TEXTO PARA DISCUSSÃO

No. 650

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Moises S. Andrade
Tiago Berriel

DEPARTAMENTO DE ECONOMIA
www.econ.puc-rio.br

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Moises S. Andrade
PUC-Rio

Tiago C. Berriel
PUC-Rio

April, 2016

Abstract

Expansionary fiscal policies have been advocated to induce output expansions and inflation in deep recession or deflationary episodes. We show that, in a fiscalist setup, an increase in deficits can trigger a stagflation by negatively affecting financial intermediation of resources to investments. Financial intermediaries collect deposits to buy government bonds and lend through nominal long-term loans. When intermediaries face financial frictions and a maturity mismatch on their assets and liabilities, a surprise inflation and/or a revaluation of bonds prices impair their net-worth reducing lending, investments, and output. Recession comes with inflation in a fiscal expansion because the fall on capital triggered on the financial sector rises production firms marginal costs. The probability of a recession is higher the greater is the maturity mismatch, the sensitivity of bonds prices to the policy rate, and the share of bonds on banks balances. These results: (1) give theoretical support for the negative relation documented between financial sector performance and inflation (2) help explaining high debt, high inflation environments coinciding with banking crisis and, more importantly, (3) expose drawbacks of fiscal inflation policies proposed to inflate and stimulate low inflation economies, where the setup stressed in this paper is more probable to be present.

*We thank without implicating Carlos Carvalho and Eduardo Loyo for their useful comments.

1 Introduction

Fiscal related issues are on the spotlight of academics and practitioners. US and Japan have been accumulating debt at an unprecedented rate, posing a few analysts to voice concerns about fiscal solvency. Europe has been experiencing a severe sovereign debt crisis for a few years. As an example among developing economies, Brazil is again going through fiscal distress, deadlocked by political and structural restrictions to tax and spending adjustments. Many DSGE models neglect the fiscal side, assuming (sometimes implicitly) that the fiscal authority is ready to rise revenues (and that agents expect these rises) to support any policy decision made by the monetary authority. Real-world examples of fiscal authorities troubled to pay its debts make clear that such hypothesis is not realistic for every scenario.

Not surprisingly, there has been a renewed interest on the Fiscal Theory of the Price Level (FTPL).[1] This literature recognizes the relevance of the fiscal side of the economy on price determination, combining in one framework the discussion about fiscal and monetary policies. In short, these theories argues that prices can be a fiscal phenomena: the value of money reflects the expected path of primary surpluses. If surpluses becomes low relative to government liabilities, there will be too much money chasing too few goods, driving prices up.

Although FTPL could be applied in several contexts, it is specially attractive to explain two scenarios: economies going through fiscal distress and/or inflation financing - particularly, fiscal stimulus financed through inflation. In a high debt environment, with a potential high tax burden, rising surpluses to cover additional debt obligations (arising, for example, from monetary policy) can be too costly and politically unfeasible.[2] In these circumstances, concerns about debt sustainability arises and agents' decisions become more sensitive to fiscal shocks. Additional deficits, for example, are more likely to trigger a run from government bonds to other assets and goods, driving prices up, than keeping the economy unaffected, as Ricardian equivalence would predict. Indeed, views of FTPL proponents suggest a close link between the FTPL and fiscal distressed economies. For example, Sims (2013) when explaining the necessity of fiscal support for monetary policy effectiveness says "This is easiest to understand in high-inflation, high nominal debt economies where fiscal policy is frozen by political deadlock or chicanery" and Loyo (1999), who offers a FTPL explanation for Brazilian hyperinflation in a similar environment. Leeper and Leith (2016) also refer to the recent Brazilian experience of unsustainable fiscal policy and high debts as suggestive evidence of the FTPL.[3]

However, FTPL models have a sharp and robust prediction in the presence of price rigidities: a fall on surpluses generates a boom of economic activity. The traditional explanation relies on wealth effects: if taxes fall and there is no expectation of an equivalent rise in the future, households

[1] See for example recent efforts of Cochrane (2011), Sims (2013), Bianchi and Melosi (2014) and Leeper and Leith (2016).

[2] Moreover, such environments are more prone to attain the top of a Laffer curve, posing limits to the amount of revenues that can be achieved from taxes.

[3] Additionally, Leeper and Leith (2016) cite the Euro Area sovereign crisis as another example of countries where the fiscal limit had been reached and the surpluses necessary to assure monetary policy actions are not guaranteed.

expends the additional resources on goods and services, stimulating production. But if fiscal accounts are unsustainable, it is not obvious that negative surplus news must induce a boom. Again, own proponents of the FTPL seem not to believe this is the case. As says Cochrane (2011):

> The large history of fiscal inflations and currency collapses does not inspire hope that a fiscal inflation always results in prosperity. The hyperinflations that follow wars (Sargent, 1992), Latin American fiscal collapses, currency crashes, or the recent hyperinflation in Zimbabwe were associated with sharp declines in economic conditions, not the spectacular booms Phillips curve would predict

One possible explanation for recessions on these economies could be a rise in default probability and risk-premia. However, it is not obvious why a country would choose (and agents would expect) default, if there is the option of deflating nominal debt. Indeed, inflation can be a convenient way to pay off debt: unlike outright default, there is no breach of contract, so there would be no reason for the government to be excluded from credit markets.

This takes us to the second scenario: inflation financing of fiscal deficits. According to what we have highlighted above, a good policy prescription for economies going through deflationary spirals goes as: peg interest rates and convince people that surpluses will not be sufficient to pay off public debt; inflation will come along with an output boom. This option may seem reckless, but is already being discussed given the difficulties to inflate economies through conventional policies.[4] A proper setup to evaluate such policy prescription is, naturally, a FTPL environment. A relevant question is: would such a policy generate the so desired demand-inflation boomlet or only stagflation?

In this paper, we build a New Keynesian model with financial frictions, and show that a fiscal deterioration can trigger a stagflation when we consider the costs of inflation to financial intermediation. In our model, financial intermediaries/banks face an agency problem similar in spirit to Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) which ties its lending capability to its balance-sheet conditions. Departing from the model of Gertler and Karadi (2011), here intermediaries collect short-term deposits to buy long-term nominal bonds and finance capital through long-term nominal loans. This introduces a maturity mismatch on banks balance-sheets that leads to an inflation exposure: although both assets and liabilities are nominal, the former are more sensible to inflation due to higher maturity.

When surpluses fall and are not expected to rise again, agents expect a rise on inflation to cover additional deficits, generating inflationary pressures today. Surprise inflation reduces the ex-post return on banks loans. When banks lend through long-term contracts, they cannot adjust the whole portfolio according to the new path of inflation and have no alternative but to keep bad loans on balance-sheets and admitting losses. Additionally, the rise on interest-rates following inflation further damages bank's balance-sheets by reducing the value of their long-term assets - in particular, their long-term holdings of government bonds. There is, thus, a 'credit deflation channel', the mirror of Fischer's debt deflation channel: while borrowers are better off with the

[4]See for example some recent articles on The Economist February edition dedicated to the theme: Economist (2016a),Economist (2016b).

surprise devaluation of debts, lenders are worse off by the same reason. Contrary to the debt deflation effect, the credit deflation makes investing entrepreneurs worse off: the fall on the value of banks assets tightens their agency problem, reduces capital finance and prompts a rise on spreads and lending rates.

For a reasonable calibration, we show that a fall on lump-sum taxes leads to a recession and that government spending multiplier, typically greater than one in FTPL models, is less than one. Recession comes with inflation because of a supply-side type shock to the economy. From the point of view of producing firms, the fall on capital is an exogenous force coming from the financial sector, not an optimal decision, as its productivity did not change. Hence, lower capital raises firms marginal costs, engendering a rise in optimal prices and inflation which helps to restore government solvency. Deficits remain financed by inflation, although at the cost of a recession.

Limited arbitrage between policy-rates and lending rates is key for these results. Without financial frictions, real interest rates typically fall to restore government budget in the presence of price rigidities. By arbitrage, all other rates fall leading to a rise on investment and consumption. In our model, while real policy rates fall, lending real rates are higher and often rise because of spreads, consistent with the fall on investments.

While the fall on banks net-worth induces a fall on investments, the fall on surpluses also raises aggregate demand. The prevailing force will depend on the degree of maturity mismatch, the sensitivity of bond prices to interest-rates and the share of bonds on banks total assets. The stronger each one of these factors, the greater the chance of a recession following a deficit shock. We further investigate the role of surprise inflation and bonds price revaluation in driving a recession, finding that each one is strong enough to generate a downturn of output.

In sum, our results show that a change on expectations of primary surpluses can be damaging to the financial sector, even without an outright default. The "inflation default" and revaluation of bonds prices reduces intermediaries net-worth and this can lead to a rise in spreads, an investment fall and a decline on output. Therefore, we provide theoretical explanation for empirical findings of a negative relationship between banks performances and inflation and a positive relation between banking crises and episodes of high inflation and high government debt. Additionally, we offer a complementary explanation to sovereign debt-banking crisis, often related to the damage on banks balance-sheets of an outright default of government bonds. In these crises, negative surplus news can feed inflation, impacting intermediaries balance-sheets and contributing to banks fragility and to the recession. As inflation grows too high, desirability and probability of a default rises, depleting even more banks net-worth (see Bocola (2016)), intensifying the recession.

Moreover, our results raises additional warnings about fiscal inflationary policies, adding to the points made in Cochrane (2011). Our results stress drawbacks of such policies that are more prone to occur just in economies experiencing low inflation, where the presence of long-term nominal contracts tend to be more common, banks tend to be more exposed to public debt as a result of QE and where protection to inflation is less needed.

Empirical relevance

In a recent paper, Cao (2014) quantifies bank's losses due to unanticipated inflation in the US. Using data from commercial banks from 1997Q3 to 2009Q3, she finds that on average 70% of banks assets and liabilities are denominated in nominal terms and have a maturity mismatch of about five years. She finds that a rise of 1% of inflation triggers losses of about 10% to 15% of Tier 1 capital, even for large banks. Such losses arise mainly from loans and leases made to the private sector, that entails more than 50% of nominal assets on banks portfolios. Another substantive loss comes from government bonds, which composes roughly 10% of banks assets. In a similar analysis, the Bank of Japan estimates that a 1% parallel rise in the yield curve leads to losses of 20% of Tier 1 capital for Japanese banks, of which around 13 to 14 percentage points comes from bond holdings losses.

In addition, there are many papers documenting that inflation can damage the financial sector institutions. Boyd et al. (2001) and Boyd and Champ (2003) find a negative relationship between inflation and financial sector performance. Demirgüç-Kunt and Detragiache (1997) and Demirgüç-Kunt and Detragiache (2005) find strong correlation between banking crisis and inflation. In a more recent paper, Boyd et al. (2015) constructs clean measures of banking crises, disentangling adverse shocks to the banking industry from restorative policies and finds that higher inflation rises the probability of systemic bank shocks.

In a comprehensive survey, Reinhart and Rogoff (2011) document a close link between high government debt, sovereign debt crisis and banking crisis in a cross-country dataset. They also find association of high indebtedness with inflation crisis, particularly for years after the first world war, when links between money and gold were weakened.

Summarizing, there is evidence that inflation can be damaging to the financial sector by its effects on maturity mismatched banks balances. There is also evidence for a relation between high inflation and poor financial sector performance and high inflation and debt environments to the occurrence of banking crisis. Our paper shows these histories can be tied together by the standard models used for policy evaluation, considering an appealing framework to study inflation financing, the fiscal determination of prices. More specifically, we show that fiscally induced inflation reduces rather than stimulates output through its impact on financial institutions balance sheets.

Related literature

Our paper fits in the extensive literature of the Fiscal Theory of the Price Level.[5] This literature typically focus on inflation and monetary policy implications of fiscal determination, relegating output implications to the side. Therefore, our paper adds to this literature by expliciting further output and inflation consequences of fiscal policy and inflation financing. In this literature, Cochrane (2011) is the work closest to ours. He shows that in the event of a fiscal shock, the government can use long-term bonds to trade all current inflation for future inflation. In a textbook New

[5] For seminal references, see Leeper (1991), Woodford (1994), Sims (1994), Cochrane (2001) and Cochrane (2005).

Keynesian model, he shows that deficits can be stagflationary on the short-run because of supply-side like effects: with the rise in future inflation and no current inflation, the Phillips Curve shifts to the left. We differ from Cochrane (2011) by the mechanism through which stagflations could arise and by taking a general-equilibrium approach to generate our impulse response functions.[6]

By considering models with financial frictions, we follow Gertler and Kiyotaki (2010) and Gertler and Karadi (2011).[7] We depart from the traditional models by allowing banks to hold government bonds and long-term nominal assets. In this sense, our paper relates to a recent literature which studies the cost of sovereign default on financial intermediaries. In a neoclassical model with financial frictions, Bocola (2016) finds that sovereign default, or even expectations of a default, can negatively affect banks balance and trigger a recession. Bi et al. (2014) extend this analysis to a New Keynesian model with bank runs, finding that defaults can also induce a disruption of interbank markets. Although our mechanism is similar to these papers, our work differs by focusing on surprise inflation, a more indirect form of default.

The idea that inflation can be damaging to the financial sector is present in previous works. Additionally to empirical works cited above, Kumhof and Tanner (2005) stress the importance of stable government debt to facilitate financial intermediation, contrasting policy prescriptions from optimal fiscal policy literature, who advocates for debt devaluation through inflation, and practitioners, who see it only as a last resort. Our paper adds to this discussion by showing that the negative relationship between inflation and financial intermediation can be rationalized by standard DSGE models and analyzing under which circumstances the damage is more severe.

Optimal fiscal policy literature advocates the use of state contingent inflation to absorb government shocks and smooth distortionary taxation. But when inflation negatively affects the financial sector, it stops being a costless way to pay off debt, even in the presence of flexible prices. A normative analysis for the option of inflation financing versus taxation is given by Cao (2015). In her paper, bankers faces collateral constraints to finance capital. A surprise inflation, by reducing the value of nominal government debt tightens banks constraints and negatively affects capital financing. She finds that government must balance inflation and distortionary taxation in the case of flexible prices and that the financial costs of inflation are substantive even in the presence of price rigidities, provided that government debt is sufficiently long-term. Differenttly, we make no normative analysis. Instead, we focus on the positive aspects of inflation financing and fiscal deficits, exploring more carefully the consequences for financial intermediation and economic activity. Our model economy also differs in some aspects. First, we adopt the more common approach to introduce financial frictions, following the lines of Gertler and Karadi (2011). Second, we add other long-term nominal assets besides long-term bonds. Our model is thus more flexible to accommodate different cases of maturity mismatch and allow inflation surprises to have substantial effects on the financial sector even when they do not affect assets prices - that is, when the monetary authority

[6]Daniel (2001) and Corsetti and Mackowiak (2006) relates FTPL models to exchange rate crisis, but the focus is also on prices and exchange rate determination, with no mention to output consequences. Loyo (2000) studies stagflation through the lens of the FTPL, but focus on monetary policy shocks.

[7]See also Bernanke et al. (1999) for frictions on the borrower side of contracts.

pegs interest rates or when maturity mismatch in bonds is null. This helps to connect the model to different countries experiences, like Japan, where banks hold great amounts of long-term government debt and the US, where banks holds a lesser amount of government debt but nevertheless have substantial maturity mismatch and exposition to a inflation default.

Guideline

In section 2, we make a brief fiscal review in a simplified New Keyenesian model, stressing key points relevant for our analysis. In section 3, we outline our model. In section 4, we build intuition to understand numerical results. In section 5, we do numerical exercises and discuss the results. Section 6 concludes.

2 Fiscal review

In this section we make a simple fiscal review to stress key points for our analysis. We focus on three issues: (i) the debt valuation equation with short-term debt and fully flexible prices; (ii) output and real rates consequences of adding price rigidities and (iii) the role of long-term bonds.

2.1 The debt valuation equation and output booms

We start with the government budget constraint written in present value form. With short term debt only, it reads as

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{s_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}/\pi_{t+s+1}}, \tag{1}$$

where B_{t-1} is the stock of government debt, P_t is the overall price index, s_t is government primary surplus, R_t is the gross interest rate on government debt and π_t is the gross inflation rate. This is the key equation for the Fiscal Theory of Price Level (FTPL). Cochrane (2005) interprets it as a market clearing condition: the real value of outstanding government debt reflects the expected value of primary surpluses covering it. If there is a fall on surpluses, the real value of government debt falls that. This revaluation can happen by two (non-excludable) ways: (1) a rise in prices P_t and (2) a fall on expected real rates. Flexible and rigid price environments will differ in the ways that this equation is satisfied under fiscal price determination. To make distinctions clear, we add a New Keynesian structure to our economy.

Households. Households consume, work and save through government bonds and maximize

$$E_t \sum_{j=0}^{\infty} \beta^j \log(C_{t+j}) - \frac{H_{t+j}^{1+\psi}}{1+\psi}$$

subject to the budget constraint

$$C_t + \frac{B_t}{R_t P_t} = \frac{W_t}{P_t} H_t + \Upsilon_t + \frac{B_{t-1}}{P_t} - s_t,$$

where C_t is consumption, W_t is the nominal wage, H_t is the work effort and Υ_t firms profits. β is the household discount factor and $1/\psi$ is the Frisch elasticity of labor supply. The first order conditions of this problem lead to the usual euler and supply of labor equations:

$$C_t^{-1} = \beta E_t \frac{R_t}{\pi_{t+1}} C_{t+1}^{-1} \tag{2}$$

$$C_t H_t^{\psi} = \frac{W_t}{P_t} \tag{3}$$

Firms and price setting. Firms produce differentiated goods through the function

$$Y_t = H_t \tag{4}$$

where Y_t is the firm production and also the aggregate level of output.[8] Firms sell their goods on competitive monopolistic markets and can face nominal price rigidities or not. In a fully flexible environment, firms will chose prices as a markup μ over marginal costs

$$P_t = \mu W_t \tag{5}$$

If they face nominal rigidities, however, they will not be able to choose prices freely. For simplicity, we assume the following structure of price rigidities: in period t firms are not allowed to adjust prices; from $t+1$ onwards, prices are fully flexible and firms will chose (optimally) to set them as in (5).

Monetary and fiscal policy and market clearing. The monetary authority acts passively, setting a constant interest rate every period and the fiscal authority sets a constant stream of surpluses

$$R_t = 1/\beta \tag{6}$$

$$s_t = s > 0 \tag{7}$$

and markets clear

$$Y_t = C_t \tag{8}$$

We now investigate the following experiment: the government unexpectedly decides to lower surpluses s. We first look to the flexible prices case.

Flexibe prices. Firms can choose freely to adjust prices, so by equation (5)

$$W_t/P_t = 1/\mu.$$

That is, real wages are constant. Combining with labor supply (3) and market clearing (8), we find that consumption and output are also constant. By the euler equation (2), we see that real interest

[8] By symmetry and our hypothesis about price setting, all firms will choose the same production.

rates are constant and the monetary policy rule (6) implies a constant inflation and interest rate. All real variables are constant, so what changes? Prices. Combining the debt valuation equation (1) and the fiscal policy (7), we find that

$$P_t = \frac{(1-\beta)B_{t-1}}{s}. \tag{9}$$

As the numerator is predetermined and s is falling, the result is a rise on overall prices. Prices are determined, even under an interest-rate peg. This is the fiscal determination of price level: prices go up or down depending on the amount of surpluses expected to soak government debt from the economy. The economic forces at play behind equilibrium work as follows: with the fall on surpluses, consumers are initially wealthier. As they try to expend additional resources on goods, firms rises prices reducing the value of consumer's assets B_{t-1}/P_t and mitigating demand pressures. With flexible prices, this process goes on until all demand pressures are eliminated and real variables return to their pre-shock values.

Rigid prices. If prices are rigid, current prices are not sufficient to lead to equilibrium. In our simplified case, this is even more pronounced: as prices are pre-set, they cannot help at all. As prices are fully flexible from $t+1$ onwards, real variables will be the same as the flexible case from $t+1$ onwards. On t prices are fixed, so output is demand determined. From the government debt valuation, we find that

$$\pi_{t+1} = 1/\beta - \frac{s}{\beta \frac{B_{t-1}}{P}}. \tag{10}$$

Inflation is rising with the fall on surpluses. As the nominal rate is constant, real rates fall inducing a rise of consumption and output.[9]

The traditional way for explaining the economic forces at play is again through wealth effects: the fall of surpluses rises households wealth and consumption demand. As prices are rigid (here, fixed), firms adjust quantities rather than prices leading to an output boom. Prices rise in the subsequent period when firms are allowed to adjust due to the increase in nominal costs, so inflation goes up. This interpretation has strong appeal, but may lead to confusion because households budget constraints do not enter explicit in the equilibrium computation and is less direct to apply when we add investment to the model. An alternative way to understand this result is as follows: when surpluses fall and are not expected to rise again, by the debt valuation equation (1) agents expect a rise in inflation to cover the additional deficits, as current prices do not adjust fully (here, do not adjust at all). The rise in inflation with no (or little) response of monetary policy implies a fall of expected real interest rates, stimulating consumption and output.

It is easy to see then what could happen if one added investment to the model: with the fall

[9] To be precise, there is a rise on the *growth rate* of consumption. In this simple model this coincides to a rise in the *level* of consumption because consumption converges to the (same) steady-state immediately after the shock and the fall of surpluses are equivalent to a fall on taxes. This coincidence could not occur if, for example, the fall of surpluses were induced by a rise of government purchases and the Frisch elasticity of labor supply was low (a high ψ). In this case, consumption could be crowded-out by government purchases since households would not be willing to work much more despite the higher wages. But in all these cases, output rises because of government purchases.

on real rates investment also rises, contributing to the rise in output. As we will show, this will no longer be necessarily true when we add financial frictions to the model. In this case, endogenous limits to arbitrage emerge and the policy real rate is no longer the same as the capital lending rate. In this case, investments can fall even with the a fall on the policy real rate, as a result of reduced capital financing, higher spreads coming from the banking sector and higher lending rates. But first, we turn to the case with long-term government bonds.

2.2 Long-term bonds and inflation postponement

In the presence of long-term bonds, the debt valuation equation reads as

$$\frac{Q_t B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{s_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}/\pi_{t+s+1}},$$

where $Q_t B_{t-1}$ is the nominal value of bonds outstanding. Now there is third way in which the debt valuation equation can be satisfied: changes in the market price of *outstanding* long-term bonds.

Analyzing the full model with long-term bonds is cumbersome and difficult to generate explicit analytical results. As the main messages of the last section do not change with the introduction of these assets, we will focus just on the consequences for inflation.[10] For this purpose, we will assume that prices are fully flexible at all periods, so real variables do not change. To make the analysis clear, we will also impose a specific structure to the bonds market, assuming only two bonds: a short-term one, paying $1 on the following period as in the last section and a long-term one, paying $1 two periods ahead. We include a short-term debt just to define a policy rate, so we additionally impose that it is in zero net-supply. Given this market structure, there is an arbitrage relation that defines the price of outstanding long-term bonds,

$$Q_t^1 = 1/R_t, \tag{11}$$

where Q_t^N is the price of a long-term bond with N periods to mature. Note that outstanding long-term bonds are equivalent to one-period short-term, as they will pay exactly $1 next period. To see the difference from the case of short-term debt only, its useful to look to the the households budget constraint in this environment

$$C_t + Q_t^2 \frac{B_t^2}{P_t} = \frac{W_t}{P_t} H_t + \Upsilon_t + Q_t^1 \frac{B_{t-1}}{P_t} - s_t,$$

where we imposed the market clearing for one-period bonds. Different from the case of short-term debt only, changes in the policy interest rate, R_t, affects directly the price of assets households already holds, not only assets they could buy. That way, changes in policy rates have a direct wealth effect on households by reducing the value of their portfolio. This falling value of long-term

[10]See Leeper and Leith (2016) for a more complete analytical treatment.

debt, in turn, is mirrored in the government budget constraint, which accordingly reads as

$$\frac{Q_t^1 B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{s_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}/\pi_{t+s+1}}. \tag{12}$$

Too see the change on the inflation trade off, we now allow nominal rates to respond to inflation according to the following rule

$$R_t = \beta^{-1}\pi_t^{\phi},\ \phi < 1. \tag{13}$$

Once again, we suppose that the economy was in zero inflation steady-state equilibrium until period t and that the government unexpectedly reduces surpluses s to a new value s' with $s' < s$. Before the shock, we had by (12) that outstanding debt equals to

$$B_{t-1} = \frac{s}{\beta(1-\beta)},$$

where we normalize the pre-shock prices to unity. Using this equation, the arbitrage relation (11) and the policy rate (13) in the debt valuation equation (12) *after* the shock, together with the fact that real rates are constant, we find the following expression for inflation at t

$$\pi_t = \left[\frac{(1-\beta)\beta B_{t-1}}{s'}\right]^{\frac{1}{1+\phi}} = \left[\frac{s}{s'}\right]^{\frac{1}{1+\phi}}, \tag{14}$$

as $s' < s$, gross inflation-rate is greater than one and is higher for smaller surpluses (that is, smaller s'). Using again the fact that real rates are constant, $R_{t+j}/\Pi_{t+j+1} = 1/\beta$, and the policy rate 13, we also find an expression for future inflation

$$\pi_{t+j} = (\pi_t)^{\phi^j}. \tag{15}$$

Expressions 14 and 15 show the differences on inflation dynamics relative to the short-term debt case. In the present case, the government has the option of trading inflation today for inflation in the future by altering the value of outstanding long-term bonds (that is, by altering R_t). A higher value of ϕ reduce inflation at t at the cost of higher future inflation. We also see what happens if the government tries to respond too strongly to inflation, for example, following the taylor principle: if $\phi > 1$, by (15), inflation takes an explosive path. If, on the contrary, the government does not let long-term debt value fluctuate, pegging the short-term interest rates ($\phi = 0$), we come back to the flexible prices short-term debt case: all revaluation happens in the moment of the shock and there is no future inflation (compare (9) to (14) with $\phi = 0$).

The mechanism goes as follows. The fall on surpluses generates demand pressures. The government then starts selling long-term debt at lower prices (that is, it permits R_t to rise), leading demand towards government bonds, alleviating pressures on goods and current inflation. But by selling long-term debt, the government transfers these demand pressures for the future, as there will be more maturing debt to chase goods tomorrow. Hence, expectations of inflation rises today and if nothing else happens, actual future inflation rises.[11] If on the other hand the government trade any amount of debt at a fixed rate, as in an interest-rate peg, it keeps prices of government bonds stable and this channel disappears.

3 Model description

Our economy is a version of Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) in which government bonds and long-term nominal loans compose banks balances. Long-term loans are introduced using a framework similar to Andreasen et al. (2013). Relative to the last section, we add capital, financial frictions and a more complex maturity structure for bonds.

3.1 Households

Households consume, work and save to maximize

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{(C_{t+j} - hC_{t+j-1})^{1-\sigma}}{1-\sigma} - \frac{H_{t+j}^{1+\psi}}{1+\psi} \right], \tag{16}$$

subject to the budget constraint

$$C_t + d_t = w_t H_t + \frac{R_{t-1}^d}{\pi_t} d_{t-1} + (1-\varsigma) \frac{R_t^b}{\pi_t} Q_{t-1} b_{t-1} - \tau_t + \Upsilon_t \tag{17}$$

where d_t is real deposits on banks, R_t^d is the nominal rate on these deposits, H_t is the work effort,w_t is the real wage, π_t is the inflation rate, τ_t are lump-sum taxes, Υ_t firms profits, b_t government bonds, Q_t and R_t^b the price and return of government bonds. The parameter σ is the inverse of the intertemporal elasticity of consumption, h is the habit persistence on consumption, $1/\psi$ is the Frisch elasticity and ς indexes government bond holders.[12] Consumption C_t is a CES aggregate over differentiated goods obtained from an expenditure minimization problem and is given by

$$C_t = \left[\int_0^1 C_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \tag{18}$$

3.2 Wholesale firms

Wholesale firms operate in competitive markets, renting capital K_t and labor H_t to produce output Y_t, according to the technology

$$Y_t = K_{t-1}^{\alpha} H_t^{1-\alpha}. \tag{19}$$

[11] Again, it is possible to interpret this mechanism through wealth effects: when government starts selling long-term debt at lower prices, the portfolio value of households holding these assets goes down, mitigating the expansion on the budget constraint generated by the fall of surpluses and reducing demand pressures.

[12] See the financial intermediaries section.

Let p_t^w be the relative price of wholesale goods, r_t the rental price of capital and w_t the real wage. Optimal factor allocations are then given by

$$\alpha p_t^w Y_t / K_{t-1} = r_t \tag{20}$$

$$(1-\alpha) p_t^w Y_t / H_t = w_t. \tag{21}$$

3.3 Entrepreneurs

At the beginning of period t, entrepreneurs take loans from financial intermediaries to finance capital for use at $t+1$. A loan signed at period t specifies the amount of capital $\tilde{k}_t$ the firms wants to finance and a fixed nominal rate R_t^l to hold until the contract matures. Following Andreasen et al. (2013), we introduce long-term loans by imposing a random maturity for debt contracts, assuming that each period loans mature with probability $1-\theta_k$. While the loan does not mature, entrepreneurs keep its capital choice constant. When a loan matures, entrepreneurs re-optimize taking a new loan.

There are two points worth noting. First, it implies an average duration of loan contracts of $\frac{1}{1-\theta_k}$ and allow us to parametrize the model through only this single parameter. Second, it implies infrequent capital adjustments as the adopted in Kiyotaki and Moore (1997) and Sveen and Weinke (2007) on the context of firm specific capital. As showed by Andreasen et al. (2013), when capital is homogeneous, infrequent capital adjustments have no effect on prices and aggregate variables in a broad class of DSGE models, including ours when we eliminate financial frictions. Thus, adopting this structure allow us to isolate the effects of maturity mismatch and financial frictions on the economy.

When ready for use, entrepreneurs rent capital to wholesale firms and sell non-depreciated capital for capital producers which is bought again at the start of the following period. All profits from these operations are rebated lump-sum to households so entrepreneurs do not accumulate net-worth. Accordingly, entrepreneurs adjusting at t choose capital $\tilde{k}_t$ to maximize the present value of profits[13]

$$\sum_{j=1}^{\infty} \beta^j \theta_k^{j-1} \frac{\lambda_{t+j}}{\lambda} \left[r_{t+j}^k \tilde{K}_t - \frac{R_t^L}{\prod_{s=0}^{j} \pi_{t+j}} p_t^k \tilde{K}_t + (1-\delta) p_{t+j}^k \tilde{K}_t \right],$$

where δ is the depreciation rate. As entrepreneurs take nominal debt to finance real assets, there is a Fischer debt-deflation channel which makes entrepreneurs better off with inflation. This effect is reflected on the negative term above and is more intense the longer is the maturity of the loan contract.

[13] Implicitly, we assume that every period entrepreneurs sells capital to capital producers at a price p_{t+j}^k and buy back at price p_t^k signed in the beginning of the loan contract, such that all capital gains goes to entrepreneurs. We do this for two reasons. First, buying at a fixed price prevents the loan value to vary with the price of capital, resembling more real-life loan arrangements. Second, allowing capital gains enable us to recover standard financial frictions models when we impose one-period loans ($\alpha_k = 0$). As all profits are rebated lump-sum to households, eliminating capital gains changes little our results.

For every optimizing entrepreneur, first order condition for capital choice is given by

$$\sum_{j=1}^{\infty} \beta^j \theta_k^{j-1} \frac{\lambda_{t+j}}{\lambda} \left[r_{t+j}^k - \frac{R_t^L}{\prod_{s=0}^{j} \pi_{t+j}} p_t^k + (1-\delta) p_{t+j}^k \right]. \tag{22}$$

Entrepreneurs not optimizing simply make $\tilde{K}_t = \tilde{K}_{t-1}$. As all optimizing entrepreneurs are homogeneous and face the same probability of adjusting capital, aggregate demand for capital is given by

$$K_t = (1-\theta_k)\tilde{K}_t + \theta_k K_{t-1}. \tag{23}$$

3.4 Financial Intermediaries

Financial intermediaries/banks are owned by households. Let l_t be the real value of loans to entrepreneurs and n_t financial intermediaries net-worth. The intermediary balance-sheet is then given by

$$l_t + \varsigma Q_t b_t = d_t + n_t. \tag{24}$$

The indicator parameter ς is introduced only to identify the presence of government bond holders. We study two polar cases: when $\varsigma = 1$, financial intermediaries hold government bonds and when $\varsigma = 0$ households are the holders of government debt. Intermediate cases follow similarly, so in order to highlight the main mechanisms we focus only on these two cases.[14]

Intermediaries revenues rev_t from loans depend on the whole portfolio of loans made to entrepreneurs. Given the maturity structure imposed, they can be written in the following recursive form

$$rev_t = (1-\theta_k) R_t^l p_t^k \tilde{K}_t + \theta_k rev_{t-1}/\pi_t. \tag{25}$$

A similar recursive form can be obtained for the total amount of loans

$$l_t = (1-\theta_k) p_t^k \tilde{K}_t + \theta_k l_{t-1}/\pi_t \tag{26}$$

These expressions simply point that revenues and loans are functions of contracts signed in the current period, in addition to the real value of contracts brought from the previous period.

As in Gertler and Karadi (2011), an agency problem between bankers and depositors requires bankers to supply their own net-worth as credit, giving rise to endogenous liquidity constraints and spreads. As banks lending rates are higher than borrowing rates, net-worth could grow to the point that banks do not need deposits to provide credit. We prevent this by assuming that each period a fraction $1-\theta_b$ of banks exits the market and transfer accumulated net-worth to households. At

[14] See section 5.4 for more on this issue.

the beginning of t, banks net-worth is given by

$$n_t = \frac{1-\chi}{\pi_t}\left[rev_{t-1} + \varsigma R_t^b Q_{t-1} b_{t-1} - R_{t-1}^d d_{t-1}\right]. \tag{27}$$

Following Andreasen et al. (2013), we introduce the proportional contribution χ on banks net-worth paid to insurance agencies. This is done for two reasons. First, these agencies guarantee that entering banks starts with the same balance-sheet composition of the existing ones, which allows aggregation and the use of a representative bank. Second, this assumption ensures that banks net-worth does not grow without bounds.[15] In any case, by matching leverage and spreads to conventional values on the literature, we calibrate χ to a quite small value, so it has little quantitative impact on our results.

Bankers objective function is to maximize the present value of expected net-worth that will be transferred to households. Their value function can be written in the following recursive form

$$V_t = \max E_t \beta \frac{\lambda_{t+1}}{\lambda_t}\left[(1-\theta_b) n_{t+1} + \theta_b V_{t+1}\right]. \tag{28}$$

That is, the value function of a banker optimizing at t is the present value of expected net-worth one period ahead in the case of exiting the industry or the value of continuation of remaining a banker.

We assume an agency problem. Each period, a banker can choose to divert a fraction of total assets to its own households. In that case, households can force intermediaries to bankruptcy, but can recover only a fraction of assets. Following Gertler and Karadi (2013), we assume this agency problem is less severe regarding government bonds than private loans, reflecting the fact that the latter is more difficult for depositors to monitor. Therefore, each period the banker has the option to divert a fraction ω of private loans and $\omega\iota$ of bonds, with $0 \leq \iota \leq 1$. Accordingly, households will supply funds to the bank, as long as the following incentive constraint holds

$$V_t \geq \omega\left[l_t + \iota\varsigma Q_t b_t\right]. \tag{29}$$

This expression says that the value of keeping operating (left side) must be greater than the value of diverting assets and going bankrupt. Banks then have to limit its demand for deposits relative to their own net-worth to keep incentives aligned to that of depositors. Throughout the paper, we assume this incentive constraint is always binding.

The optimization problem of bankers is then to choose l_t,$Q_t b_t$ and d_t to solve (28) subject to the balance-sheet condition (24), the laws of motion (25)-(27) and the incentive constraint (29). We solve this problem by first guessing that the value function has the following form

$$V_t = \nu_t^k l_t + \nu_t^b \varsigma Q_t b_t - \mu_t d_t \tag{30}$$

[15]This setup contrasts with Gertler and Karadi (2011), who assumes that retired banks are replaced by new banks with a sufficiently low net-worth

where each coefficient represents the marginal value of varying the respective variable. By guess and verify, we find these coefficients to be

$$\nu_t^k = E_t \Omega_{t+1} \frac{rev_t / l_t}{\pi_{t+1}} \tag{31}$$

$$\nu_t^b = E_t \Omega_{t+1} \frac{R_{t+1}^b}{\pi_{t+1}} \tag{32}$$

$$\mu_t = E_t \Omega_{t+1} \frac{R_t^d}{\pi_{t+1}} \tag{33}$$

where Ω_{t+1} is the adjusted stochastic discount factor, given by

$$\Omega_{t+1} = E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\beta}{(1-\chi)} \left[(1-\theta_b) + \theta_b \left((lev_{t+1}^k + \iota lev_{t+1}^b)(\nu_{t+1}^k - \mu_{t+1}) + \mu_{t+1} \right) \right] \tag{34}$$

where $lev_t^k = \frac{l_t}{n_t}$ and $lev_t^b = \frac{\varsigma Q_t b_t}{n_t}$ are the leverage ratios for private loans and bonds, respectively. These expressions simply say that the marginal gains of financing capital or the government are their expected average present-value of real returns. The marginal cost of collecting more deposits, in turn, is the expected payment by doing so. By the presence of financial frictions, banker's stochastic factor is not the same as household's and must be adjusted by the term in brackets.

Letting ϑ_t denote the Lagrange multiplier on the incentive constraint, substituting the balance-sheet condition (24) for d_t in the guess for the value function and using the expressions above, the first order conditions for l_t and $Q_t b_t$ imply

$$E_t \Omega_{t+1} \left(\frac{rev_t / l_t}{\pi_{t+1}} - \frac{R_t^d}{\pi_{t+1}} \right) = \omega \frac{\vartheta_t}{1+\vartheta_t} \tag{35}$$

$$E_t \Omega_{t+1} \left(\frac{R_{t+1}^b}{\pi_{t+1}} - \frac{R_t^d}{\pi_{t+1}} \right) = \varsigma \omega \iota \frac{\vartheta_t}{1+\vartheta_t} \tag{36}$$

So when the compatibility constraint binds ($\vartheta_t > 0$), a spread between deposit rates and lending rates arise. The spread on government bonds returns will be lower if the agency friction is less intense on these assets ($\iota < 1$) and absent if they are owned by households ($\varsigma = 0$).[16] We can combine these two equations to obtain the following arbitrage relation that must hold whenever banks holds bonds

$$E_t \Omega_{t+1} \left(\frac{R_{t+1}^b}{\pi_{t+1}} - \frac{R_t^d}{\pi_{t+1}} \right) = \varsigma \iota E_t \Omega_{t+1} \left(\frac{rev_t / l_t}{\pi_{t+1}} - \frac{R_t^d}{\pi_{t+1}} \right). \tag{37}$$

Finally, we can use the definitions for leverage and the guess for the value function to rewrite

[16] Arbitrage between bonds and deposits when households are the holders of government bonds comes from the households problem. In this case, the arbitrage relation would involve the household discount factor, not the bankers' one. In a linearized model, the setup with ς yields the same results.

the incentive constraint as

$$lev_t^k + \iota lev_t^b = \frac{\mu_t}{\gamma - (\nu_t^k - \mu_t)}. \tag{38}$$

3.5 Capital producers

At the end of period t, capital producers buys non-depreciated capital and investment goods to produce new capital. In the beginning of $t+1$ and before shocks are realized, capital is sold again to entrepreneurs. As noted before, in order to keep capital gains with entrepreneurs and to prevent the value of loans to vary with capital prices, capital producers buys capital for the market price and resell it by the price defined in the entrepreneurs' loan contract. Thus, the present-value of profits is given by

$$E_t \sum_{j=0}^{\infty} \frac{\lambda_{t+1}}{\lambda_t} \beta^j \left[\upsilon_{t+j} - (1-\delta) p_{t+j}^k K_{t+j-1} - I_{t+j} \right], \tag{39}$$

where aggregate investment I_t is a bundle of retail goods similar to the consumers, p_t^k is the real price of capital and υ_t is an aggregate given by

$$\upsilon_t = (1-\theta_k) \sum_{j=0}^{\infty} \theta_k^j \tilde{K}_{t-j} \frac{P_{t-j}^k}{P_t} = (1-\theta_k) p_t^k \tilde{K}_t + \theta_k \frac{\upsilon_{t-1}}{\pi_t}, \tag{40}$$

As in Christiano et al. (2005), capital production involves investment flow adjustment costs and follows the technology

$$K_t = (1-\delta) K_{t-1} + I_t - S(I_t/I_{t-1}) I_t \tag{41}$$

where $S(.)$ is the adjustment cost function, satisfying:$S(0) = 1$, $S'(0) = 0$ and $S''(0) = \kappa > 0$.[17]

Capital producers objective is to choose K_t,$\tilde{K}_t$, υ_t and I_t to maximize the discounted value of profits (39) subject to (40), (41) and capital demand from entrepreneurs (23). The optimization is described on the appendix.

3.6 Retailers

Retailers buys goods from wholesale producers and differentiate it through a linear technology to sell at competitive monopolistic markets. We introduce price rigidities following Calvo (1983): each period only 1-θ_p firms are allowed to choose prices freely. Firms able to optimize choose prices to maximize discounted profits

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \frac{\lambda_{t+1}}{\lambda_t} Y_{t+j}(i)(\tilde{p}_t - mc_{t+j})$$

[17] Particularly, we follow Christiano et al. (2015) adopting the following functional form for $S(.)$: $S\left(\frac{I_t}{I_{t-1}}\right) = 1/2\left[\exp\left(\sqrt{\kappa}(I_t/I_{t-1} - 1)\right) + \exp\left(-\sqrt{\kappa}(I_t/I_{t-1} - 1)\right) - 2\right]$. On a linearized solution, any function with the properties in the text gives identical results.

subject to the demand curves

$$Y_t(i) = \left[\frac{\tilde{p}_t}{P_t}\right]^{-\eta} (C_t + G_t + I_t)$$

where $mc_t = p_t^w$ is the real marginal cost and $\tilde{p}_t$ is the new price. Solution to this problem leads to

$$\tilde{p}_t = \frac{\eta}{\eta - 1} \frac{\sum\limits_{j=0}^{\infty} (\beta\xi)^j \frac{\lambda_{t+j}}{\lambda_t} Y_{t+j} mc_{t+j}}{\sum\limits_{j=0}^{\infty} (\beta\xi)^j \frac{\lambda_{t+j}}{\lambda_t} Y_{t+j}}. \tag{42}$$

By the Calvo scheme and agents baskets of retail goods, inflation is given by

$$\pi_t^{1-\eta} = (1 - \theta_p)(\tilde{p}_t \pi_t)^{1-\eta} + \theta_p. \tag{43}$$

3.7 Government, policies and market clearing

The government collects taxes and issues debt to finance purchases and repay maturing debt. Following Cochrane (2001), Woodford (2001) and Eusepi and Preston (2011), we introduce long-term bonds imposing a geometric payment structure: a bond sold at the price Q_t pays \$1 in t, \$1+ρ in $t+1$, \$1+ρ^2 in $t+2$ and so on. Equivalently, as Leeper and Leith (2016) show, this is equivalent to assume a geometric decay rate for a portfolio of zero-coupon bonds. This setup implies an average maturity of outstanding bonds of $\frac{1}{1-\beta\rho}$, allowing us to control maturity through ρ. Given this maturity structure, we can define the ex-post return for bonds as

$$R_t^b = \frac{1 + \rho Q_t}{Q_{t-1}} \tag{44}$$

and write the period government budget constraint as

$$G_t + Q_t b_t = \tau_t + \frac{1 + \rho Q_t}{\pi_t} b_{t-1}, \tag{45}$$

where government consumption G_t is a bundle equal to the consumer's and follows the exogenous process

$$\log(G_t) = (1 - \rho_g) \log(G) + \rho_g \log(G_{t-1}) + \varepsilon_t^G, \tag{46}$$

where G is the steady-state value of government purchases, ρ_g an autoregressive coefficient and ε_t^g a zero mean normal distributed random shock.

Monetary policy is described by the following rule:

$$\log(R_t^d) = \log(1/\beta) + \rho_r \log(R_{t-1}) + (1 - \rho_r)(\phi_\pi \log(\pi_t)). \tag{47}$$

As we will focus on the case for fiscal price level determination, rather than interpreting the above rule as an explicit inflation-targeting, we will interpret it as an 'inflation postponement' rule. As we showed on section 2, letting $\phi_\pi > 0$ the government can trade inflation along time by allowing asset prices changes to restore government solvency. The higher the value of ϕ_π, the lower the inflation coming today and the higher the inflation tomorrow. On the other hand, when $\phi_\pi = 0$, the government is willing to trade all quantities of bonds at a constant rate so asset prices remains constant, eliminating this channel. Therefore, we let $\phi_\pi > 0$ whenever we want to study the cases of inflation affecting government bonds prices directly.[18]

Following Leeper (1991), the fiscal rule is specified by a tax response to the stock of government debt

$$\log(\tau_t) = \log(\tau) + \phi_\tau \log(Q_{t-1} b_{t-1}) + \epsilon_t, \tag{48}$$

where τ is the steady-state value of taxes and ϵ_t is a stochastic process following

$$\epsilon_t = \rho_\tau \epsilon_{t-1} + \varepsilon_t^\tau. \tag{49}$$

Finally, goods market clearing requires that

$$Y_t = (C_t + G_t + I_t)\o_t \tag{50}$$

where $\o_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\eta} di$ is a measure of price dispersion.

4 Forces at play

Before going to our numerical exercises, in this section we provide intuition for the key results in our model. There are two offsetting forces at play in a fall on surpluses financed through inflation. First, the usual aggregate demand effect described in section 2: when surpluses fall, households start expecting negative rates to cover the additional deficits and, thus, raise consumption, stimulating production. On the other hand, inflation default negatively affects financial intermediaries net-worth, reducing capital finance, raising spreads and discouraging investments. Which of these forces prevail depends on: (i) the maturity mismatch of the banking sector, (ii) the size of government debt and its relevance on banks portfolio, and (iii) how the central bank time this inflationary default.

Suppose that banks hold government debt and that the agency problem is the same between private loans and bonds. Then, the incentive constraint is written as

$$l_t + Q_t b_t \leq lev_t n_t \tag{51}$$

where $lev_t = \frac{\mu_t}{\omega - (\nu_t^k - \mu_t)}$ is the overall leverage of financial intermediaries. This expression is essentially a liquidity constraint. A fall on banks net-worth tightens this constraint, reducing credit for both capital purchases and for the government. Intuitively, the fall on banks net-worth rises the incentive to divert assets relatively to keeping financial operations. As a consequence, bank reduce

[18] In the model with financial frictions, spreads in government bonds prices can vary even in a riskless econonomy with $\phi_\pi = 0$.

their lending credit. This tightening rises the shadow value of relaxing the incentive constraint and by equations (35)-(36), rises the spread on lending rates. Therefore, this limit to arbitrage implies that lending real rates can be positive even if deposit/policy real rates remains negative. The fall in credit and the rise on real rates decrease investments.

We see how surprise inflation affects banks net-worth by looking at the net-worth accumulation equation:

$$n_t = \frac{1-\chi}{\pi_t}\left[rev_{t-1} + \varsigma(1+\rho Q_t)b_{t-1} - R^d_{t-1}d_{t-1}\right]. \tag{52}$$

A surprise inflation can negatively affect banks net-worth by reducing the value of loans revenues or the value of long-term bonds. At the same time, deposits value fall with inflation, offsetting these effects. As banks are leveraged, typically the negative effect prevails. And the longer the maturity of assets relative to liabilities, the stronger is this negative effect.[19] To make this clear, we look to the net-worth equation when banks holds no bonds and loans are only one-period:

$$n_t = \frac{1-\chi}{\pi_t}\left[R^l_{t-1}p_{t-1}K_{t-1} - d_{t-1}\right] = \frac{(1-\chi)n_{t-1}}{\pi_t}\left[lev_{t-1}(R^l_{t-1} - R^d_{t-1}) + R^d_{t-1}\right]. \tag{53}$$

In the second equality, as spreads are positive and rates predetermined, we see that a surprise inflation negatively affects banks. But for one-period loans and a reasonable calibration, we find these effects to be almost irrelevant for the rest of the economy. The reason is that with one-period loans, in response to inflation banks can immediately revise all their loans to entrepreneurs, switching to new loans with higher rates and facing limited losses. When contracts are long-term, however, this will not be possible and banks will have to stick with some contracts and suffers the losses from inflation. This can be clearly seen by the law of motion of bank's revenues when contracts have more than one period:

$$rev_t = (1-\theta_k)R^l_t p^k_t \tilde{K}_t + \theta_k(1-\theta_k)\frac{R^l_{t-1}}{\pi_t}p^k_{t-1}\tilde{K}_{t-1} + \theta_k^2(1-\theta_k)\frac{R^l_{t-2}}{\pi_t\pi_{t-1}}p^k_{t-2}\tilde{K}_{t-2} + ... \tag{54}$$

This expression shows the other side of the traditional Fischer's debt-deflation channel for debtors: if intermediaries engage in long-term nominal loans, revenues are diminished when there is a surprise inflation and banks cannot immediately adjust their contracts with the private sector. The longer the average duration of loans, the greater the impact of a surprise inflation. If intermediaries (or lenders in general) have borrowing constraints limiting the credit they can provide, this fall on profits leads to higher spreads which, in contrast to the conventional effect of the debt-deflation channel, makes entrepreneurs worse off and discourage investments. So the presence of a debt-deflation channel comes with a 'credit-deflation channel' which, as we will show, can even countervail the positive effect on entrepreneurs.

[19] This is not true if loans are denominated in real terms, as in Gertler and Karadi (2011) where banks buys equity stakes from firms. As a great portion of banks financing is made through nominal loans, we will stick with this case to stress its consequences. But we note that, even in the case of real loans, revaluation of nominal assets (long-term bonds, for example) still have relevant effects on banks positions in the event of a surprise inflation.

If inflation surprises are a problem, the government can trade it for bond prices surprises by allowing the interest-rate to vary, reducing the value of outstanding debt. In this case, although the total amount of inflation is similar, the movement is anticipated, allowing intermediaries to adjust smoothly to the shock. However, as we see by (52), when banks hold long-term government bonds or assets with comparable payment structure, the fall on bond prices reduces banks net-worth, leading to the same negative effects on capital financing. The fall on bank's assets comes either from the fall in the price of government bonds or any other type of long-term nominal assets. If banks were allowed to hold other types of long-term assets, by arbitrage (even if limited) there would be a fall on their values as well. Nonetheless, note again the role of maturity: when assets are one-period only ($\rho = 0$), this channel becomes irrelevant and if loans are relatively short-term, effects on intermediaries net-worth are mild.

If a fall on investment is such that the economy enters in a recession, there could be deflationary preassures, contradicting the primary reason for the recession. This is not true: the fall on capital acts a supply side shock to the economy, rising the present value of firms marginal costs. Specifically, the fall on capital is not an optimal allocation, as there is no change on its productivity. Rather, from the point of view of producing firms, it is a result of external factors coming from the financial sector and the government. Therefore, the inflation necessary to finance government deficits still comes, but now at an output cost.

5 Numerical analysis

Numerical solution to the model is obtained through a first-order approximation around a deterministic steady-state in which leverage constraint binds. We explore impulse response functions to deficit shocks coming from taxes and government purchases. We first show and comment the impulse responses for the full model for each case. We isolate each channel detailed in the last section, by further separating the analysis in two cases: (1) banks holding only long-term loans and no bonds and (2) holding one-period loans and long-term bonds.

5.1 Calibration

Table 1 describes our calibration. We take conventional values for the discount factor β, the capital share on the production function α, the depreciation rate δ and the government expenditure share. As in Gertler and Karadi (2011) and Gertler and Karadi (2013), the parameters in the banking sector χ, θ_b, ω and ι are calibrated to imply a leverage ratio of four and a steady-state spread for loans and bonds of one hundred basis points and fifty basis points, respectively. Real economy parameters are obtained from Christiano et al. (2015), who estimate a model with financial and labor frictions using US data. The only exception is the Frisch elasticity $1/\psi$, for which we use the value proposed by Chetty et al. (2011) based on macro and micro estimates.[20] As mentioned before, conditional on fiscal price determination, the lump-sum taxes response to debt ϕ_τ does not

[20] There is no direct analogue for this parameter in Christiano et al. (2015).

have relevant effects on the economy, so we keep it at zero. For exogenous deficit shocks, we assume an autoregressive coefficient of 0.9. Lastly, we experiment values for the parameters controlling the duration of loans and bonds, θ_k and ρ, the inflation response ϕ_π and the debt-to-GDP ratio in order to analyze their effects.

Table 1: Calibration

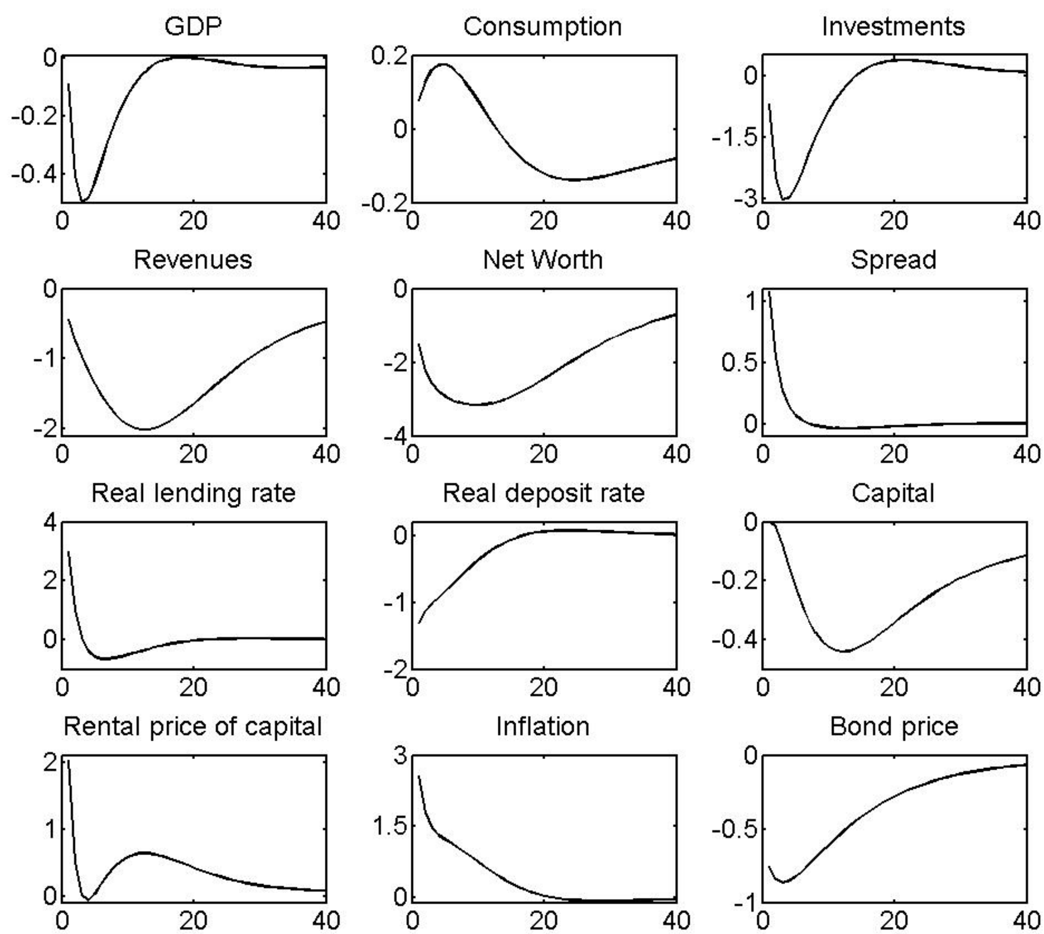
Parameter	Value	Description
β	0.99	Households discount factor
h	0.89	Habits persistence
$1/\psi$	0.75	Frisch elasticity
α	0.33	Capital share
δ	0.025	Depreciation rate
κ	12.07	Investment adjustment costs
θ_p	0.74	Probability of kepping prices fixed
η	4.36	Elasticity of substitution
χ	0.018	Insurance agencies contribution
ω	0.33	Fraction of assets can be diverted
θ_b	0.92	Probabilty of remaining a banker
ι	0.5	Intensity of bonds agency friction
ϕ_τ	0	Taxes response to debt
G/Y	0.2	Staeady-state share of government purchases
ρ_r	0.8	Policy rate smoothing parameter
ϕ_π	Free	Policy rate response to inflation
α_k	Free	Loans maturity parameter
ρ_b	Free	Bonds maturity parameter
$\frac{Qb}{4Y}$	Free	Steady-state debt-to-GDP ratio

5.2 Fully-specified model

For the full model, we calibrate the free parameters as follows. According to the evidence in Cao (2014), we choose α_k and ρ_b to generate an average duration of four years for loans and five years for bonds. The debt-to-GDP ratio is set at 80%, the average level for the G7 economies at the end of 2015 and the policy-rate coefficient is set at $\phi_\pi = 0.2$.

Figure 1 shows the impulse response functions to a fall on taxes and a rise in government purchases of 1% percent of GDP. The surprise rise on inflation following a fall on taxes has two negative effects on banks net-worth. First, it reduces banks real revenues on impact (the inflation term in 27) and even more afterwards as they cannot adjust a fraction of loans according to the new inflation path (the cumulative inflation in 54). Second, the rise in the policy-rate push bonds price down, reducing the value of banks assets. The result is a rise in spreads and a fall on capital financing, which leads to a fall on investments and output. The rise on bond spreads acts as a financial accelerator channel by pushing bonds prices down even more. Consumption rise on impact because of the fall on taxes and real deposit rates, but fall later as output (and hence, income) keeps below its steady-state.[21] Despite the recession, inflation comes up as result of a supply-side like shock to the economy: the fall on capital coming from the financial sector rises the costs of capital for producing firms, increasing marginal costs and inflation.

Figure 1: Impulse response to a tax shock, full model



[21]The rise in consumption following this type of shock is a common result under fiscal price-level determination. Here it is reinforced by an also common result of negative net-worth shocks with financial frictions: following the fall on net-worth, banks reduce demands for deposits to satisfy the compatibility constraint, pressing deposit rates down. There are ways to address this countercyclical movement of consumption (see for example Bigio (2015)), but as this is not central for our analysis we abstract from these features here.

Figure 2 shows the impulse response functions to a rise in government purchases of 1% percent of GDP. Not surprisingly, a direct aggregate stimulus from the government combined with rigid prices leads to a rise on output. But all the effects on the financial sector behind the fall on investments in the previous case are also present here. This explain why the fiscal multiplier, typically well above one in models with fiscal price-level determination, is less than one here. Government purchases crowd-out investments, as the rise in inflation and fall on bonds prices reduce capital finance. Consumption rises little and fall relatively faster than the previous case, also as a result of a crowding out effect.[22] Accordingly, fiscal multiplier becomes less than one and eventually turn negative as government purchases return to steady-state with consumption and investment below steady-state. Inflation rises more than in the previous case since there is also the rise in government demand pressing prices upwards.

These exercises show that a change on expectations of surpluses covering the public debt can be damaging to the financial sector, even if they do not imply an outright default (which we explicitly excluded here). The surprise 'inflation default' and revaluation of bonds prices reduce intermediaries net-worth and this alone can lead to a rise in spreads, fall on investments and recession. This gives theoretical support for articles finding negative relationship between banks performances and inflation and a positive relation between banking crises and episodes of high inflation and high government debt. Outright sovereign default, as in Bocola (2016), is the common channel associated with the impairing of the financial sector coming from an unsustainable fiscal policy. Instead of claiming that our channel is the right explanation, we think these alternatives can reinforce each other: before or even jointly with an outright default, negative news about surpluses in the presence of high debt and unsustainable fiscal policy feeds inflation, which hurts bank's balance-sheets and contributes to the recession. Furthermore, as inflation grows intolerable, default starts being a more desirable and even the only option (see Uribe (2006)), rising its probability and impairing banks net-worth even more, as in Bocola (2016).

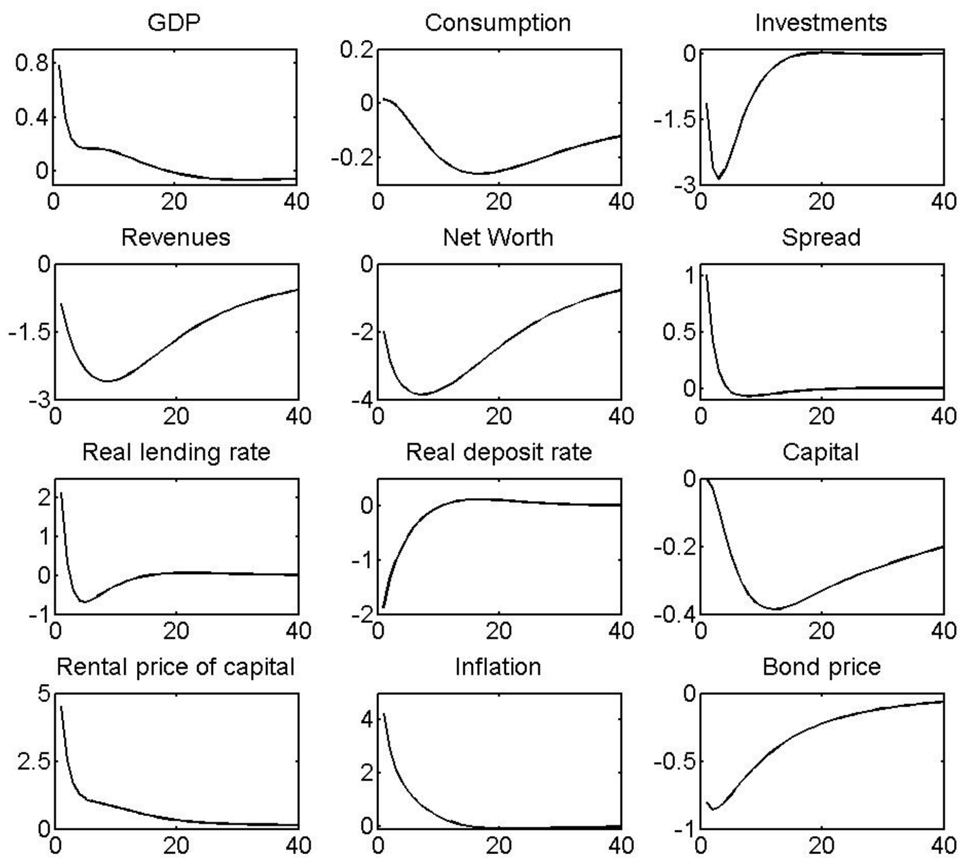
These results are also additional warnings against fiscal policy financed through inflation, adding to the points made in Cochrane (2011). This includes the 'helicopter drop' policies being proposed more recently as a solution to low inflation environments. A 'helicopter drop' is similar to our exercise for taxes: the government makes a promise that it will be reckless, transferring money to people and (at least implicitly) promising not to rise taxes to soak up that money - or not to issue bonds to be paid with greater taxes on the future. By definition, for such a policy to 'stimulate' at all it must involve a non-Ricardian fiscal policy as in the FTPL.[23] We do not argue that such

[22] The initial response of consumption is sensitive to the Frisch elasticity. The intuition is as follow: the rise in output following the government spending shock rises prices and the needing for work. To expand consumption, consumers will have to work more but if the elasticity is low, they will not be willing to vary much of their work even with higher wages. In this case, they optimally choose to work little and consume less relative to the case of high Frisch elasticity.

[23] There are at least other three possibilities worth noting by which such a policy could not be innocuous. First, if consumers had a finite horizon they would react if the taxes rise happens beyond their expected horizon (see Richter (2015)). Second, if consumers were rule-of-thumb or liquidity constrained, such a policy would be a relief to their income and could stimulate consumption (see Gali et al. (2004) for a treatment on rule-of-thumb consumers). And third, if people had less than rational expectations about fiscal policy. Two very interesting works on this

a policy would lead necessarily to a recession, as there are other countervailing forces we do not consider in our model. But these results nevertheless stress drawbacks of such a policy that are more prone to happen just in economies experiencing low inflation, where the presence of long-term nominal contracts tend to be more common, banks tend to be more exposed to public liabilities as a result of QE and where protection to inflation is historically less needed.

Figure 2: Impulse response to a government spending shock, full model



5.3 The credit deflation channel

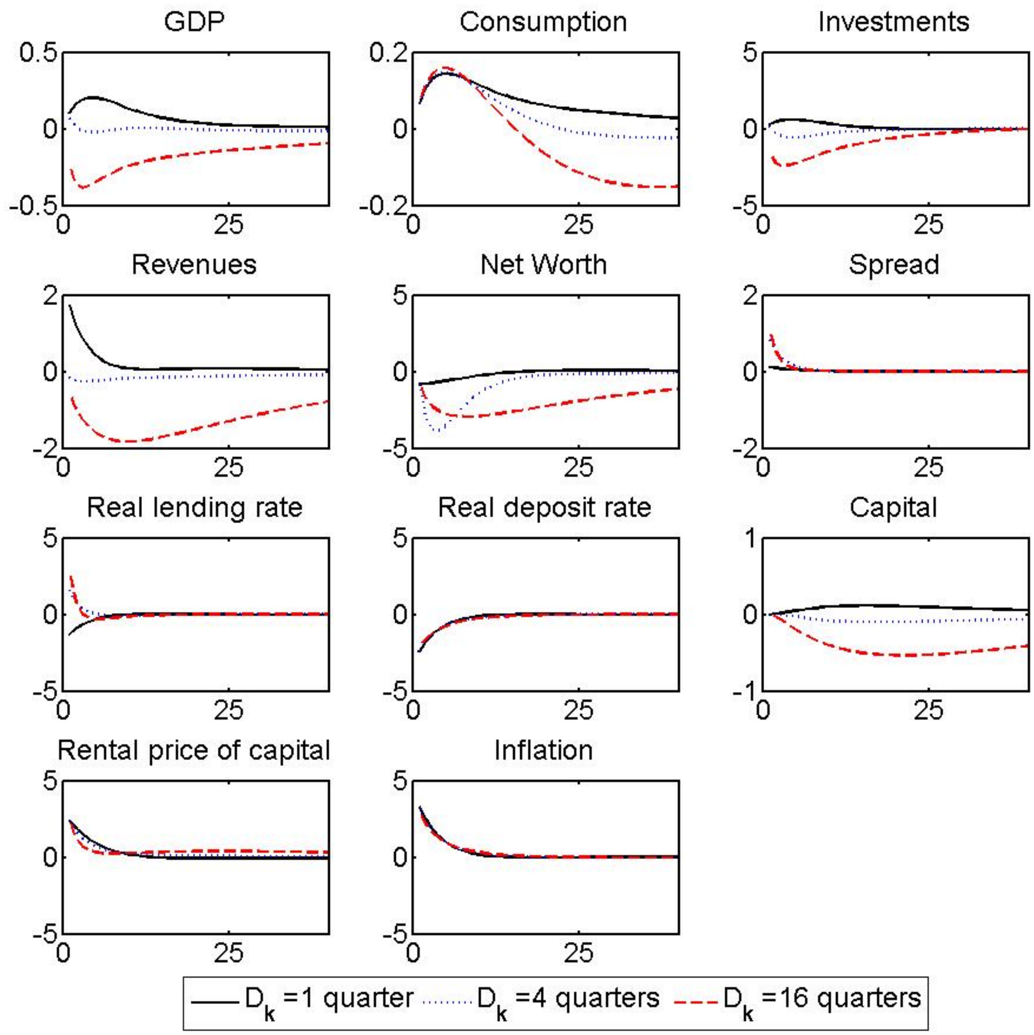
We isolate the effect of inflation on banks loans revenues (the 'credit-deflation' channel) by removing bonds from bank's balance-sheets, setting the policy response $\phi_\pi = 0$ and assuming one-period bonds only ($\rho = 0$), such that asset prices do not move. The debt-to-GDP ratio is irrelevant here since asset prices are fixed and banks hold no bonds, so we keep the value of 80% of GDP.

Figure 3 shows the impulse response to a fall of taxes of 1% of GDP for different scenarios

third possibility are Eusepi and Preston (2013), in which non-Ricardian behavior arises as a result of learning on expectations formation, and **?**, in which agents are non-Ricardian due to myopic planning horizon.

of loans average duration. These figures make clear the role played by the long-term structure of banks loans in driving our results: if loans are relatively short-term, the aggregate demand effect of the tax fall is more important than the negative effect on banks net-worth, generating a rise on output, consumption and investments. With an average maturity of four years, the negative effects are predominant and output falls by the same reasoning as stressed on the previous section.

Figure 3: Impulse response to a tax shock with different loan durations and no bonds



If the government wants to reduce inflation surprises, it can raise interest-rates. Figure 4 shows the impulse-response for the same shock keeping the loan duration at four years. We vary the maturity of bonds and let interest-rates responsive to inflation. We initially set $\phi_\pi = 0.2$ to compare to the interest-rate peg scenario. The thick line and dotted line shows the impulse response with only short-term government debt and different policy rules. From the figure, we see that increasing interest rates is counterproductive: recession is greater on impact. The reason is simple: with fiscal price-level determination, rising interest-rates rises inflation, not the opposite. With more surprise inflation and higher interest-rates, output falls more on impact. The figure is different if there are long-term bonds on the market, as the dashed line shows. In this case, the value of bonds fall and the government trades current and future inflation. As inflation is expected, banks can adjust loans more effectively reducing the negative effects of the surprise inflation: output and bank's net worth fall less, inflation is smaller and spreads are lower than the two other scenarios.

Figure 4: Impulse response to a tax shock, inflation postponement scenario

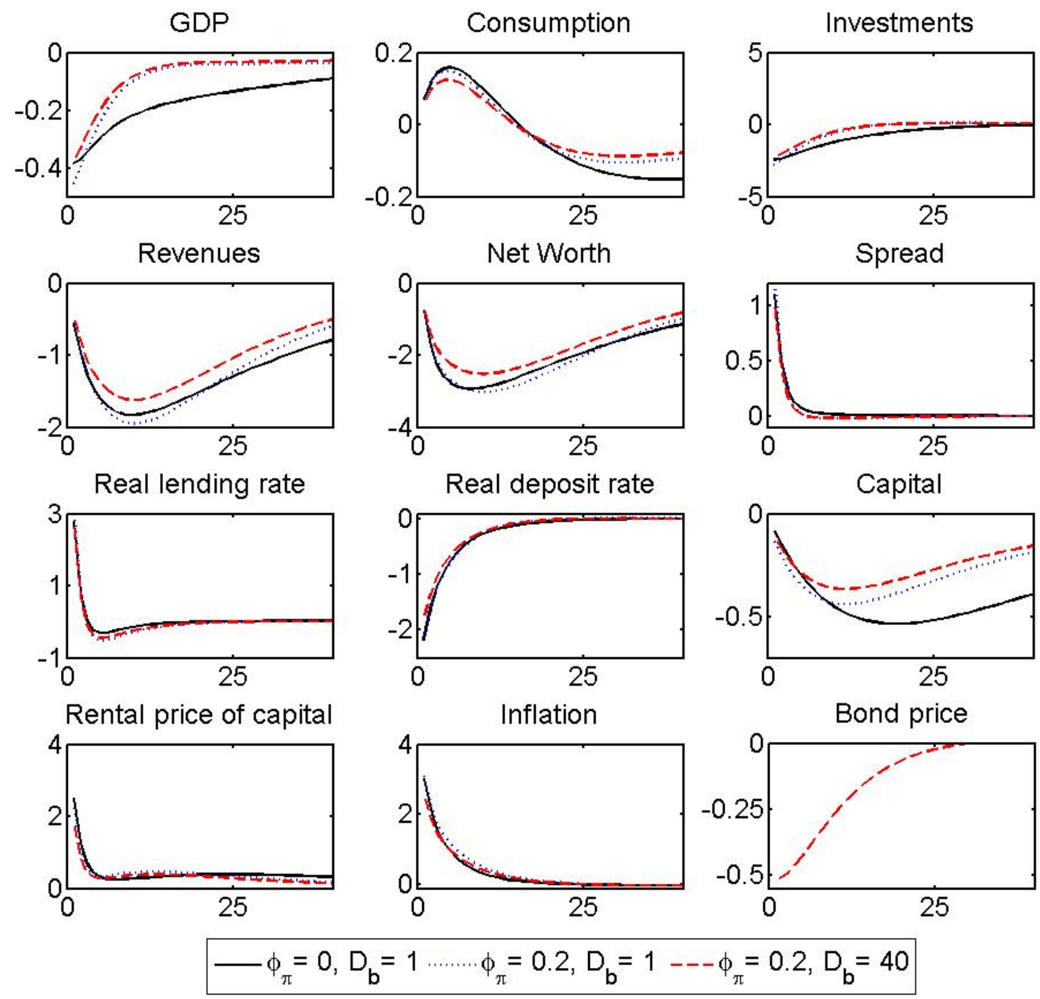
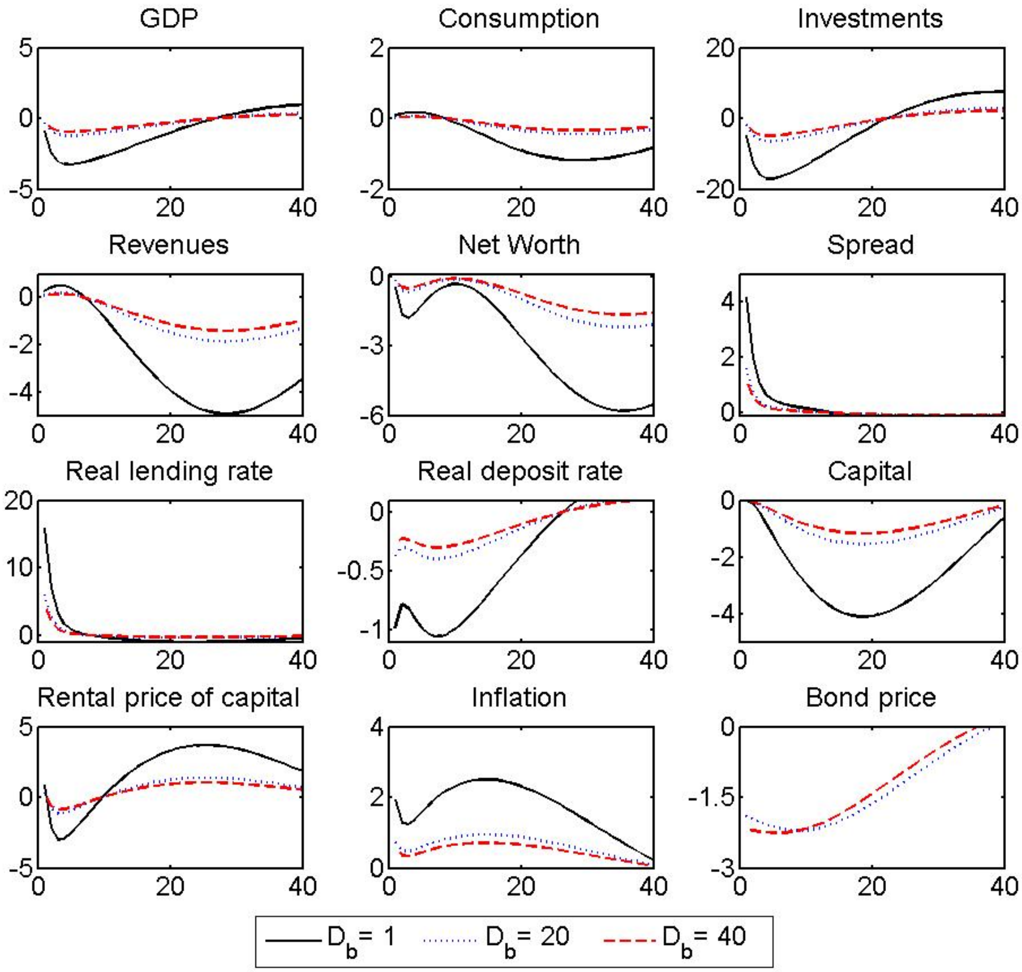


Figure 5 shows the same exercise supposing the government devote more effort to inflation postponement, which we capture by letting $\phi_\pi = 0.9$. In the thick line government bonds are one-period only, so all inflation come as a surprise. The dotted and dashed lines denotes scenarios of bonds average maturity of 4 and 10 years, respectively. This exercise makes clear the potential gains from inflation postponement: if the maturity structure is relatively long, the fall on net-worth, investments and output is greatly mitigated relative to the inflation surprise case.

However, a key hypothesis in this section is that banks do not hold bonds or more broadly, do not hold assets exposed to surprise revaluation. When they do, the fall on asset prices following the surplus shock negatively affect bank's net-worth and hence, capital financing. We explore this channel in the next section.

Figure 5: Impulse response to a tax shock inflation postponement, different bond maturities

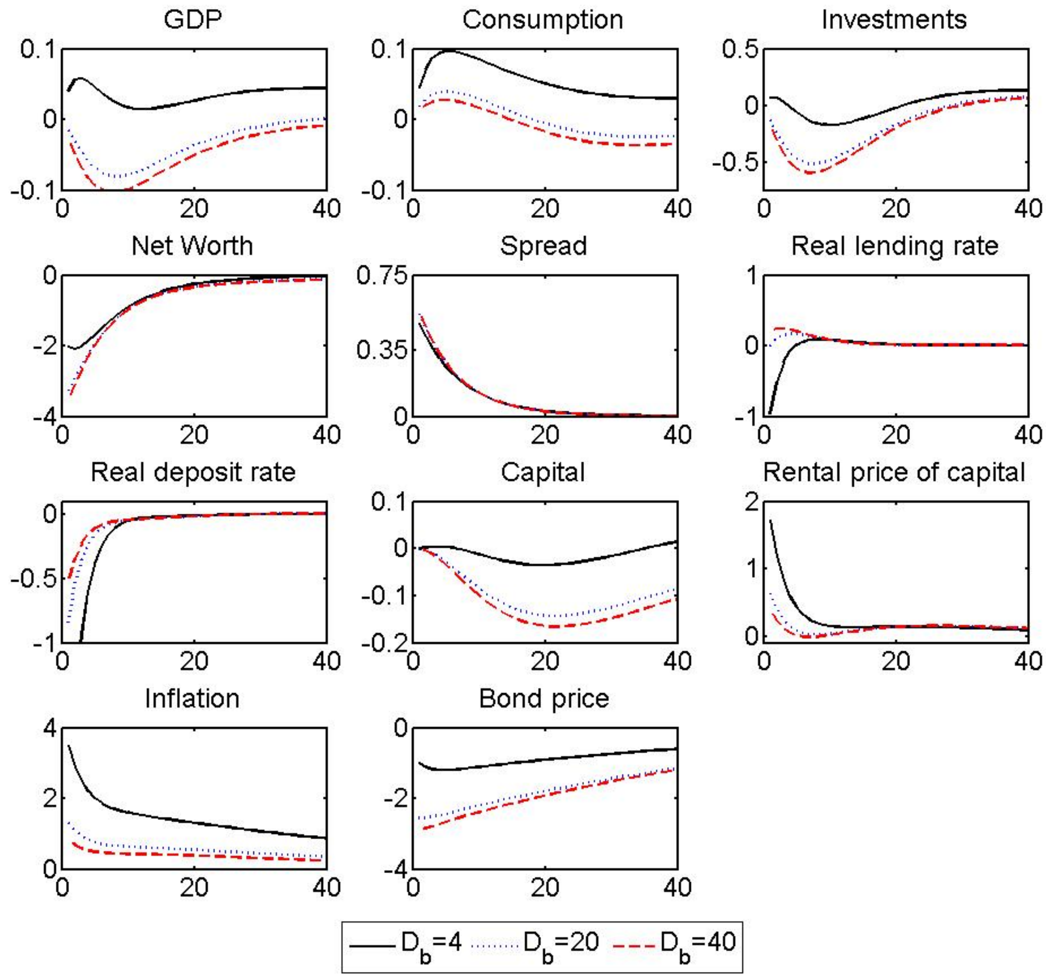


5.4 The asset price channel

In this section, we assume only short-term loans $\alpha_k = 0$ and an interest-rate response of $\phi_\pi = 0.9$. As before, we initially fix the debt-to-GDP ratio at 80%. For our model, this implies that bonds composes 28% of banks assets. We will discuss and test other values for this this parameter.

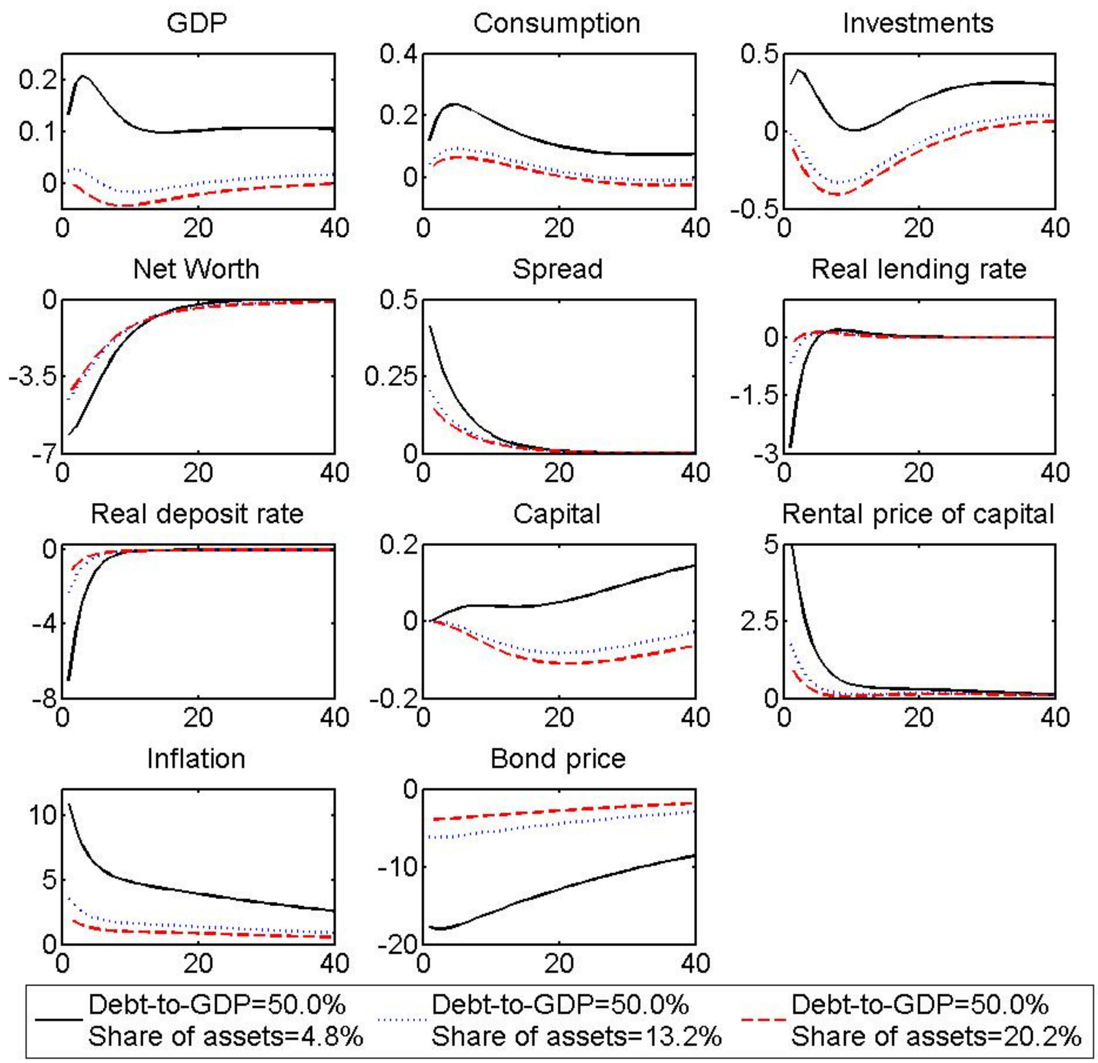
Figure 6 shows the impulse response for a fall on taxes of 1% of GDP for different bonds maturities. With bonds of short maturity, the aggregate demand effect predominates, raising output. There is a fall on net worth and rise in spreads, but insufficient to generate a rise in real lending rates and a fall on capital financing great enough to compensate for the demand effect. Inflation is higher due to the rise in output and the small response of bonds prices. The figure changes when government bonds increase in maturity: with five and ten year average maturity, the fall on net worth following the surprise revaluation of asset prices reduces capital financing and investments. Spreads and real lending rates rise, consistent with the fall on investments. Inflation is lower and smoother, but comes with a recession.

Figure 6: Impulse response to a tax shock, different bond maturities, no long-term loans



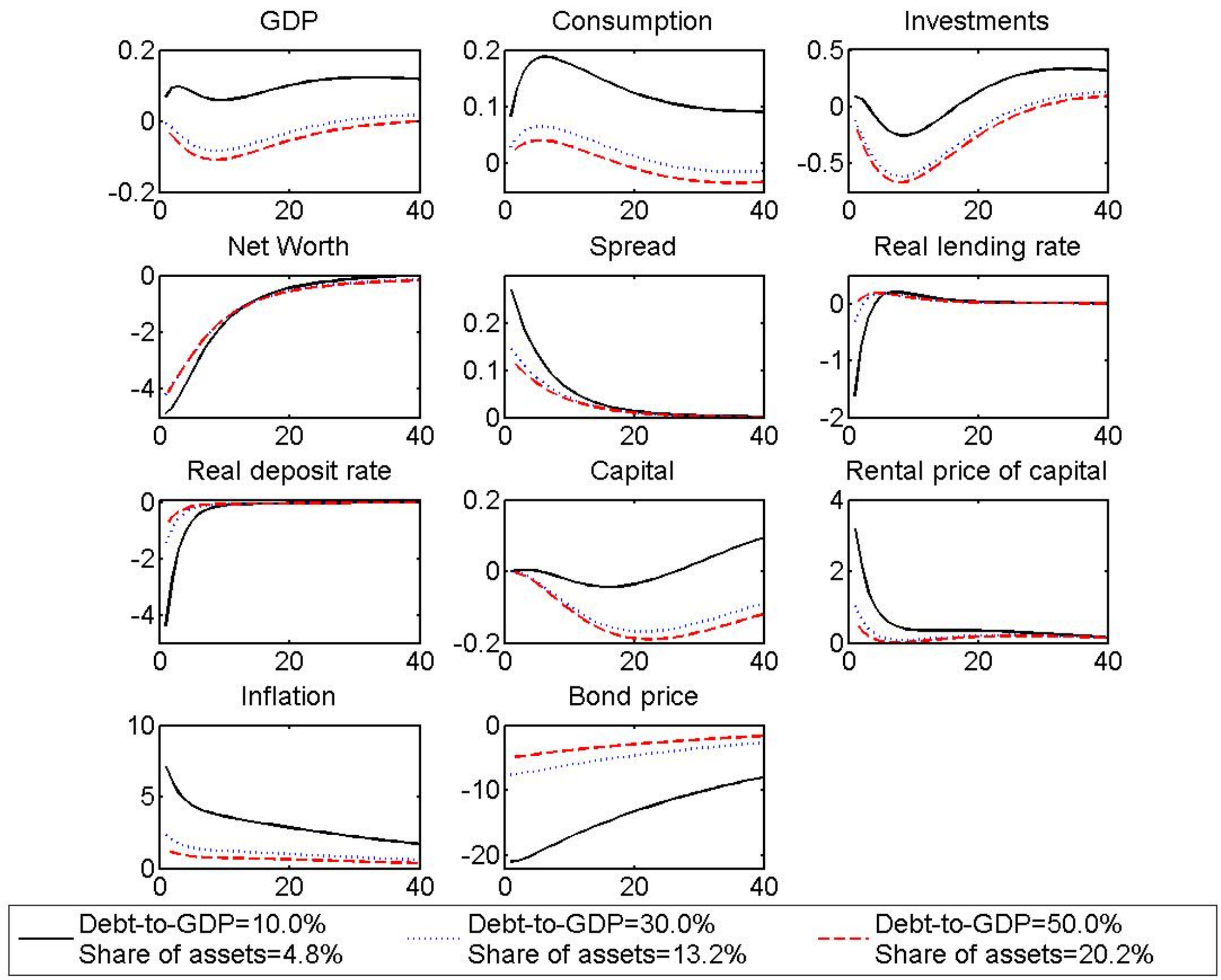
These results are sensitive to the debt-to-GDP ratio, or more precisely, to the fraction of bonds composing intermediaries assets. The average share of bank's holdings of government bonds is heterogeneous across countries. Gennaioli et al. (2014) find they compose on average 13% of bank's assets for emerging economies and are typically greater for countries that faced banking crises. Cao (2014) estimate such holdings to be approximately 10% for US banks at the end of 2008. In Japan, such holdings reached more than 20% of assets on 2013, but are smoothly being reduced as a result of efforts from Japanese policy makers. In Figure 7 and 8 we do the tax reduction exercise for different Debt-to-GDP scenarios, chosen to imply different fractions of bank's assets. We also consider average maturity of government bonds of five and ten years. As these figures show, the extent of the damage of bond prices to the financial sector depends on its share of banks assets and the sensitivity of prices to interest-rates. With a low fraction of assets, the damage in bank's balance-sheets is not sufficient to compensate for the aggregate demand effect coming from the fall on taxes. As the fractions and average maturity grows, the negative effects coming from the financial sector starts to predominate.

Figure 7: Impulse response to a tax shock different debt-to-GDP, no long-term loans, average bond duration of 5 years



Two points are worth noting. First, in our model we assumed that banks hold no other assets beyond bonds. If other assets with payment structure similar to those of government bonds were introduced (corporate bonds, for example), similar effects on banks net-worth would arise. For this reason, in the full model we decided for a higher calibration for the bonds share (28%) implied by the debt-to-GDP ratio of 80%. Second, in our model either banks or households hold government bonds. It is possible to make the two of them holding public debt at the same time, but we note

Figure 8: Impulse response to a tax shock different debt-to-GDP, no long-term loans, average bond duration of 10 years



that, in this case, the sensitivity of bonds prices tend to be higher, as arbitrage relations tends to be stronger. So, for the same maturity and share of banks assets, bonds prices will tend to have more effect on banks net-worth.[24]

6 Conclusion

We based our analysis on the Fiscal Theory of Price Level to study fiscal led inflationary processes. We showed that surprise inflation engendered by a deficit shock damages financial intermediation when intermediaries have a maturity mismatch on assets and liabilities. This reduce capital finance and raise spreads, leading the economy to stagflation. This helps explaining experiences of high debt, high inflation environments who suffered from banking crisis and also exposes potential

[24]A similar reasoning applies to the degree of agency friction on bonds, ι.

drawbacks of fiscal inflationary polices proposed to low inflation countries as a way of stimulus.

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A Appendix

A.1 Capital producers optimization problem

Capital producers objective is to choose v_t, I_t, K_t and $\tilde{K}_t$ to maximize expected flow of profits

$$\sum_{j=0}^{\infty} \frac{\lambda_{t+1}}{\lambda_t} \beta^j \left[v_{t+j} - (1-\delta) p_t^k K_{t-1+j} - I_{t+j} \right]$$

subject to

$$\begin{aligned} v_t &= (1-\theta_k) p_t^k \tilde{K}_t + \theta_k \frac{v_{t-1}}{\pi_t} \\ K_t &= (1-\delta) K_{t-1} + I_t - S(I_t/I_{t-1}) I_t \\ K_t &= (1-\theta_k) \tilde{K}_t + \theta_k K_{t-1} \end{aligned}$$

for each t. Associating multipliers $u_{1,t}, q_t, u_{2,t}$ for each restriction, first order conditions are given by:

$$\begin{aligned} \partial v_t &: 1 = u_{1t} - \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \theta_k u_{1t+1} \\ \partial I_t &: 1 = q_t \left(1 - S\left(\frac{I_t}{I_{t-1}} \right) - S'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} S'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \\ \partial K_t &: q_t - u_{2t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(q_{t+1}(1-\delta) - \theta_k u_{2t+1} - (1-\delta) p_{t+1}^k \right) \\ \partial \tilde{K}_t &: u_{1t} p_t^k = u_{2t} \end{aligned}$$

Here, q_t is the Tobin's q, representing the marginal value of changing the stock of capital to be used at $t+1$. It differs from the price of installed capital p_t^k, because of infrequent capital adjustments, but both coincide at steady-state.

A.2 Model summary

Equilibrium in the model can be computed from the following set of equations.

Households

$$(C_t - hC_{t-1})^{-1} = \lambda_t + h\beta(E_t C_{t+1} - C_t)^{-1} \tag{A.1}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{R_t^d}{\pi_{t+1}} \tag{A.2}$$

$$H_t^{\psi} = w_t \lambda_t \tag{A.3}$$

Wholesale firms

$$r_t^k = p_t^w \alpha Y_t / K_{t-1} \tag{A.4}$$

$$w_t = p_t^w (1-\alpha) Y_t / H_t \tag{A.5}$$

$$Y_t = H_t^{1-\alpha} K_{t-1}^{\alpha} \tag{A.6}$$

Entrepreneurs

$$1 = u_{1t} - \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \theta_k u_{1t+1} \tag{A.7}$$

$$1 = q_t \left(1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \tag{A.8}$$

$$q_t - u_{2t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left(q_{t+1}(1-\delta) - \theta_k u_{2t+1} - (1-\delta) p_{t+1}^k\right) \tag{A.9}$$

$$u_{1t} p_t^k = u_{2t} \tag{A.10}$$

Capital producers

$$1 = u_{1t} - \beta \frac{\lambda_{t+1}}{\lambda_t} \theta_k u_{1t+1} \tag{A.11}$$

$$1 = q_t \left(1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right) + \beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \tag{A.12}$$

$$q_t - u_{2t} = \beta \frac{\lambda_{t+1}}{\lambda_t}\left(q_{t+1}(1-\delta) - \theta_k u_{2t+1} - (1-\delta) p_{t+1}^k\right) \tag{A.13}$$

$$u_{1t} p_t^k = u_{2t} \tag{A.14}$$

$$K_t = (1-\delta) K_{t-1} + I_t - S\left(\frac{I_t}{I_{t-1}}\right) I_t \tag{A.15}$$

Financial intermediaries

$$\nu_t^k = E_t \Omega_{t+1} \frac{rev_t / l_t}{\pi_{t+1}} \tag{A.16}$$

$$\nu_t^b = E_t \Omega_{t+1} \frac{R_{t+1}^b}{\pi_{t+1}} \tag{A.17}$$

$$\mu_t = E_t \Omega_{t+1} \frac{R_t^d}{\pi_{t+1}} \tag{A.18}$$

$$\Omega_{t+1} = E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\beta}{(1-\chi)} \left[(1-\theta_b) + \theta_b \left[(lev_{t+1}^k + \iota lev_{t+1}^b)(\nu_{t+1}^k - \mu_{t+1}) + \mu_{t+1}\right]\right] \tag{A.19}$$

$$\nu_t^b - \mu_t = \varsigma \iota (\nu_t^k - \mu_t) \tag{A.20}$$

$$lev_t^k + \iota lev_t^b = \frac{\mu_t}{\gamma - (\nu_t^k - \mu_t)} \tag{A.21}$$

$$lev_t^k = \frac{l_t}{n_t} \tag{A.22}$$

$$lev_t^b = \frac{Q_t b_t}{n_t} \tag{A.23}$$

$$n_t = \frac{1-\chi}{\pi_t}\left[rev_{t-1} + \varsigma R_t^b Q_{t-1} b_{t-1} - R_{t-1}^d d_{t-1}\right] \tag{A.24}$$

$$rev_t = (1-\theta_k) R_t^l p_t^k \tilde{K}_t + \theta_k rev_{t-1}/\pi_t \tag{A.25}$$

$$l_t = (1-\theta_k) p_t^k \tilde{K}_t + \theta_k l_{t-1}/\pi_t \tag{A.26}$$

$$l_t + Q_t b_t = d_t + n_t \tag{A.27}$$

Retailers

$$\tilde{p}_t = \frac{\eta}{\eta-1}\frac{x_{1t}}{x_{2t}} \tag{A.28}$$

$$x_{1t} = \lambda_t y_t p_t^w + \beta\theta_p (\pi_{t+1})^{\eta} x_{1t+1} \tag{A.29}$$

$$x_{2t} = \lambda_t y_t + \beta\theta_p (\pi_{t+1})^{\eta-1} x_{2t+1} \tag{A.30}$$

$$\pi_t^{1-\eta} = (1-\theta_p)(\tilde{p}_t \pi_t)^{1-\eta} + \theta_p \tag{A.31}$$

Government and Market clearing

$$R_t^b = \frac{1+\rho Q_t}{Q_{t-1}} \tag{A.32}$$

$$G_t + Q_t b_t = \tau_t + \frac{1+\rho Q_t}{\pi_t} b_{t-1} \tag{A.33}$$

$$\log(R_t^d) = \log(1/\beta) + \rho_r \log(R_{t-1}) + (1-\rho_r)(\phi_\pi \log(\pi_t)) \tag{A.34}$$

$$\log(\tau_t) = \log(\tau) + \phi_\tau \log(Q_{t-1} b_{t-1}) + \epsilon_t \tag{A.35}$$

$$Y_t = (C_t + G_t + I_t)\diameter_t \tag{A.36}$$

$$\diameter_t = (1-\theta_p)\tilde{p}_t^{-\eta} + \theta_p \pi_t^{\eta} \diameter_{t-1} \tag{A.37}$$

Exogenous process

$$\log(G_t) = (1-\rho_g)\log(G) + \rho_g \log(G_{t-1}) + \varepsilon_t^G \tag{A.38}$$

$$\epsilon_t = \rho_\tau \epsilon_{t-1} + \varepsilon_t^\tau \tag{A.39}$$