition, in the case of persistent shocks only, the calibration of the multisector model produces RER dynamics that resemble the data along various dimensions of the underlying impulse response functions (i.e., $\mathcal{HL}$, $\mathcal{QL}$, and $\mathcal{UL}$).

A comparison of the three versions of the multisector economies (the first block of Table 1), however, shows that the ability of the multisector model to generate empirically plausible RER dynamics disappears when the policy rule features a strong interest rate smoothing component, as evidenced by the third and fourth columns of Table 1. The last three columns of Table 1 show that RER in the one-sector versions of the model also drop dramatically in the presence of a high degree of policy inertia.

It is clear from these results that the nature of monetary policy matters a great deal for the

[5] Measures of real exchange rate volatility over real GDP volatility yield similar results.

[6] Following Chari et al. (2002), the first-order autocorrelations and standard deviations are based on HP-filtered model-generated data. We simulate 100 replications of each economy with 30,100 observations each. After dropping the first 100 observations, we average each series over three-month periods to obtain a quarterly series (10000 observations), to which we apply a Hodrick-Prescott filter with bandwidth 1600.

dynamics of the RER in both the multisector and one-sector economies. While the version of the multisector model in which there is no policy inertia is able to produce empirically plausible real exchange rate persistence measures and match other properties of the impulse response function, the versions of the model in which the policy rule features interest rate smoothing fail to do so.

Notice that up-life ($\mathcal{UL}$) is nonzero only in the multisector model with persistent monetary shocks only (second column). As we will explain in the next section, this is a key factor behind the failure of the other versions of the model to generate empirically plausible RER dynamics.

4 The role of monetary policy

Engel (2012) and Benigno (2004) find that real exchange rate persistence increases with the degree of policy inertia (ρ_i). Table 1, on the other hand, shows cases under which the inclusion of policy inertia *decreases* RER persistence. In this section we investigate this apparent discrepancy further. Using the calibrated models, we first show that whether RER persistence increases or decreases with the degree of policy inertia depends on the persistence of monetary policy shocks. We then use a simplified version of the model to show analytically that RER persistence may indeed decrease in the presence of policy inertia.

4.1 Comparative statics in the calibrated models

Figure 1 presents comparative statics results when varying the degree of policy inertia (ρ_i) in the calibrated models.[7] Dashed lines indicate one-sector economies, whereas solid lines correspond to the 3-sector models.

The two top charts show results in the case of i.i.d. monetary policy shocks ($\rho_v = 0$). The left chart reports cumulative impulse responses ($\mathcal{CIR}$) and half-lives ($\mathcal{HL}$), and the right chart reports up-lives ($\mathcal{UL}$), and the first autocorrelation of the RER. In line with Engel (2012) and Benigno (2004), persistence increases with the degree of policy inertia (ρ_i) when monetary shocks are serially uncorrelated. This holds for all measures of persistence and in both the multisector and one-sector economies.

The bottom two charts show that results can change dramatically when monetary policy shocks are persistent ($\rho_v = 0.975$). In that case, increasing the degree of policy inertia *decreases* RER persistence in both the multisector and one-sector economies. Note also that only the multisector model with a low degree of policy inertia is capable of generating non-zero up-lives (bottom right chart). In other words, the other specifications fail to produce a hump-shaped impulse response function. As the next section discusses, this is key to understanding the mechanism through which

[7] All other parameters are held constant at the values described in Section 3.

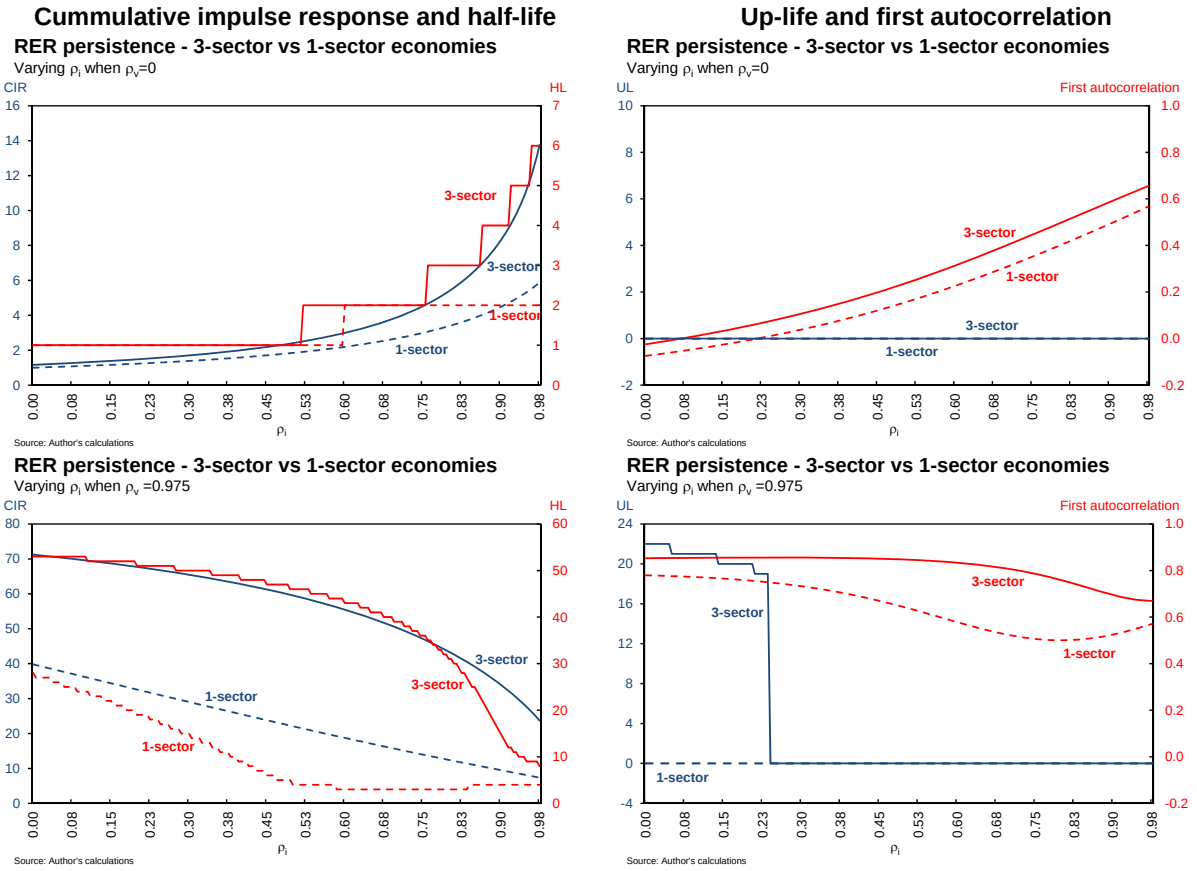

Figure 1: Real exchange rate persistence in response to monetary shocks when varying the degree of policy inertia (ρ_i)

different monetary policy specifications affect RER dynamics in these models.

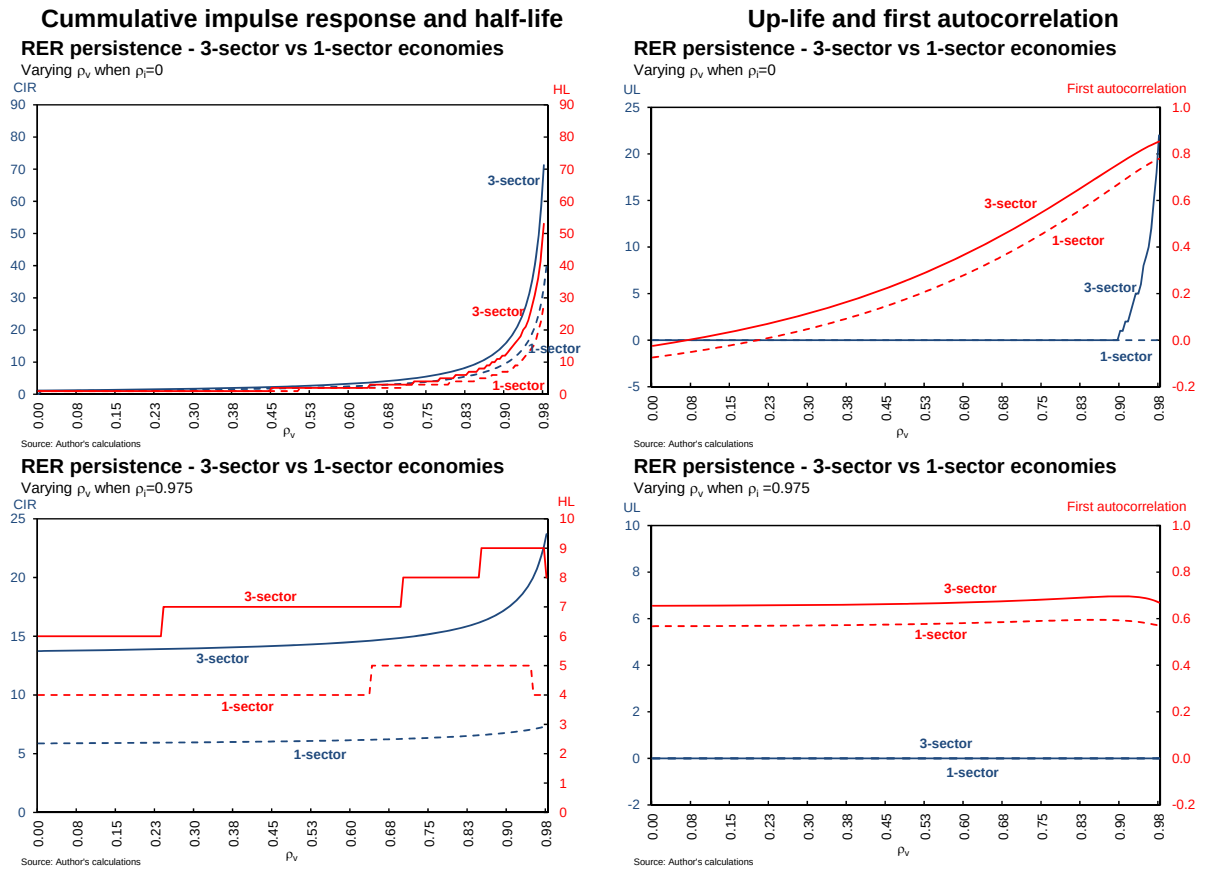

Figure 2: Real exchange rate persistence in response to monetary shocks when varying monetary shock persistence (ρ_v)

For completeness, Figure 2 reports comparative statics results when varying the persistence of policy shocks (ρ_v) in the calibrated models. As in Figure 1, dashed lines indicate one-sector economies, whereas solid lines correspond to the 3-sector models. The top charts consider the case without policy inertia ($\rho_i = 0$), while the bottom assumes instead $\rho_i = 0.975$. The main takeaway from a comparison between the top and the bottom charts is that models with persistent shocks only can generate much more RER persistence. Once policy inertia is introduced, all measures of persistence drop substantially and the impulse response functions fail to exhibit a hump-shaped response to monetary shocks.

Figures 1 and 2 show that RER persistence may decrease with the degree of policy inertia when monetary policy shocks are persistent. The different values for ρ_i and ρ_v used to construct each figure may, however, imply substantially different levels of persistence in nominal interest rates. Hence, we redo our analysis adjusting the values of ρ_i and ρ_v to obtain the same level of nominal interest rate

persistence as in the baseline specification ($\rho_i = 0$, $\rho_v = 0.975$). More specifically, for each ρ_i in $[0, 1)$, we set ρ_v such that nominal interest rate persistence, as measured by the cumulative impulse response ($\mathcal{CIR}$), equals that in the baseline specifications of the multisector and one-sector economies.

Figure 3 presents the results. The left panel shows measures of RER persistence ($\mathcal{CIR}$ and $\mathcal{HL}$) as a function of the degree of policy inertia (ρ_i), with shock persistence (ρ_v) adjusted as described above. The panel shows that, as the degree of policy inertia increases, RER persistence declines. The right panel of Figure 3 presents analogous results for one-sector economies.

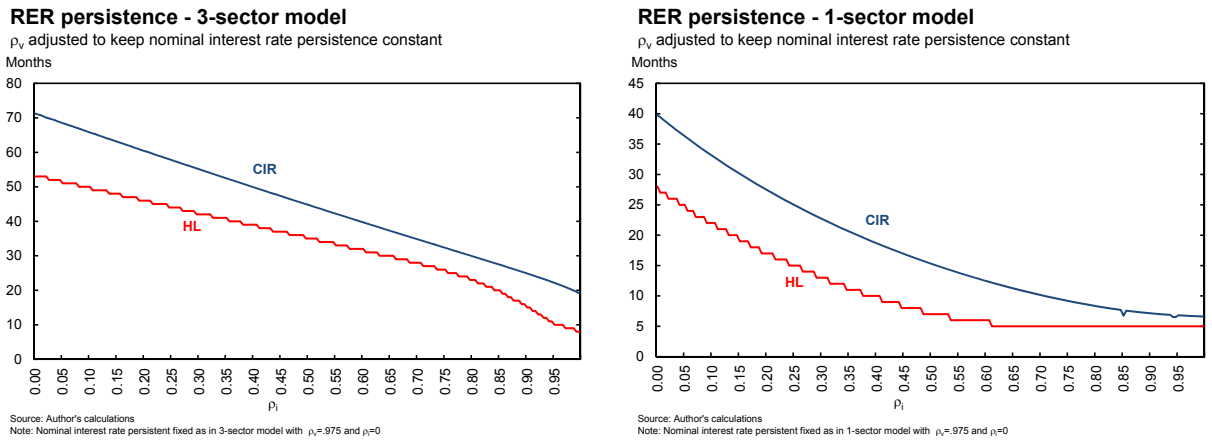

Figure 3: Real exchange rate persistence in response to monetary shocks as a function of policy inertia (ρ_i), with shock persistence (ρ_v) adjusted to keep nominal interest rate persistence constant

4.2 Analytical results

We now make a set of simplifying assumptions to obtain some analytical results. In particular, we consider a one-sector economy with no home bias ($\omega = 0.5$), constant returns to labor ($\chi = 1$), and infinite Frisch elasticity of labor supply ($\gamma = 0$). These simplifications allow us to write the model with three equations for each country; a Phillips curve, an aggregate demand (derived from each countries' Euler equation), and an interest rate rule. As in Engel (2012), we can then solve the model by rewriting it in terms of deviations between Home and Foreign variables, where for any variable x, $dx_t \equiv x_t - x_t^*$ – i.e., the difference between each variable's Home and Foreign counterparts. These simplifications yield:

$$d\pi_t = \delta q_t + \beta E_t d\pi_{t+1} \tag{16}$$

$$\beta di_t = E_t q_{t+1} - q_t + E_t d\pi_{t+1}, \tag{17}$$

where $\delta = \frac{\bar{\alpha}(1-\beta(1-\bar{\alpha}))}{1-\bar{\alpha}}$ and $\bar{\alpha} = \Sigma_{s=1}^{S} f_s \alpha_s$ (as defined in Section 2). Equation (16) corresponds to the difference between Home and Foreign's Phillips curves. Equation (17) obtains as the difference

between Home and Foreign's Euler equations.

Assuming the same interest rate rule for both countries, we further simplify equation (15) to yield:

$$di_t = \rho_i di_{t-1} + \phi_\pi d\pi_t + v_t, \tag{18}$$

where, as described in equation (15), v_t follows an AR(1) process with parameter $\rho_v \in [0, 1)$. The Appendix provides details of the derivation of these three equations.

Proposition 1 *The solution to the simplified three-equation model (equations 16-18) takes the form:*

$$\begin{aligned} q_t &= \varphi_{qv} v_t + \gamma_q di_{t-1} \\ d\pi_t &= \varphi_{\pi v} v_t + \gamma_\pi di_{t-1}, \end{aligned}$$

where φ_{qv}, $\varphi_{\pi v}$, γ_π, γ_q are negative coefficients.

Corollary 1 *Under the assumptions above, the cumulative impulse response function of the real exchange rate equals:*

$$\mathcal{CIR}(q) = \frac{1}{1-\rho_v}\left(1 + \frac{\gamma_q}{\varphi_{qv}} \frac{(1+\phi_\pi \varphi_{\pi v})}{(1-\rho_i - \phi_\pi \gamma_\pi)}\right). \tag{19}$$

In the absence of policy inertia ($\rho_i = 0$), $\gamma_q = \gamma_\pi = 0$, and, hence, $\mathcal{CIR}(q) = \frac{1}{1-\rho_v}$. When $\rho_i > 0$, $\mathcal{CIR}(q)$ is given by (19). From **Proposition 1**, $\gamma_q/\varphi_{qv} > 0$, and $(1 - \rho_i - \phi_\pi \gamma_\pi) > 0$. Hence, whether $\mathcal{CIR}(q) \gtrless \frac{1}{1-\rho_v}$ hinges on whether $(1 + \phi_\pi \varphi_{\pi v}) \gtrless 0$. One can show that $(1 + \phi_\pi \varphi_{\pi v}) < 0$ whenever the persistence of monetary shocks (ρ_v) satisfies $\rho_v > \frac{(1+\beta+\delta) - \sqrt{(1+\beta+\delta)^2 - 4\beta}}{2\beta} > 0$. This result leads to the following corollary.

Corollary 2 *Whenever $\rho_v > \frac{(1+\beta+\delta) - \sqrt{(1+\beta+\delta)^2 - 4\beta}}{2\beta} > 0$, introducing policy inertia ($\rho_i > 0$) lowers RER persistence, as measured by $\mathcal{CIR}(q)$, relative to the case of persistent monetary policy shocks only.*

For the simplified model used to obtain analytical results in this section, and the remaining parameter values assumed in Section 3, **Corollary 2** implies that policy inertia decreases RER persistence (as measured by $\mathcal{CIR}(q)$) as long as $\rho_v > 0.5$ (at a quarterly frequency – roughly 0.79 at a monthly frequency). Hence, for standard parameter values and an empirically plausible degree of monetary shock persistence, policy inertia may decrease RER persistence.

Needless to say, the expression for the ρ_v-threshold in **Corollary 2** is only valid under the simplifying assumptions used in this section (one-sector economy, no home bias, $\chi = 1$, $\gamma = 0$, and $\phi_y = 0$). Deviations from an economy with those characteristics will change the threshold for ρ_v, and can potentially make it a function of other parameters. The quantitative results of the previous sections show that the lessons obtained with the simplified model also hold in our calibrated models.

4.3 Discussion

The results of the previous sections show that policy inertia can hamper the ability of sticky-price models to generate high levels of RER persistence, and it can even decrease persistence. But why does the source of interest rate persistence matter so much?

The reason can be traced back to Steinsson (2008). He argues that the ability of a model to produce hump-shaped RER dynamics is key to matching the degree of persistence seen in the data. He also concludes that one-sector sticky-price models struggle to induce hump-shaped RER dynamics in response to monetary shocks.

Our results corroborate his conclusions. Indeed, as can be seem from Table 1, the version of the model that succeeds in producing enough RER persistence in response to monetary shocks also generates pronounced hump-shaped RER dynamics – as can be seen from the nonzero up-lives ($\mathcal{UL}$). In contrast, the models with a high degree of policy inertia fail to generate hump-shaped RER dynamics, and also fail to produce enough RER persistence (see Table 1 and Figure 1). In addition, our calibrated one-sector models fail to generate hump-shaped RER dynamics in response to monetary shocks – even in the absence of policy inertia. The same is not true, however, of our calibrated multisector model, which is able to generate nonzero up-lives in response to such shocks (Table 1 and Figure 1) when the degree of policy inertia is not too high.

But how can the multisector model generate hump-shaped RER dynamics in response to monetary shocks? Let us revisit Steinsson's (2008) deconstruction of the mechanism that induces such dynamics. He departs from the well-known result that, in open economy models with complete markets and standard preferences, there is a close relationship between relative consumptions and the real exchange rate, as implied by equation (6). Thus, understanding RER dynamics in response to monetary shocks amounts to understating the response of consumption differentials across countries. Due to home bias in consumption, this response is well approximated by the response of consumption in the country where the monetary policy shock hit. The consumption Euler equation (2) implies a relationship between expected real interest rates and consumption. Solving (the loglinearized version of) equation (2) forward yields:

$$c_t = -\sigma^{-1} E_t \sum_{j=0}^{\infty} \left(i_{t+j} - E_{t+j} \pi_{t+1+j} \right).$$

Steinsson (2008) concludes that, for the response of consumption to a given shock to be hump-shaped, the response of nominal interest rates and expected inflation must be such that the real interest rate changes sign during the transition back to the steady state. How can this happen in response to, say, an expansionary monetary policy shock ($v_t < 0$)? To gain some intuition, let us analyze the responses of inflation, the nominal and real interest rates, and the aggregate real exchange

rate in one-sector and multisector versions of the model, with and without policy inertia.

Figure 4 shows the results using our calibrated one-sector model. The left column corresponds to the version of the model with persistent monetary policy shocks only, and the right column corresponds to the model with policy inertia. Irrespective of whether the policy rule exhibits inertia, in response to an expansionary monetary policy shock, expected inflation increases more than the nominal interest rate, and, hence, the real interest rate drops. It then reverts monotonically back to steady state. In this case the RER does not exhibit hump-shaped dynamics (and neither does consumption – not shown). This is consistent with Steinsson (2008).

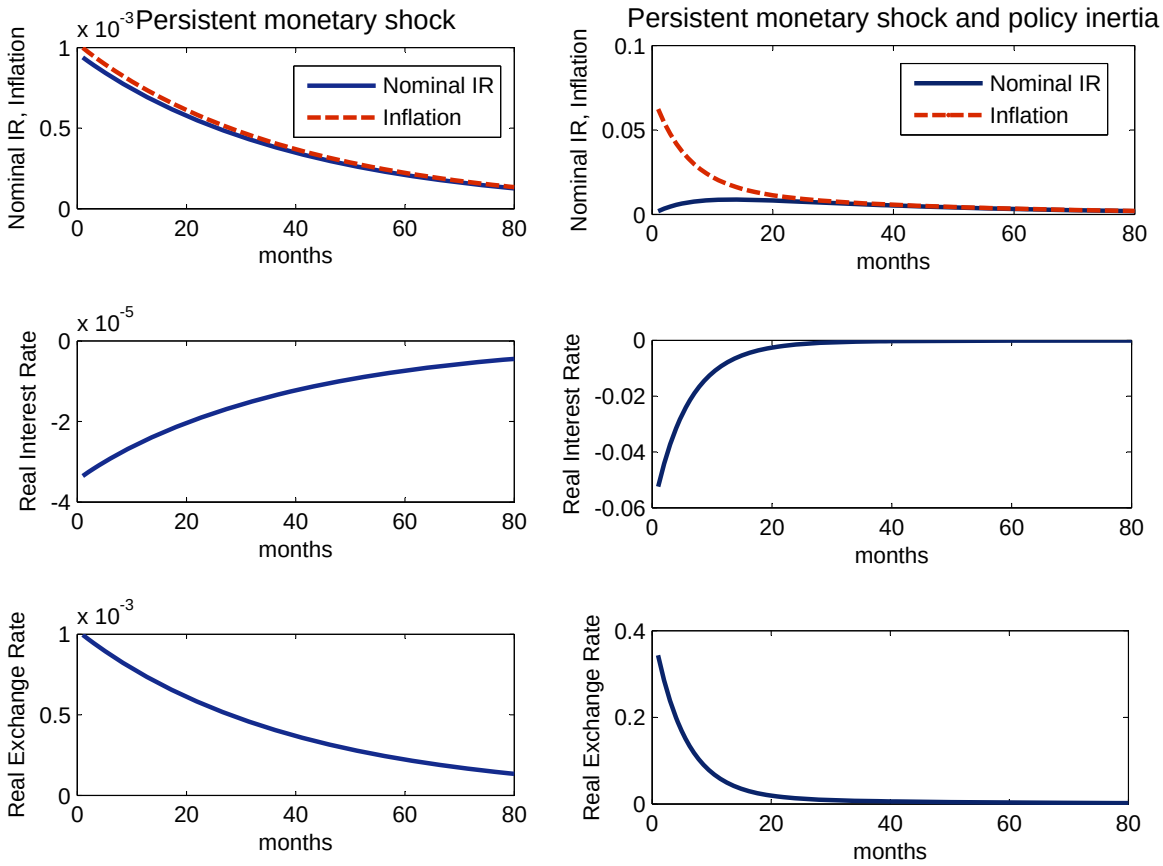

Figure 4: Impulse response functions of inflation, nominal interest rate, real interest rate, and aggregate real exchange rates to an expansionary monetary policy shock at Home – calibrated one-sector model

Figure 5 shows analogous results using our calibrated multisector model. Again, the left column corresponds to the version of the model with persistent monetary policy shocks only, and the right column corresponds to the model with policy inertia. With persistent monetary policy shocks only, aggregate dynamics differ noticeably from the one-sector economy case. Initially, the nominal interest rate increases more than expected inflation, and thus the real interest rate actually increases in

the short run. This happens because the endogenous increase in inflation induced by the monetary expansion is stronger than in the one-sector model, leading to a larger increase in the nominal interest rate. As a result, in response to a persistent expansionary policy shock, the real interest rate increases at first, and then falls below steady state before converging back to its initial level. This is precisely the response that Steinsson (2008) concludes is necessary to induce hump-shaped consumption – and thus RER – dynamics. However, this result entails a non-standard response of the real interest rate to the monetary shock.

In contrast, the results obtained with a high degree of policy inertia, reported on the right column of Figure 5, show that the nominal interest rate does not move as much, inflation increases substantially more in response to an expansionary policy shock, and thus the real interest rate drops before reverting back to steady state. This monotonic dynamics imply the absence of a hump-shaped RER response.

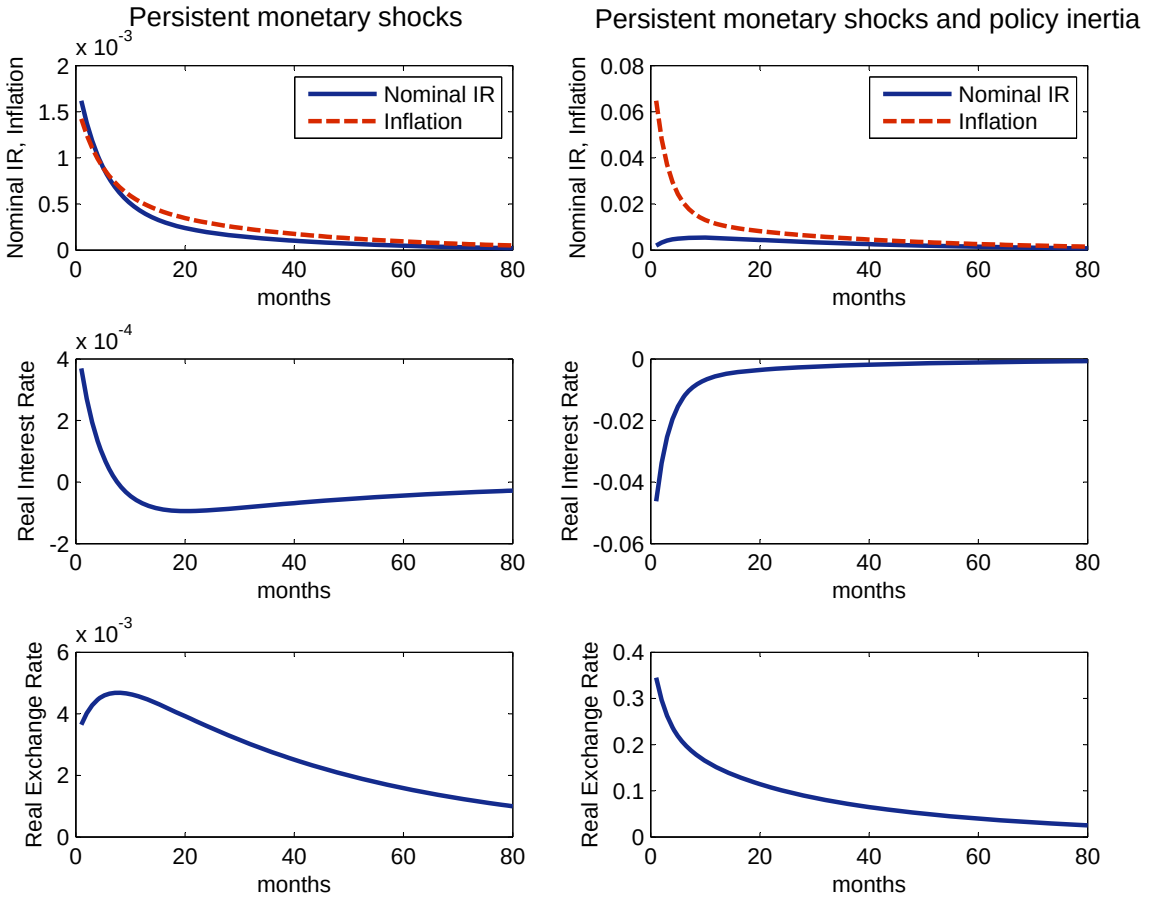

Figure 5: Impulse response functions of inflation, nominal interest rate, real interest rate, and aggregate real exchange rates to an expansionary monetary policy shock at Home – calibrated multisector model

5 Revisiting Eichenbaum and Evans (1995)

Thus far, our approach has followed the literature and compared the models' predictions to *unconditional* moments of real exchange data, obtained by fitting simple autoregressive processes to those data. While this is the common practice in the literature, one may question the comparison of unconditional data moments with model-implied moments generated by monetary shocks only. In this section, we redo the analysis reported in Table 1, focusing on empirical moments conditional on identified monetary policy shocks.

To obtain these conditional moments, we follow Eichenbaum and Evans (1995) and estimate the effects of shocks to the Federal Funds Rate on the RER of a few countries, using vector autoregressions (VARs). In particular, we consider their specification in which dynamic responses are obtained from a 7-variable VAR that includes data on U.S. industrial production (Y^{US}), the U.S. consumer price level (P^{US}), foreign output (Y^{For}), the foreign interest rate (R^{For}), the federal funds rate (FFR), the ratio of U.S. nonborrowed reserves to total reserves ($NBRX$), and the real exchange rate (RER). Real exchange rates are defined as the relative price of the foreign good in terms of the U.S. good, and hence, an increase in RER_t denotes a depreciation of the U.S. dollar. All variables are in logarithms except R^{For} and FFR. Impulse response functions are calculated assuming a Wold ordering of $\left\{Y^{US}, P^{US}, Y^{For}, R^{For}, FFR, NBRX, RER\right\}$. The monetary policy shock is identified as the component of the innovation in FFR_t that is orthogonal to Y_t^{US}, P_t^{US}, Y_t^{For}, and R_t^{For}. Our sample of countries includes the United Kingdom, Japan, Switzerland, France, Germany, and Italy. The monthly data covers the sample period of Jan-1974 to Dec-2007. For each country, we select the number of lags in the VAR based on the Bayesian Information Criterion (BIC), which yields 2 lags for all countries in our sample.[8]

Table 2: Conditional empirical moments and alternative monetary policy rules

		Multisector economy			One-sector economy		
Persistence measures:	Data:	$\rho_v = 0.9865$ $\rho_i = 0$	$\rho_v = 0$ $\rho_i = 0.9865$	$\rho_v = 0.9865$ $\rho_i = 0.9865$	$\rho_v = 0.9865$ $\rho_i = 0$	$\rho_v = 0$ $\rho_i = 0.9865$	$\rho_v = 0.9865$ $\rho_i = 0.9865$
$\mathcal{HL}$	104	102	7	8	51	4	4
$\mathcal{QL}$	123	156	20	25	102	8	9
$\mathcal{UL}$	62	45	0	0	0	0	0
$\mathcal{CIR}$	166	139	16	24	74	6	7
ρ_1	0.8	0.9	0.7	0.6	0.8	0.6	0.5
Volatility: $\frac{\sigma_q}{\sigma_y}$	1.9	3.4	4.0	4.0	3.1	3.9	3.9

[8] Other information criteria, such as Akaike Information Criterion (AIC), yield up to 3 lags. Impulse response functions obtained with 3 lags do not differ significantly from those obtained with 2 lags.

Table 2 reports the cross-country median RER moments conditional on monetary shocks, in addition to the results for different versions of the model. Details for individual countries are available in the Appendix. In analogy to the exercise of Table 1, the parameter ρ_v of the multisector economy with persistent monetary shocks only (second column) is calibrated to roughly match the moments of RERs conditional on identified monetary shocks. Given the value of this parameter, the other model simulations, reported in columns 3 to 7, are meant to illustrate the effects of policy inertia. All other parameters are unchanged relative to Section 3.

To obtain the first-order autocorrelation (ρ_1) and volatility of real exchange rates conditional on monetary shocks, we use the identified shock obtained from the estimated structural VAR to construct counterfactual series for the real exchange rate and the U.S. industrial production that are driven only by monetary shocks. The conditional first-order autocorrelation and standard deviations are, then, based on the HP-filtered versions of the resulting RER and U.S. industrial production series, averaged over three-month periods to obtain quarterly data. Note that, differently from the relative unconditional volatility reported in Table 1, the relative conditional volatility corresponds to the ratio between the standard deviations of the constructed RER and industrial production (instead of consumption) series. Therefore, the model-implied volatility measures of columns 2 to 7 also correspond to the ratio between standard deviations of RER and domestic output.[9]

A comparison between the first columns of Tables 1 and 2 show that, conditional on monetary shocks, RER persistence measures are even larger than the unconditional ones. The cross-country median half life of deviations of RER from parity, for example, reaches 8 years, well above the (unconditional) "consensus" of 3-5 years. While both the time sample and the set of countries vary between these two aforementioned columns, unconditional persistence measures using the same dataset used to construct the statistics reported in Table 2 yield similar statistics to the unconditional moments reported in Table 1.[10]

Despite the differences between the conditional and the unconditional measures of persistence reported in Tables 1 and 2, when focusing on the model-implied moments, the main lessons from Table 1 remain. Multisector models significantly increase real exchange rate persistence when compared to their one-sector counterparts. More importantly, a comparison among the three versions of the multisector economies reported in Table 2 confirms that the ability of the multisector model to generate empirically plausible RER dynamics disappears when the policy rate features a strong interest rate smoothing component.

[9] In unconditional terms, the median real exchange rate (HP-filtered, quarterly) standard deviation relative to the standard deviation of U.S. industrial production (HP-filtered, quarterly) equals 2.9.

[10] More specifically, estimating a simple autoregressive (AR) process on the same sample used to estimate the VAR yields median $\mathcal{HL}$, $\mathcal{QL}$, $\mathcal{UL}$, and $\mathcal{CIR}$ of 44, 70, 17 and 59 months, respectively. For each country, we estimate AR processes with 2 lags, as indicated by the BIC.

6 Conclusion

In this paper we study how different monetary policy rules affect RER dynamics in sticky-price models. We do so by entertaining a policy rule that encompasses both persistent monetary shocks and policy inertia. We find that the source of interest rate persistence matters a great deal. When subjected to persistent monetary shocks, a multisector model with heterogeneous price stickiness can produce volatile and persistent RER. In particular, it can induce hump-shaped RER dynamics that resemble the patterns documented in the data. One-sector versions of the model economy with the same average frequency of price changes fail to do so.

When the monetary policy rule displays a strong enough degree of policy inertia, even the multi-sector sticky-price model fails to generate enough RER persistence in response to monetary shocks. This result highlights the importance of the empirical debate on the source of the high degree of interest rate persistence observed in the data – whether it stems from persistent shocks, or from policy inertia.

Our focus on different specifications of the interest rate rule followed by the monetary authority allows us to analyze how it affects the policy transmission mechanism, and the associated implications for RER dynamics. However, whether a specific model manages to produce plausible RER dynamics in response to monetary policy shocks will also depend on other parts of the transmission mechanism. For example, it is well known that the standard consumption Euler equation – a building block of many models of the monetary transmission mechanism – receives only weak empirical support from the data (e.g., Fuhrer and Rudebusch 2004). This poses important challenges for macroeconomic models (e.g., Cochrane 2008). In our multisector model with persistent monetary shocks only, for example, the hump-shaped response of the RER to a shock is associated with a non-monotonic response of the real interest rate. Hence, it appears that the literature that tries to make sense of the PPP puzzle through the lens of sticky-price DSGE models would also benefit from additional research on this important block of the transmission mechanism of monetary policy.

References

[1] Benigno, G. (2004), "Real Exchange Rate Persistence and Monetary Policy Rules," *Journal of Monetary Economics* 51: 473-502.

[2] Bergin, P. and R. Feenstra (2001), "Pricing-to-Market, Staggered Contracts, and Real Exchange Rate Persistence," *Journal of International Economics* 54: 333–359.

[3] Calvo, G. (1983), "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics* 12: 383-98.

[4] Carvalho, C. and F. Nechio (2011), "Aggregation and the PPP Puzzle in a Sticky Price Model," *American Economic Review* 101: 2391-2424.

[5] Chari, V., P. Kehoe, and E. McGrattan (2002), "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?," *Review of Economic Studies* 69*:* 533-63.

[6] Cochrane, J. (2008), "Financial Markets and the Real Economy," in R. Mehra: Handbook of the Equity Risk Premium, Elsevier.

[7] Coibion, O. and Y. Gorodnichenko (2012), "Why Are Target Interest Rate Changes So Persistent?," *American Economic Journal: Macroeconomics*, 4(4): 126-62.

[8] Eichenbaum, M. and C. Evans (1995), "Some Empirical Evidence on the Effects of Monetary Shocks on Exchange Rates," *Quarterly Journal of Economics* 110: 975-1010.

[9] Engel, C. (2012), "Real Exchange Rate Convergence: The Roles of Price Stickiness and Monetary Policy," manuscript.

[10] Fuhrer, J. and G. Rudebusch (2004), "Estimating the Euler Equation for Output," *Journal of Monetary Economics* 51: 1133-1153.

[11] Johri, A. and A. Lahiri (2008), "Persistent Real Exchange Rates," *Journal of International Economics* 76: 223-236.

[12] Kollman, R. (2001), "The Exchange Rate in a Dynamic-Optimizing Business Cycle Model with Nominal Rigidities: A Quantitative Investigation," *Journal of International Economics* 55: 243-262.

[13] Martinez-Garcia, E. and J. Søndergaard (2008), "The Real Exchange Rate in Sticky Price Models: Does Investment Matter?," forthcoming *Macroeconomic Dynamics*.

[14] Nakamura, E. and J. Steinsson (2008), "Five Facts About Prices: A Reevaluation of Menu Cost Models," *Quarterly Journal of Economics* 123: 1415-1464.

[15] Rogoff, K. (1996), "The Purchasing Power Parity Puzzle," *Journal of Economic Literature* 34: 647-668.

[16] Rudebusch, G. (2002), "Term structure evidence on interest rate smoothing and monetary policy inertia," *Journal of Monetary Economics*, 49: 1161-1187.

[17] Steinsson, J. (2008), "The Dynamic Behavior of the Real Exchange Rate in Sticky-Price Models," *American Economic Review* 98: 519-533.

[18] Taylor, J. (1993), "Discretion versus policy rules in practice," *Carnegie-Rochester Conference Series on Public Policy* 39: 195-214.

A Appendix

A.1 Details on our update of Eichenbaum and Evans (1995)

In Section 5, we revisited the results of Eichenbaum and Evans (1995), extending their sample and estimating the response of real exchange rates to identified monetary shocks. In this appendix we provide additional details on the estimation results.

We consider Eichenbaum and Evans (1995)'s specification in which dynamic responses are obtained from a 7-variable VAR that includes data on U.S. industrial production (Y^{US}), the U.S. consumer price level (P^{US}), foreign output (Y^{For}), the foreign interest rate (R^{For}), the federal funds rate (FFR), the ratio of U.S. nonborrowed reserves to total reserves ($NBRX$), and the real exchange rate (RER). Real exchange rates are defined as the relative price of the foreign good in terms of the U.S. good, and hence, an increase in RER_t denotes a depreciation of the U.S. real exchange rate. All variables are in logarithms except R^{For} and FFR.

Our sample of countries includes the United Kingdom, Japan, Switzerland, France, Germany, and Italy. The monthly data covers the sample period of Jan-1974 to Dec-2007. Seasonally adjusted industrial production and consumer price index data for each country are obtained from the International Financial Statistics (IFS) database. Foreign interest rates are measured by the interbank rates for all countries except for the United Kingdom and Japan, for which we use 3-month treasury bill rates. Foreign interest rates were obtained from the St. Louis Fed FRED database. Nominal exchange rate series were collected from Bloomberg database. Nonborrowed reserves (Adjusted Nonborrowed Reserves Plus Extended Credit) and total reserves (Adjusted Reserves of Depository Institutions) were obtained from FRED. Impulse response functions are calculated assuming a Wold ordering of $\left\{Y^{US}, P^{US}, Y^{For}, R^{For}, FFR, NBRX, RER\right\}$. The monetary policy shock is identified as the component of the innovation in FFR_t that is orthogonal to Y_t^{US}, P_t^{US}, Y_t^{For}, and R_t^{For}. For each country, we select the number of lags in the VAR based on the Bayesian Information Criterion (BIC), which yields 2 lags for all countries in our sample.

Figure 6 shows the impulse response functions of real exchange rates of each country in response to a shock to the Federal Funds Rate. Based on these impulse response functions, Table 3 provides real exchange rate persistence and volatility measures for each country.

The results of Figure 6 corroborate the finding of Eichenbaum and Evans (1995) that the response of real exchange rates to monetary shocks are long-lasting and hump-shaped. The later two properties are also present in unconditional moments of the real exchange rate (e.g., Chari et al. 2002, and Steinsson 2008). The results of Table 3, however, suggest much larger persistence and smaller volatility than the unconditional moments that the literature has focused on. While Eichenbaum and Evans (1995) included the estimated impulse response functions, they did not report real exchange rate

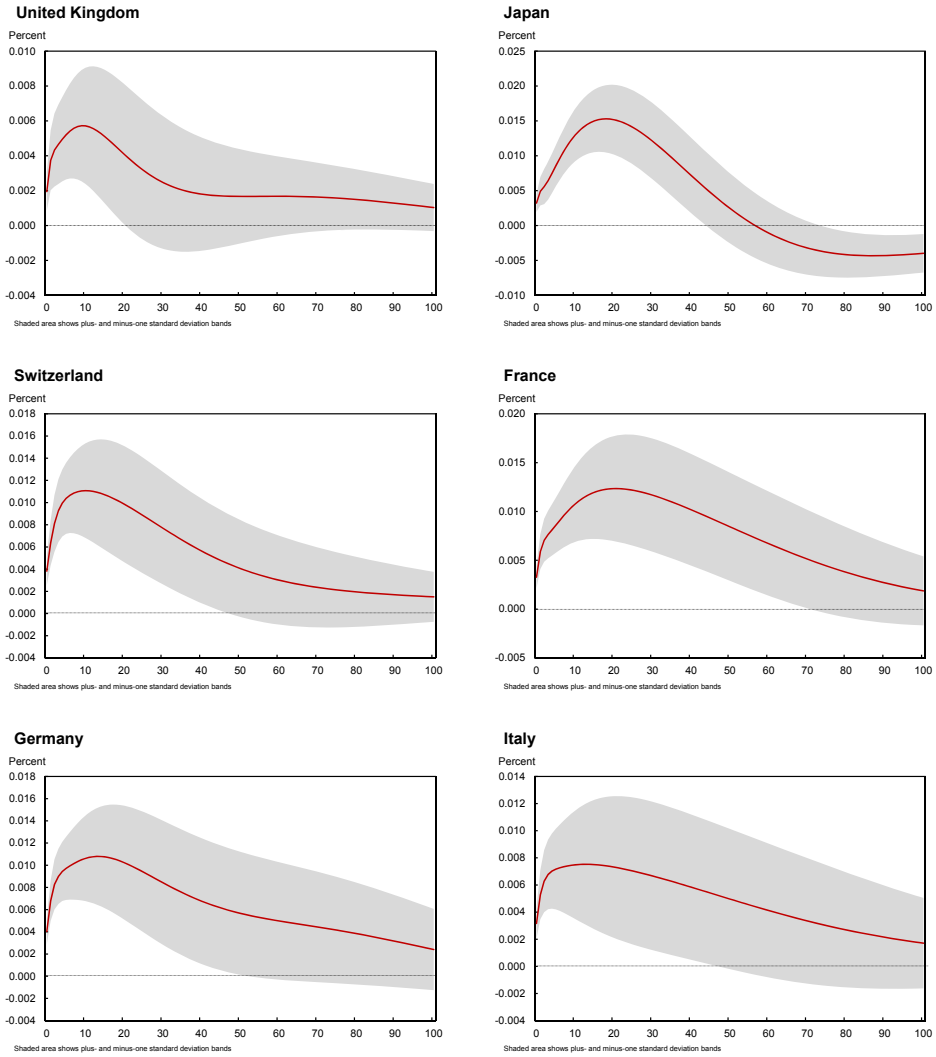

Figure 6: Impulse response functions of aggregate real exchange rate in response to shocks to the Federal Funds Rate

Table 3: Real exchange rate onditional moments by country

Persistence measures:	United Kingdom	Japan	Switzerland	Germany	France	Italy
$\mathcal{HL}$	103	53	82	106	104	104
$\mathcal{QL}$	124	55	153	121	121	137
$\mathcal{UL}$	36	48	51	77	84	72
$\mathcal{CIR}$	140	39	163	168	248	177
ρ_1	0.8	0.8	0.8	0.8	0.9	0.8
Volatility:						
$\frac{\sigma_q}{\sigma_y}$	1.8	2.6	2.0	1.4	1.9	1.5

moments implied by their estimation. For a closer comparison to their results, we constrained our sample from Jan-1974 to May-1990, as in Eichenbaum and Evans (1995), reestimate the VARs, and calculate implied measures of persistence and volatility. In that sample, the impulse response functions yield median $\mathcal{HL}$, $\mathcal{QL}$, $\mathcal{UL}$, and $\mathcal{CIR}$ of 62, 65, 55 and 130 months, respectively. The median standard deviation of the counterfactual RER (driven by monetary shocks only, as described in Section 5) relative to the standard deviation of industrial production (also conditional on monetary shocks) equals 2.5.[11]

A.2 The approximating 3-sector economy

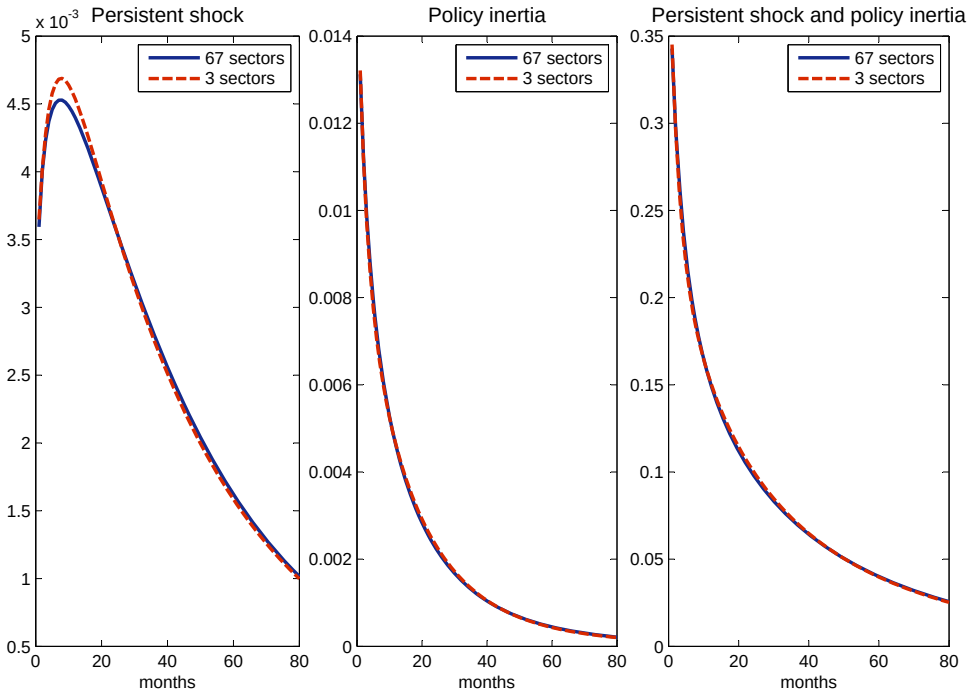

Figure 7: Impulse response functions of aggregate real exchange rate in baseline 3- and 67-sector economies

Here we show that a model with three sectors, with suitably chosen degrees of price stickiness and sectoral weights, provides an extremely good approximation to the original 67-sector economy. We choose the sectoral weights and frequencies of price changes to match the following moments of the distribution of price stickiness from our baseline parametrization with 67 sectors: average frequency of price changes ($\overline{\alpha} = \sum_{s=1}^{S} f_s \alpha_s$), cross-sectional average of the expected durations of price spells ($\overline{d} \equiv \sum_{s=1}^{S} f_s \alpha_s^{-1}$), cross-sectional standard deviation of the expected durations of price spells

[11] All VARs are estimated with 1 lag as suggested by the Baysian Information Criteria. Eichenbaum and Evans (1995) includes 6 lags in each VAR. Persistence measures obtained when estimating VARs with 6 lags (instead of 1 lag) are unchanged. The standard deviation of the counterfactual RER relative to the counterfactual industrial production is smaller, at 1.5.

($\sigma_d = \sqrt{\sum_{s=1}^{S} f_s \left(\alpha_s^{-1} - \overline{d}\right)^2}$), skewness of the cross-sectional distribution of expected durations of price spells ($\mathcal{S}_d = \frac{1}{\sigma_d^3} \sum_{s=1}^{S} f_s \left(\alpha_s^{-1} - \overline{d}\right)^3$), and kurtosis of the cross-sectional distribution of expected durations of price spells ($\mathcal{K}_d = \frac{1}{\sigma_d^4} \sum_{s=1}^{S} f_s \left(\alpha_s^{-1} - \overline{d}\right)^4$).[12]

We present our findings in Figure 7. It shows the impulse response functions of the aggregate real exchange rate to a nominal shock in Home in our baseline multisector economy, and in the approximating three-sector economy obtained with the moment-matching exercise described above. The three charts report the impulse response functions that correspond to the 3-sector model as reported in the second, third and fourth columns of Table 1 along with their 67-sector version variant.[13] These charts shows that the three-sector economy provides a very good approximation to our multisector economy, which justifies our use of the approximating model to save on computational time.

A.3 Simplified one-sector economy

To obtain an analytical solution, we simplify our two-country multisector economy. We abstract from heterogeneity and assume a one-sector economy with no home bias ($\omega = 0.5$), constant returns to labor ($\chi = 1$), and infinite Frisch elasticity of labor supply ($\gamma = 0$). As in Engel (2012), we rewrite all variables as the difference between the Home and Foreign counterparts.

A.3.1 Prices and outputs

A one-sector economy yields:

$$\begin{aligned} p_t &= \omega p_{H,t} + (1-\omega)\, p_{F,t} \\ p_t^* &= \omega p_{F,t}^* + (1-\omega)\, p_{H,t}^* \end{aligned}$$

$$p_{Ht} = \alpha x_{H,t} + (1-\alpha)\, p_{H,t-1}$$

$$p_{F,t} = \alpha x_{F,t} + (1-\alpha)\, p_{F,t-1}$$

$$p_{H,t}^* = \alpha x_{H,t}^* + (1-\alpha)\, p_{H,t-1}^*$$

$$p_{F,t}^* = \alpha x_{F,t}^* + (1-\alpha)\, p_{F,t-1}^*.$$

And for the intermediate outputs, we have:

$$y_{H,j,t+s} = y_{t+s} - \theta \left[p_{H,j,t} - p_{H,t+s}\right] - \rho \left[p_{H,t+s} - p_{t+s}\right]$$

$$y_{F,j,t+s} = y_{t+s} - \theta \left[p_{F,j,t} - p_{F,t+s}\right] - \rho \left[p_{F,t+s} - p_{t+s}\right]$$

[12] We have 5 degrees of freedom (2 weights and 3 frequencies of price change) to match 5 moments from the distribution of price stickiness in the 67-sector economy.

[13] The values of all other parameters are the same as in the baseline model.

$$y^*_{H,j,t+s} = y^*_{t+s} - \theta\left[p^*_{H,j,t} - p^*_{H,t+s}\right] - \rho\left[p^*_{H,t+s} - p^*_{t+s}\right]$$

$$y^*_{F,j,t+s} = y^*_{t+s} - \theta\left[p^*_{F,j,t} - p^*_{F,t+s}\right] - \rho\left[p^*_{F,t+s} - p^*_{t+s}\right]$$

for all times $t+s$ for which the price set at t is still in effect.

For the output aggregations, we have:

$$\begin{aligned} y_t &= \omega y_{H,t} + (1-\omega)\, y_{F,t} \\ y^*_t &= \omega y^*_{F,t} + (1-\omega)\, y^*_{H,t} \\ \omega y_{H,t} + (1-\omega)\, y^*_{H,t} &= \chi n_t = n_t \\ \omega y^*_{F,t} + (1-\omega)\, y_{F,t} &= \chi n^*_t = n_t \end{aligned}$$

$$\begin{aligned} y_{H,t} &= \int_0^1 y_{H,j,t} dj \\ y_{F,t} &= \int_0^1 y_{F,j,t} dj \\ y^*_{F,t} &= \int_0^1 y^*_{F,j,t} dj \\ y^*_{H,t} &= \int_0^1 y^*_{H,j,t} dj \end{aligned}$$

The marginal cost is such that (recall $\chi = 1$):

$$mc_t = w_t - p_t = \sigma c_t + \gamma n_t$$

Loglinearizing the interest rate gives us:

$$i_t = E_t\left(\sigma\left(c_{t+1} - c_t\right) + p_{t+1} - p_t\right)$$

For the real exchange rate, we have:

$$q_t = \sigma\left(c_t - c^*_t\right)$$

Loglinearizing the equation for the prices set by firms when they are called to adjust:[14]

$$x_{H,t} = \left(1 - \beta\left(1 - \alpha_k\right)\right) E_t \sum_{j=0}^{\infty} \beta^j \left(1-\alpha\right)^j \left(p_{t+j} + mc_{t+j}\right)$$

[14] Although marginal costs are potentially firm-specific due to (potentially) decreasing returns to scale, in each sector firms that adjust face the same conditional distribution for all future variables that matter for price setting, including marginal costs. Thus we simplify the notation using a price that is common to all adjusting firms, and also a common marginal cost.

$$x_{F,t} = (1-\beta(1-\alpha_k))\, E_t \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j \left(p_{t+j} + q_{t+j} + mc^*_{t+j}\right)$$

$$x^*_{H,t} = (1-\beta(1-\alpha))\, E_t \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j \left(p^*_{t+j} - q_{t+j} + mc_{t+j}\right)$$

$$x^*_{F,t} = (1-\beta(1-\alpha))\, E_t \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j \left(p^*_{t+j} + mc^*_{t+j}\right)$$

A.3.2 Phillips curves

Using $x_{H,t} - p_{H,t} = \frac{1-\alpha}{\alpha}\pi_{H,t}$:

$$\begin{aligned}
x_{H,t} &= (1-\beta(1-\alpha_k))\, E_t \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j (p_{t+j} + mc_{t+j}) \\
&= (1-\beta(1-\alpha))\, E_t \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j (p_{t+j} + \sigma c_{t+j} + \gamma n_{t+j}) \\
&\Rightarrow \\
\frac{1-\alpha}{\alpha}\pi_{H,t} &= (1-\beta(1-\alpha))\, E_t \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j (p_{t+j} + \sigma c_{t+j} + \gamma n_{t+j} - p_t) \\
&= (1-\beta(1-\alpha))\, E_t \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j \left(p_{t+j} + \sigma c_{t+j} + \gamma n_{t+j} - \left[p_{H,t+j} - \Sigma_{i=1}^{j} E_t \pi_{H,t+i}\right]\right) \\
&= \frac{\alpha(1-\beta(1-\alpha))}{1-\alpha}(p_t - p_{H,t} + \sigma c_t + \gamma n_t) + \beta(1-\alpha) E_t \pi_{H,t+1} \\
&\quad + (1-\beta(1-\alpha))\, E_t \sum_{j=1}^{\infty} \beta^j (1-\alpha)^j (p_{t+j} - p_{H,t+j} + \sigma c_{t+j} + \gamma n_{t+j}) + \Sigma_{i=2}^{\infty} \beta^i (1-\alpha)^i E_t \pi_{H,t+i}
\end{aligned}$$

$$\begin{aligned}
\pi_{H,t} &= \frac{\alpha(1-\beta(1-\alpha))}{1-\alpha}(p_t - p_{H,t} + \sigma c_t + \gamma n_t) + \beta E_t \pi_{H,t+1} \\
&= \frac{\alpha(1-\beta(1-\alpha))}{1-\alpha}\left(\frac{1}{\rho}(y_{H,t} - y_t) + \sigma c_t + \gamma n_t\right) + \beta E_t \pi_{H,t+1}
\end{aligned}$$

Using $x_{F,t} - p_{F,t} = \frac{1-\alpha}{\alpha}\pi_{F,t}$:

$$\begin{aligned}
x_{F,t} &= (1-\beta(1-\alpha_k))\, E_t \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j \left(p_{t+j} + q_{t+j} + mc^*_{t+j}\right) \\
&\Rightarrow \\
\pi_{F,t} &= \frac{\alpha(1-\beta(1-\alpha))}{1-\alpha}(p_t - p_{F,t} + \sigma c^*_t + \gamma n^*_t + q_t) + \beta(1-\alpha) E_t \pi_{F,t+1} \\
&= \frac{\alpha(1-\beta(1-\alpha))}{1-\alpha}\left(\frac{1}{\rho}(y_{F,t} - y_t) + \sigma c^*_t + \gamma n^*_t + q_t\right) + \beta E_t \pi_{F,t+1}
\end{aligned}$$

Using $x^*_{H,t} - p^*_{H,t} = \frac{1-\alpha}{\alpha}\pi^*_{H,t}$:

$$\begin{aligned}
x^*_{H,t} &= (1-\beta(1-\alpha))\, E_t \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j \left(p^*_{t+j} - q_{t+j} + mc_{t+j}\right) \\
&\Rightarrow \\
\pi^*_{H,t} &= \frac{\alpha(1-\beta(1-\alpha))}{1-\alpha}\left(p^*_t - p^*_{H,t} + \sigma c_t + \gamma n_t - q_t\right) + \beta E_t \pi_{H,t+1} \\
&= \frac{\alpha(1-\beta(1-\alpha))}{1-\alpha}\left(\frac{1}{\rho}\left(y^*_{H,t} - y^*_t\right) + \sigma c_t + \gamma n_t - q_t\right) + \beta E_t \pi_{H,t+1}
\end{aligned}$$

$$\begin{aligned}
x^*_{F,t} &= (1-\beta(1-\alpha))\, E_t \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j \left(p^*_{t+j} + mc^*_{t+j}\right) \\
&\Rightarrow \\
\pi^*_{F,t} &= \frac{\alpha(1-\beta(1-\alpha))}{1-\alpha}\left(p^*_t - p^*_{F,t} + \sigma c^*_t + \gamma n^*_t\right) + \beta E_t \pi^*_{F,t+1} \\
&= \frac{\alpha(1-\beta(1-\alpha))}{1-\alpha}\left(\frac{1}{\rho}\left(y^*_{F,t} - y^*_t\right) + \sigma c^*_t + \gamma n^*_t\right) + \beta E_t \pi^*_{F,t+1}
\end{aligned}$$

From these equations we get:

$$\begin{aligned}
\pi_t &= \omega \pi_{H,t} + (1-\omega)\,\pi_{F,t} \\
&= \omega\left[\left(\frac{1}{\rho}\left(y_{H,t} - y_t\right) + \sigma c_t + \gamma n_t\right) + \beta E_t \pi_{H,t+1}\right] \\
&\quad + (1-\omega)\left[\left(\frac{1}{\rho}\left(y_{F,t} - y_t\right) + \sigma c^*_t + \gamma n^*_t + q_t\right) + \beta E_t \pi_{F,t+1}\right]
\end{aligned}$$

$$\begin{aligned}
\pi^*_t &= \omega \pi^*_{F,t} + (1-\omega)\,\pi^*_{H,t} \\
&= \omega\left[\left(\frac{1}{\rho}\left(y^*_{F,t} - y^*_t\right) + \sigma c^*_t + \gamma n^*_t\right) + \beta E_t \pi^*_{F,t+1}\right] \\
&\quad + (1-\omega)\left[\left(\frac{1}{\rho}\left(y^*_{H,t} - y^*_t\right) + \sigma c_t + \gamma n_t - q_t\right) + \beta E_t \pi_{H,t+1}\right]
\end{aligned}$$

$$\begin{aligned}
\pi_t - \pi^*_t &= \omega\left[\frac{\alpha(1-\beta(1-\alpha))}{1-\alpha}\left(\frac{1}{\rho}\left(y_{H,t} - y_t\right) + \sigma c_t + \gamma n_t\right) + \beta E_t \pi_{H,t+1}\right] \\
&\quad -\omega\left[\frac{\alpha(1-\beta(1-\alpha))}{1-\alpha}\left(\frac{1}{\rho}\left(y^*_{F,t} - y^*_t\right) + \sigma c^*_t + \gamma n^*_t\right) + \beta E_t \pi^*_{F,t+1}\right] \\
&\quad + (1-\omega)\left[\frac{\alpha(1-\beta(1-\alpha))}{1-\alpha}\left(\frac{1}{\rho}\left(y_{F,t} - y_t\right) + \sigma c^*_t + \gamma n^*_t + q_t\right) + \beta E_t \pi_{F,t+1}\right] \\
&\quad - (1-\omega)\left[\frac{\alpha(1-\beta(1-\alpha))}{1-\alpha}\left(\frac{1}{\rho}\left(y^*_{H,t} - y^*_t\right) + \sigma c_t + \gamma n_t - q_t\right) + \beta E_t \pi_{H,t+1}\right]
\end{aligned}$$

Note that:

$$\begin{aligned} y_t &= \omega y_{H,t} + (1-\omega)\, y_{F,t} \\ y_t^* &= \omega y_{F,t}^* + (1-\omega)\, y_{H,t}^* \\ &\Rightarrow \\ \omega\,(y_{H,t} - y_t) &= -(1-\omega)\,(y_{F,t} - y_t) \\ \omega\left(y_{F,t}^* - y_t^*\right) &= -(1-\omega)\left(y_{H,t}^* - y_t^*\right) \end{aligned}$$

Call: $\frac{\alpha(1-\beta(1-\alpha))}{1-\alpha} = \delta$. And we can rewrite $\pi_t - \pi_t^*$:

$$\begin{aligned} \pi_t - \pi_t^* &= \omega\left[\delta\left(\sigma c_t + \gamma n_t\right) + \beta E_t \pi_{H,t+1}\right] \\ &\quad -\omega\left[\delta\left(\sigma c_t^* + \gamma n_t^*\right) + \beta E_t \pi_{F,t+1}^*\right] \\ &\quad +(1-\omega)\left[\delta\left(\sigma c_t^* + \gamma n_t^* + q_t\right) + \beta E_t \pi_{F,t+1}\right] \\ &\quad -(1-\omega)\left[\delta\left(\sigma c_t + \gamma n_t - q_t\right) + \beta E_t \pi_{H,t+1}\right] \end{aligned}$$

Recall that $q_t = \sigma\left(c_t - c_t^*\right)$, and hence:

$$\begin{aligned} \pi_t - \pi_t^* &= (2\omega - 1)\,\delta q_t + \\ &\quad (2\omega - 1)\,\gamma\delta\left(n_t - n_t^*\right) \\ &\quad +\beta E_t\left(\pi_{t+1} - \pi_{t+1}^*\right) \\ &\quad +(1-\omega)\,\delta q_t \\ &\quad +(1-\omega)\,\delta q_t \\ &= \delta q_t + (2\omega - 1)\,\gamma\delta\left(n_t - n_t^*\right) + \beta E_t\left(\pi_{t+1} - \pi_{t+1}^*\right) \end{aligned}$$

Assuming there is no home bias, $\omega = (1-\omega)$, and setting $\gamma = 0$ yield:

$$\pi_t - \pi_t^* = \delta q_t + \beta E_t\left(\pi_{t+1} - \pi_{t+1}^*\right)$$

A.3.3 Euler equations (demand side):

The Euler equations for Home and Foreign equal:

$$\begin{aligned} i_t &= \sigma E_t c_{t+1} - \sigma c_t + E_t \pi_{t+1} \\ i_t^* &= \sigma E_t c_{t+1}^* - \sigma c_t^* + E_t \pi_{t+1}^* \end{aligned}$$

$$
\begin{aligned}
i_t - i_t^* &= \sigma E_t c_{t+1} - \sigma c_t + E_t \pi_{t+1} \\
&\quad - \left(\sigma E_t c_{t+1}^* - \sigma c_t^* + E_t \pi_{t+1}^*\right) \\
&= \sigma E_t \left(c_{t+1} - c_{t+1}^*\right) - \sigma \left(c_t - c_t^*\right) + E_t \left(\pi_{t+1} - \pi_{t+1}^*\right) \\
&= E_t q_{t+1} - q_t + E_t \left(\pi_{t+1} - \pi_{t+1}^*\right)
\end{aligned}
$$

A.3.4 Monetary policy

We simplify monetary policy such that the interest rate rule for Home equals:

$$i_t = \rho_i i_{t-1} + \phi_\pi \pi_t + v_t,$$

where we assume that the exogenous component of the interest rate follows an AR(1) process:

$$
\begin{aligned}
v_t &= \rho_v v_{t-1} + \varepsilon_t^v \\
\rho_v &\in [0, 1)
\end{aligned}
$$

We assume throughout that monetary policy in Foreign follows the same rule as in Home, and that monetary shocks are uncorrelated across the two countries.

A.3.5 Proofs of propositions and corollaries

Proposition 1 *The solution to the simplified three-equation model (equations 16-18) takes the form:*

$$
\begin{aligned}
q_t &= \varphi_{qv} v_t + \gamma_q di_{t-1} \\
d\pi_t &= \varphi_{\pi v} v_t + \gamma_\pi di_{t-1},
\end{aligned}
$$

where φ_{qv}, $\varphi_{\pi v}$, γ_π, γ_q are negative coefficients.

Proof. Consider the model as deviations between Home and Foreign, where any variable labeled as dx_t corresponds to $dx_t = x_t - x_t^*$. Replicating the main text equations (16) and (17):

$$
\begin{aligned}
d\pi_t &= \delta q_t + \beta E_t d\pi_{t+1} \\
di_t &= E_t q_{t+1} - q_t + E_t d\pi_{t+1}
\end{aligned}
$$

Since both countries have the same policy rule, and we focus on Home monetary shocks, the interest

rate equation (18) equals:

$$\begin{aligned} di_t &= \rho_i di_{t-1} + \phi_\pi d\pi_t + v_t, \\ \text{where} &: v_t = \rho_v v_{t-1} + \varepsilon_t^v \Rightarrow E_t\left(v_{t+1}\right) = \rho_v v_t \\ \rho_v &\in [0,1) \end{aligned}$$

Replacing (18) on (17):

$$\begin{aligned} \rho_i di_{t-1} + \phi_\pi d\pi_t + v_t &= E_t q_{t+1} - q_t + E_t d\pi_{t+1} \\ +q_t + \phi_\pi d\pi_t &= E_t q_{t+1} + E_t d\pi_{t+1} - \rho_i di_{t-1} - v_t \end{aligned}$$

Our system of equations:

$$\begin{aligned} q_t + \phi_\pi d\pi_t &= +E_t q_{t+1} + E_t d\pi_{t+1} - \rho_i di_{t-1} - v_t \\ -\delta q_t + d\pi_t &= +\beta E_t d\pi_{t+1} \end{aligned}$$

$$\begin{pmatrix} 1 & +\phi_\pi \\ -\delta & 1 \end{pmatrix} \begin{pmatrix} q_t \\ d\pi_t \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & \beta \end{pmatrix} \begin{pmatrix} E_t q_{t+1} \\ E_t d\pi_{t+1} \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} \left(\rho_i di_{t-1} + v_t\right)$$

And our system of equations equals:

$$\begin{aligned} \begin{pmatrix} q_t \\ d\pi_t \end{pmatrix} &= \Omega \begin{pmatrix} 1 & 1-\beta\phi_\pi \\ \delta & \beta+\delta \end{pmatrix} \begin{pmatrix} E_t q_{t+1} \\ E_t d\pi_{t+1} \end{pmatrix} + \Omega \begin{pmatrix} -1 \\ -\delta \end{pmatrix} \left(\rho_i di_{t-1} + v_t\right) \\ \Omega &= \frac{1}{1+\delta\phi_\pi} \end{aligned}$$

Guess the solution will take the form:

$$\begin{aligned} q_t &= \varphi_{qv} v_t + \gamma_q di_{t-1} \\ d\pi_t &= \varphi_{\pi v} v_t + \gamma_\pi di_{t-1} \end{aligned}$$

Starting from q_t and substituting for $d\pi_t$ and di_t:

$$\begin{aligned} q_t &= -\phi_\pi d\pi_t + E_t q_{t+1} + E_t d\pi_{t+1} - \rho_i di_{t-1} - v_t \\ &= -\phi_\pi \left[\varphi_{\pi v} v_t + \gamma_\pi di_{t-1}\right] + E_t\left[\varphi_{qv} v_{t+1} + \gamma_q di_t\right] + E_t\left[\varphi_{\pi v} v_{t+1} + \gamma_\pi di_t\right] - \rho_i di_{t-1} - \beta v_t \\ &= \left[-\phi_\pi\varphi_{\pi v} + \varphi_{qv}\rho_v + \gamma_q\phi_\pi\varphi_{\pi v} + \gamma_q + \varphi_{\pi v}\rho_v + \gamma_\pi\phi_\pi\varphi_{\pi v} + \gamma_\pi - 1\right] v_t \\ &\quad + \left[-\phi_\pi\gamma_\pi + \gamma_q\rho_i + \gamma_q\phi_\pi\gamma_\pi + \gamma_\pi\rho_i + \gamma_\pi\phi_\pi\gamma_\pi - \rho_i\right] di_{t-1} \end{aligned}$$

Now for $d\pi_t$ and substituting for q_t and di_t:

$$\begin{aligned} d\pi_t &= \delta q_t + \beta E_t d\pi_{t+1} \\ &= \beta E_t d\pi_{t+1} + \delta \left[\varphi_{qv} v_t + \gamma_q di_{t-1}\right] \\ &= \beta \left[\varphi_{\pi v} v_{t+1} + \gamma_\pi di_t\right] + \delta \left[\varphi_{qv} v_t + \gamma_q di_{t-1}\right] \\ &= \left[\beta \varphi_{\pi v} \rho_v + \beta \gamma_\pi \phi_\pi \varphi_{\pi v} + \beta \gamma_\pi + \delta \varphi_{qv}\right] v_t \\ &\quad + \left[\beta \gamma_\pi \rho_i + \beta \gamma_\pi \phi_\pi \gamma_\pi + \delta \gamma_q\right] di_{t-1} \end{aligned}$$

Matching coefficients, we have 4 equations and 4 variables:

$$\begin{aligned} \varphi_{qv} &= -\phi_\pi \varphi_{\pi v} + \varphi_{qv} \rho_v + \gamma_q \phi_\pi \varphi_{\pi v} + \gamma_q + \varphi_{\pi v} \rho_v + \gamma_\pi \phi_\pi \varphi_{\pi v} + \gamma_\pi - 1 \\ \varphi_{\pi v} &= \beta \varphi_{\pi v} \rho_v + \beta \gamma_\pi \phi_\pi \varphi_{\pi v} + \beta \gamma_\pi + \delta \varphi_{qv} \\ \gamma_q &= -\phi_\pi \gamma_\pi + \gamma_q \rho_i + \gamma_q \phi_\pi \gamma_\pi + \gamma_\pi \rho_i + \gamma_\pi \phi_\pi \gamma_\pi - \rho_i \\ \gamma_\pi &= \beta \gamma_\pi \rho_i + \beta \gamma_\pi \phi_\pi \gamma_\pi + \delta \gamma_q \end{aligned}$$

■

Corollary 1 *Under the assumptions above, the cumulative impulse response function of the real exchange rate equals:*

$$\begin{aligned} \mathcal{CIR}(q) &= \frac{1}{1-\rho_v} + \frac{1}{(1-\rho_v)} \frac{\gamma_q}{\varphi_{qv}} \frac{(\phi_\pi \varphi_{\pi v} + 1)}{(1 - (\rho_i + \phi_\pi \gamma_\pi))} \\ &= \underbrace{\frac{1}{1-\rho_v}}_{>0} \left(1 + \underbrace{\frac{\gamma_q}{\varphi_{qv}}}_{>0} \frac{\overbrace{(\phi_\pi \varphi_{\pi v} + 1)}^{\gtrless 0}}{\underbrace{(1 - \rho_i - \phi_\pi \gamma_\pi)}_{>0}} \right) \end{aligned}$$

Proof. Using the above equations, we can calculate the cumulative impulse response function of q_t following a unit monetary policy shock.

$$CIR(q_t) = \sum_{n=1}^{N} \frac{q_n}{q_1},$$

where:

$$\begin{aligned} q_t &= \varphi_{qv} v_t + \gamma_q di_{t-1}, \\ d\pi_t &= \varphi_{\pi v} v_t + \gamma_\pi di_{t-1}, \\ i_t &= \rho_i i_{t-1} + \phi_\pi \pi_t + v_t. \end{aligned}$$

Solving the equation for interest rate forward:

$$\begin{aligned} i_{t+n} &= (\rho_i + \phi_\pi\gamma_\pi)^{n+1} i_{t-1} + (\phi_\pi\varphi_{\pi v} + 1)\sum_{i=0}^{n}(\rho_i + \phi_\pi\gamma_\pi)^i \rho_v^{n-i} v_t \\ i_{n-1} &= (\rho_i + \phi_\pi\gamma_\pi)^{n} i_{-1} + (\phi_\pi\varphi_{\pi v} + 1)\sum_{i=0}^{n-1}(\rho_i + \phi_\pi\gamma_\pi)^i \rho_v^{n-1-i} v_1 \end{aligned}$$

Replacing in the equation for q_n:

$$\begin{aligned} q_n &= \varphi_{qv} v_n + \gamma_q i_{n-1} \\ &= \varphi_{qv}\rho_v^{n-1} v_1 + \gamma_q i_{n-1} \\ where &: \\ i_{n-1} &= (\rho_i + \phi_\pi\gamma_\pi)^{n} i_{-1} + (\phi_\pi\varphi_{\pi v} + 1)\sum_{i=0}^{n-1}(\rho_i + \phi_\pi\gamma_\pi)^i \rho_v^{n-1-i} v_1 \\ &= (\phi_\pi\varphi_{\pi v} + 1)\sum_{i=0}^{n-1}(\rho_i + \phi_\pi\gamma_\pi)^i \rho_v^{n-1-i} \end{aligned}$$

The cumulative impulse response is given by:

$$\begin{aligned} \underset{N\to\infty}{CIR}(q_t) \rightarrow \sum_{n=1}^{\infty}\frac{q_n}{q_1} &= \sum_{n=1}^{N\to\infty}\frac{\varphi_{qv}\rho_v^{n-1} + \gamma_q(\phi_\pi\varphi_{\pi v}+1)\sum_{i=0}^{n-1}(\rho_i+\phi_\pi\gamma_\pi)^i\rho_v^{n-1-i}}{\varphi_{qv}} \\ &= \frac{\frac{\varphi_{qv}}{1-\rho_v} + \gamma_q(\phi_\pi\varphi_{\pi v}+1)\sum_{n=1}^{\infty}\left[\rho_v^{n-1}\frac{1-\left(\frac{\rho_i+\phi_\pi\gamma_\pi}{\rho_v}\right)^n}{1-\frac{\rho_i+\phi_\pi\gamma_\pi}{\rho_v}}\right]}{\varphi_{qv}} \\ &= \frac{1}{1-\rho_v} + \frac{1}{(1-\rho_v)}\frac{\gamma_q}{\varphi_{qv}}\frac{(\phi_\pi\varphi_{\pi v}+1)}{(1-(\rho_i+\phi_\pi\gamma_\pi))} \end{aligned}$$

$$\begin{aligned} CIR(q_t) &= \frac{1}{1-\rho_v} + \frac{1}{(1-\rho_v)}\frac{\gamma_q}{\varphi_{qv}}\frac{(\phi_\pi\varphi_{\pi v}+1)}{(1-(\rho_i+\phi_\pi\gamma_\pi))} \\ &= \underbrace{\frac{1}{1-\rho_v}}_{>0}\left(1 + \underbrace{\frac{\gamma_q}{\varphi_{qv}}}_{>0}\underbrace{\frac{\overbrace{(\phi_\pi\varphi_{\pi v}+1)}^{\gtrless 0}}{(1-\rho_i-\phi_\pi\gamma_\pi)}}_{>0}\right). \end{aligned}$$

■

Corollary 2 *Whenever* $\rho_v > \frac{(1+\beta+\delta)-\sqrt{(1+\beta+\delta)^2-4\beta}}{2\beta} > 0$, *introducing policy inertia* ($\rho_i > 0$) *lowers RER persistence, as measured by* $CIR(q)$, *relative to the case of persistent monetary policy shocks only.*

Proof. When $\rho_i = 0$, $\gamma_q = \gamma_\pi = 0$, which implies $CIR(q) = \frac{1}{1-\rho_v}$.

Since φ_{qv}, $\varphi_{\pi v}$, γ_π, γ_q are negative coefficients, the only term that doesn't have a definite sign is

$(\phi_\pi \varphi_{\pi v} + 1)$. Since $\phi_\pi > 1$, this term will be negative when $\varphi_{\pi v} < -\frac{1}{\phi_\pi}$.

Departing from the solutions for $\varphi_{\pi v}$ and φ_{qv}:

$$\begin{aligned}
\varphi_{qv} &= -\phi_\pi \varphi_{\pi v} + \varphi_{qv}\rho_v + \gamma_q \phi_\pi \varphi_{\pi v} + \gamma_q + \varphi_{\pi v}\rho_v + \gamma_\pi \phi_\pi \varphi_{\pi v} + \gamma_\pi - 1 \\
\varphi_{\pi v} &= \beta \varphi_{\pi v}\rho_v + \beta\gamma_\pi \phi_\pi \varphi_{\pi v} + \beta\gamma_\pi + \delta\varphi_{qv}
\end{aligned}$$

For $\rho_v = 0$:

$$\begin{aligned}
\varphi_{qv} &= \left(\gamma_q + \gamma_\pi - 1\right)\phi_\pi \varphi_{\pi v} + \left(\gamma_q + \gamma_\pi - 1\right) \\
\varphi_{\pi v} &= \beta\gamma_\pi \phi_\pi \varphi_{\pi v} + \beta\gamma_\pi + \delta\varphi_{qv} \\
&= \beta\gamma_\pi \phi_\pi \varphi_{\pi v} + \beta\gamma_\pi + \delta\left(\gamma_q + \gamma_\pi - 1\right)\phi_\pi \varphi_{\pi v} + \delta\left(\gamma_q + \gamma_\pi - 1\right) \\
&= \left[\beta\gamma_\pi + \delta\left(\gamma_q + \gamma_\pi - 1\right)\right]\phi_\pi \varphi_{\pi v} + \left[\beta\gamma_\pi + \delta\left(\gamma_q + \gamma_\pi - 1\right)\right]
\end{aligned}$$

$$\begin{aligned}
\varphi_{\pi v} &= A\phi_\pi \varphi_{\pi v} + A \\
\left(1 - A\phi_\pi\right)\varphi_{\pi v} &= A \\
\varphi_{\pi v} &= \frac{A}{\left(1 - A\phi_\pi\right)} \\
\text{where } A &= \left[\beta\gamma_\pi + \delta\left(\gamma_q + \gamma_\pi - 1\right)\right]
\end{aligned}$$

Note that A is negative since γ_π is negative, $\rho_i < 1$, and $\delta > 0$. Therefore,

$$\varphi_{\pi v} = \frac{A}{\left(1 - A\phi_\pi\right)} > -\frac{1}{\phi_\pi}$$

Hence, when $\rho_v = 0$, we find that $\varphi_{\pi v}$ is always larger than $-\frac{1}{\phi_\pi}$.

More generally,

$$\begin{aligned}
\varphi_{qv} &= -\phi_\pi \varphi_{\pi v} + \varphi_{qv}\rho_v + \gamma_q \phi_\pi \varphi_{\pi v} + \gamma_q + \varphi_{\pi v}\rho_v + \gamma_\pi \phi_\pi \varphi_{\pi v} + \gamma_\pi - 1 \\
&\Rightarrow \\
\left(1 - \rho_v\right)\varphi_{qv} &= \left(\gamma_q + \gamma_\pi - 1\right)\phi_\pi \varphi_{\pi v} + \rho_v \varphi_{\pi v} + \left(\gamma_\pi + \gamma_q - 1\right)
\end{aligned}$$

$$\begin{aligned}
\varphi_{\pi v} &= \beta\rho_v \varphi_{\pi v} + \beta\gamma_\pi + \beta\gamma_\pi \phi_\pi \varphi_{\pi v} + \delta\varphi_{qv} \\
&\Rightarrow \\
\left(1 - \rho_v\right)\varphi_{\pi v} &= \left[\left(1 - \rho_v\right)\beta + \delta\right]\rho_v \varphi_{\pi v} + \left[\left(1 - \rho_v\right)\beta\gamma_\pi + \delta\left(\gamma_q + \gamma_\pi - 1\right)\right]\phi_\pi \varphi_{\pi v} \\
&\quad + \left[\left(1 - \rho_v\right)\beta\gamma_\pi + \delta\left(\gamma_\pi + \gamma_q - 1\right)\right]
\end{aligned}$$

$$
\begin{aligned}
\left\{(1-\rho_v)-\left[(1-\rho_v)\beta+\delta\right]\rho_v\right\}\varphi_{\pi v} &= \left[(1-\rho_v)\beta\gamma_\pi+\delta\left(\gamma_q+\gamma_\pi-1\right)\right]\phi_\pi\varphi_{\pi v} \\
&\quad +\left[(1-\rho_v)\beta\gamma_\pi+\delta\left(\gamma_\pi+\gamma_q-1\right)\right] \\
\left\{(1-\rho_v)-\left[(1-\rho_v)\beta+\delta\right]\rho_v\right\}\varphi_{\pi v} &= B\phi_\pi\varphi_{\pi v}+B \\
\varphi_{\pi v} &= \frac{B}{\left\{(1-\rho_v)-\left[(1-\rho_v)\beta+\delta\right]\rho_v-B\phi_\pi\right\}} \\
\text{where } B &= \left[(1-\rho_v)\beta\gamma_\pi+\delta\left(\gamma_\pi+\gamma_q-1\right)\right]<0
\end{aligned}
$$

Since $\varphi_{\pi v}<0$, we know that the denominator of the above expression is positive.

$$
\begin{aligned}
\frac{B}{\left\{(1-\rho_v)-\left[(1-\rho_v)\beta+\delta\right]\rho_v-B\phi_\pi\right\}} &< -\frac{1}{\phi_\pi} \\
B\phi_\pi &< -(1-\rho_v)+\left[(1-\rho_v)\beta+\delta\right]\rho_v+B\phi_\pi \\
+\beta\rho_v^2-(1+\beta+\delta)\rho_v+1 &< 0
\end{aligned}
$$

Solving the above expression for ρ_v:

$$
\rho_v=\frac{(1+\beta+\delta)\pm\sqrt{(1+\beta+\delta)^2-4\beta}}{2\beta}
$$

And $\varphi_{\pi v}<-\frac{1}{\phi_\pi}$, as long as:

$$
\frac{(1+\beta+\delta)-\sqrt{(1+\beta+\delta)^2-4\beta}}{2\beta}<\rho_v<1,
$$

where $\frac{\bar{\alpha}(1-\beta(1-\bar{\alpha}))}{1-\bar{\alpha}}=\delta$.

Under our parameterization, $\varphi_{\pi v}<-2/3$ when $\rho_v>0.75$. Note that $\frac{(1+\beta+\delta)+\sqrt{(1+\beta+\delta)^2-4\beta}}{2\beta}>1$, since $\Delta=(1+\beta+\delta)^2-4\beta>0$ and $\frac{(1+\beta+\delta)}{2\beta}>1$. ■

Departamento de Economia PUC-Rio
Pontifícia Universidade Católica do Rio de Janeiro
Rua Marques de Sâo Vicente 225 - Rio de Janeiro 22453-900, RJ
Tel.(21) 31141078 Fax (21) 31141084
www.econ.puc-rio.br
flavia@econ.puc-rio.br