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Abstract

Ordinary Least Squares (OLS) estimation of monetary policy rules produces potentially inconsistent estimates of policy parameters. The reason is that central banks react to variables, such as inflation and the output gap, that are endogenous to monetary policy shocks. Endogeneity implies a correlation between regressors and the error term – hence, an asymptotic bias. In principle, Instrumental Variables (IV) estimation can solve this endogeneity problem. In practice, however, IV estimation poses challenges, as the validity of potential instruments depends on various unobserved features of the economic environment. We argue in favor of OLS estimation of monetary policy rules. To that end, we show analytically in the three-equation New Keynesian model that the asymptotic OLS bias is proportional to the fraction of the variance of regressors due to monetary policy shocks. Using Monte Carlo simulations, we then show that this relationship also holds in a quantitative model of the U.S. economy. Since monetary policy shocks explain only a small fraction of the variance of regressors typically included in monetary policy rules, the endogeneity bias tends to be small. For realistic sample sizes, OLS outperforms IV. Finally, we estimate a standard Taylor rule on different subsamples of U.S. data and find that OLS and IV estimates are quite similar.

JEL classification codes: E52, E58, E50, E47

Keywords: Taylor rule, OLS, GMM, endogeneity bias, weak instruments, New Keynesian models

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1 Introduction

The macroeconomics literature frequently summarizes a central bank's reaction function with an interest rate rule, such as the ones introduced in Taylor (1993, 1999). Such policy rules serve as good representations of how the monetary authority adjusts its policy instrument (typically a short term interest rate) in response to deviations of inflation and/or economic conditions (output or unemployment, for example) from their objectives.

Estimation of a central bank's reaction function poses some challenges, however. Ordinary Least Squares (OLS) estimation of monetary policy rules produces potentially inconsistent estimates of policy parameters. This is so because central banks react to variables that are endogenous to monetary policy shocks. Endogeneity implies a correlation between regressors and the error term – hence, an asymptotic bias. In principle, estimation by Instrumental Variables (IV) or Generalized Method of Moments (GMM) can solve this endogeneity problem (*e.g.*, Clarida, Galí and Gertler, 2000). In practice, however, finding suitable instruments can be challenging, as their validity depends on details of the economic environment. For example, persistent monetary shocks tend to invalidate the common practice in the literature of using lagged endogenous variables as instruments.

In this paper, we argue in favor of OLS estimation of policy rules. To do so, we first show analytically in a three-equation New Keynesian (NK) model that the asymptotic OLS estimation bias is proportional to the fraction of the variance of regressors due to monetary policy shocks. Since there is ample evidence that such shocks explain only a small fraction of the variance of regressors typically included in estimation of monetary policy rules (*e.g.*, Leeper, Sims and Zha, 1996, Christiano, Eichenbaum and Evans, 1999), our analytical finding suggests that the endogeneity bias is likely to be small. To illustrate the properties of single-equation estimates of policy parameters in finite samples, we resort to Monte Carlo simulations and explore different parameterizations of the basic NK model. More specifically, we generate artificial data from economies for which we know the "true" policy parameters, and compare them to single-equation OLS and GMM estimates.

We then quantify estimation biases using Monte Carlo simulations of the Smets and Wouters (2007) model as a laboratory. Our results suggest that endogeneity does induce some bias in the estimation of interest rate rules by OLS. For empirically relevant sample sizes, however, OLS estimates outperform GMM estimates. OLS biases are close to those obtained with GMM, but the estimates are more precise. More importantly, when we look at the economic implications of estimation biases, we find them to be unimportant, in the sense that replacing the true policy rule in the model with the one estimated by OLS does not materially change the dynamics of the model. The impulse response functions (IRFs) produced by the model under the policy rule estimated by single-equation OLS are close to the true IRFs. In addition, the range of IRFs produced by the model under the various policy rules estimated by OLS in the Monte Carlo exercise is narrower than the corresponding range obtained when using policy rules estimated by GMM. In a sense, this result suggests an "economic irrelevance" of the OLS estimation bias.[1]

Finally, we perform an empirical analysis by estimating an interest rate rule by OLS and GMM using U.S. data. In particular, we build on Clarida, Galí and Gertler (2000) and estimate an interest rate rule for subsamples corresponding to different Federal Reserve chairs. We find that OLS and GMM estimated coefficients and the associated IRFs – estimated with the local projection method proposed by Jordà (2005) – are quite close to each other.

The literature on Taylor rule estimation is quite large, covering debates about whether monetary policy in the U.S. has changed over time in terms of satisfying the Taylor principle (*e.g.*, Taylor, 1999, Judd and Rudebusch, 1998, Clarida, Galí and Gertler, 2000, Orphanides, 2004), and whether persistence in interest rates stems from policy inertia or persistent monetary shocks (*e.g.*, Rudebusch, 2002, and Coibion and Gorodnichenko, 2012), among others.

Our paper does not focus on a particular issue pertaining to Taylor rules, but, rather, sheds light on the costs and benefits of estimation by OLS or IV. Hence, our contribution is closer to papers that focus on issues related to estimation of Taylor rules. Cochrane (2011) argues that Taylor rule parameters are not identified in the baseline NK model. Sims (2008) shows that Cochrane (2011)'s finding is not a generic implication of NK models, but is rather the result of a particular assumption regarding the policy rule. He shows that, under assumptions usually made in the literature, policy parameters are identified. Closest to our paper, de Vries and Li (2013) investigate the magnitude of the IV estimation bias when monetary shocks are serially correlated and lags of inflation and output gap are endogenous to monetary shocks, and thus, are not valid instruments. They find that the endogeneity problem due to serial correlation does not lead to a large bias in the conventional estimation of Taylor rules based on the three-equation NK model. We focus on OLS estimation, and use IV estimation only as a comparison. We show analytically, in the canonical NK model, and by simulation in a larger model, that the OLS bias depends on the fraction of the variance of endogenous regressors explained by monetary policy shocks. Because this fraction tend to be small, OLS bias need not be a material problem. Finally, our argument can be loosely related to "identification through heteroskedasticity" (Rigobon, 2003). That identification strategy explores time-variation in the relative volatility of structural shocks, whereas our argument hinges of the fact that the structural shock that shifts the macroeconomic equation of interest explains only a small fraction of the endogenous variables used as regressors.

The paper is organized as follows. Section 2 derives analytically the OLS bias for the policy rule parameter in a three-equation NK model. It also presents Monte Carlo analyses of the bias in that simple model. Section 3 quantifies estimations biases under OLS and IV methods using simulated data from the Smets and Wouters (2007) model. Section 4 compares the performance of OLS and IV empirically. Section 5 concludes.

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[1]We thank the referee for suggesting this interpretation of our findings.

2 OLS bias in the three-equation New Keynesian model

We are interested in the estimation of interest rate rules such as the ones described in Taylor (1993) and Taylor (1999). According to such rules, policy interest rates respond to various macroeconomic variables. Estimation of policy rules may, however, lead to inconsistent estimates, as the Taylor rule equation typically involves endogenous variables that are determined in macroeconomic equilibrium. This section uses basic versions of the three-equation NK model to illustrate some of the challenges involved in single-equation estimation of Taylor rules. We first explore the simplest version of the model and derive analytical results on the asymptotic OLS bias. We then use simulations of a slightly more involved version of the model to develop additional intuition on the estimation challenges posed by endogeneity, as well as to illustrate how the biases behave under realistic sample sizes.

2.1 Analytical results

We begin our analysis with the simplest possible three-equation NK model, described in Galí (2008, Chapter 3). Equilibrium inflation, output, and the policy interest rate evolve as functions of technology and monetary shocks. The simplicity of the model allows us to obtain an analytical expression for the asymptotic bias of the OLS estimator of the Taylor rule parameter and develop intuition on the main point of the paper.

The model consists of: (i) a Phillips curve, equation (1), that relates inflation, π_t, to the current output gap, $\tilde{y}_t$, and to expected inflation $E_t(\pi_{t+1})$; (ii) a dynamic IS curve, equation (2), that relates the output gap to the expected output gap $E_t(\tilde{y}_{t+1})$ and to the gap between the ex-ante real interest rate, $i_t - E_t(\pi_{t+1})$, and the natural rate of interest, r_t^n; and (iii) a simplified policy rule, equation (3), that relates the nominal interest rate, i_t, to inflation and includes a monetary shock, v_t. The natural interest rate is determined by the dynamics of output in the model's flexible-price equilibrium, which is a function of the technology shock, a_t. Technology and monetary shocks follow autoregressive processes. Appendix A provides additional details of the model, which we summarize below using standard notation:

$$\pi_t = \beta E_t\left(\pi_{t+1}\right) + \kappa \tilde{y}_t, \tag{1}$$

$$\tilde{y}_t = E_t\left(\tilde{y}_{t+1}\right) - \frac{1}{\sigma}\left(i_t - E_t\left(\pi_{t+1}\right) - r_t^n\right), \tag{2}$$

$$i_t = \phi_\pi \pi_t + v_t, \tag{3}$$

with $a_t = \rho_a a_{t-1} + \epsilon_t^a$, $v_t = \rho_v v_{t-1} + \epsilon_t^v$.

The policy parameter of interest is ϕ_π. Assuming the Taylor principle holds ($\phi_\pi > 1$), the unique bounded solution of this model can be obtained by the method of undetermined coefficients – details in Appendix A.4. In equilibrium, inflation is given by

$$\pi_t = -\kappa \Lambda_v v_t - \sigma \psi_{ya}^n (1-\rho_a) \kappa \Lambda_a a_t, \tag{4}$$

where $\Lambda_j = \frac{1}{(1-\beta\rho_j)\sigma(1-\rho_j)+\kappa(\phi_\pi-\rho_j)}$ for $j = v, a$, and ψ_{ya}^n is another function of the model's structural parameters (see Appendix A.4).

From this solution, it follows that the asymptotic bias of the single-equation OLS estimate of ϕ_π (denoted by $\hat{\phi}_\pi^{OLS}$) is given by:

$$plim\ \hat{\phi}_\pi^{OLS} - \phi_\pi = -\frac{1}{\kappa \Lambda_v} \gamma_v, \tag{5}$$

where $plim$ denotes probability limit and

$$\gamma_v = \frac{(\kappa \Lambda_v)^2 var(v_t)}{(\kappa \Lambda_v)^2 var(v_t) + \left(\sigma \psi_{ya}^n (1-\rho_a) \kappa \Lambda_a\right)^2 var(a_t)}$$

is the fraction of the variance of inflation that is due to monetary policy shocks.

Equation (5) shows that the asymptotic OLS bias is proportional to the fraction of the variance of π_t due to monetary shocks. The composite parameter Λ_v is positive and, hence, the asymptotic bias is negative. The economic intuition behind this result is simple. An expansionary monetary policy shock – *i.e.*, a negative innovation to v_t – increases inflation (see equation (4)) and this leads to an endogenous policy response according to the policy parameter ϕ_π. Because the policy shock and the endogenous policy response go in opposite directions, in equilibrium the interest rate appears to respond less intensely to movements in π_t – hence, the downward bias in $\hat{\phi}_\pi^{OLS}$.[2]

The key insight produced by this illustrative model is that the OLS bias depends on the fraction of the variance of inflation that is due to monetary shocks. If these shocks explain only a small fraction of variation of inflation and other macroeconomic variables to which central banks respond, then the OLS bias may be small. Whether or not this is the case depends also on other structural parameters of the economy, as illustrated by equation (5).

The simple NK model allows additional analytical insight into the nature of the OLS bias. For example, one can study how the strength of the policy response to inflation affects the bias. There are two – possibly opposite – effects, which can be seen from equations (4) and (5). On the one hand, a stronger policy response to inflation (*i.e.*, a higher ϕ_π) diminishes its

[2]That the sign of the OLS bias is necessarily negative depends on the assumption that $\phi_\pi > 1$, and hence that the model has a unique bounded equilibrium. If this is not the case, one can find equilibria in which the OLS bias is positive. This is possible because multiplicity allows for equilibria in which the dynamics of the economy change considerably relative to the region with a determinate equilibrium. Hence, the nature of the OLS bias may also change. Future research on the properties of OLS estimates of the Taylor rule in the presence of multiplicity may contribute to the literature that aims to distinguish between policy rules that ensure determinacy and those that allow for multiple equilibria (*e.g.*, Clarida, Galí and Gertler, 2000).

sensitivity to shocks, by reducing the two composite parameters Λ_v and Λ_a in equation (4). On the other hand, the share of the variance of inflation that is due to monetary policy shocks (γ_v) may increase or decrease, depending on the relative effect of a stronger policy response on those two composite parameters. Under the plausible assumption that monetary shocks are less persistent than technology shocks ($\rho_v < \rho_a$), one can show that the OLS bias increases in absolute value with the strength of the policy response to inflation (*i.e.*, the bias becomes more negative as ϕ_π increases; proof in Appendix A.6).[3]

To sidestep endogeneity problems with OLS, a common strategy is to estimate the Taylor rule by instrumental variables – usually by Generalized Method of Moments (GMM) – with lagged endogenous variables as instruments. The matrix representation of the simple model above helps in the understanding of that strategy:

$$X_t = A\Pi(A'A)^{-1}AX_{t-1} + A\epsilon_t, \tag{6}$$

where $X_t = \begin{pmatrix} y_t \\ \pi_t \end{pmatrix}$, $A = \begin{pmatrix} \psi_{\pi v} & -\psi_{\pi a}\sigma\psi^n_{ya}(1-\rho_a) \\ \psi_{yv} & -\psi_{ya}\sigma\psi^n_{ya}(1-\rho_a) \end{pmatrix}$, $\Pi = \begin{pmatrix} \rho_v & 0 \\ 0 & \rho_a \end{pmatrix}$, $\epsilon_t = \begin{pmatrix} \epsilon^v_t \\ \epsilon^a_t \end{pmatrix}$, and the $\psi_{\bullet\bullet}$ parameters in A are functions of the model's structural parameters (see Appendix A.4).

In this simple model, the rank condition for identification is satisfied if and only if $\rho_v \neq 0$ and $\rho_a \neq 0$, as the determinant of the matrix $M = A\Pi(A'A)^{-1}A$ equals $\rho_v\rho_a$. Therefore, identification requires some persistence in the shocks. In this simple environment, this creates a problem for the standard practice in the literature of using lagged endogenous variables as instruments. The reason is that, in general, when shocks are persistent, instruments and shocks may be correlated, hampering the asymptotic properties of GMM estimators. In this simple model with two shocks, this will be the case whenever the monetary shock is persistent, which turns out to be a requirement for the rank condition to be satisfied.[4]

In addition to the issues raised above, the rank condition is not sufficient for reliable estimation and inference, as estimates may still suffer from weak instrument problems. Stock, Wright and Yogo (2002) show that weak instruments lead to poor parameter identification and asymptotic results become a poor guide to the actual sampling distributions.[5]

In short, single-equation estimation of interest rate rules poses a few challenges. On the one

[3]We thank our referee for raising this conjecture and encouraging us to analyze it.

[4]With additional shocks, the rank condition may be verified even if monetary shocks are *i.i.d.*, so lagged endogenous variables may be valid instruments (see Section 2.2).

[5]The strength of identification implied by a given set of instruments can be assessed using a concentration parameter (*e.g.*, Mavroeidis, 2005 and Stock and Yogo, 2005), which measures the instruments' signal-to-noise ratio. More precisely, the concentration parameter is a measure of the variation of the endogenous regressors that is explained by the instrumental variables after controlling for any exogenous regressors, relative to the variance of the residuals of the first-stage regression in a two-stage least squared approach. For the simple model above, the first-stage regression, as described in equation (6), implies a concentration parameter $\Sigma^{1/2}\Pi' Z_t Z_t' \Pi\Sigma^{1/2}$, where $\Sigma^{1/2}$ is the covariance matrix of the vector of errors from the first-stage regression, and Z_t contains the instruments X_{t-1}. The strength of the instruments can be measured by the smallest eigenvalue of that matrix.

hand, if one uses OLS to estimate Taylor rule parameters, equation (5) illustrates a natural source of bias, due to endogeneity. On the other hand, when using GMM one should worry about instrument validity and strength. In particular, the standard practice of using lagged endogenous variables as instruments may result in unreliable GMM estimates if the monetary policy shock displays some persistence, and theoretical asymptotic results may be a poor guide to the actual sampling distributions.

2.2 Simulation analysis

In order to complement the analytical insights produced by the simple model and to assess the behavior of OLS and GMM in small samples, we turn to simulations of the three-equation NK model. To that end, we augment the simple model described in equations (1)-(3) in two dimensions. First, we add an unanticipated and possibly persistent "cost-push" shock, $u_t = \rho_u u_{t-1} + \varepsilon_t^u$, which enters the Phillips curve additively. Adding a third shock to the model allows us to parameterize it so the GMM rank condition is satisfied without the need for persistent monetary policy shocks. Second, we allow for the possibility of policy smoothing by including a first-order autoregressive term in the Taylor rule, with coefficient ρ_i: $i_t = \rho_i i_{t-1} + (1-\rho_i)\phi_\pi \pi_t + v_t$. This allows us to study the OLS bias when interest rate persistence stems from policy smoothing rather than persistent monetary policy shocks, as argued by Coibion and Gorodnichenko (2012). The resulting augmented model is described in Appendix A.3.

To calibrate this simple model, we set its structural parameters to standard values used in the literature. The relative scales of nonpolicy (technology and cost-push) shocks do not affect the OLS bias, so we fix them at arbitrary values. We then use estimates from Smets and Wouters (2007) to select the scale of monetary policy shocks to be compatible with the variance of inflation in equilibrium (details in Appendix A.7). Unless stated otherwise, in all simulations presented below we discipline the scale of monetary shocks as just described.

As discussed before, the persistence of shocks in the model matters for identification when relying on GMM with lagged dependent variables as instruments. Hence, before we turn to the comparison between OLS and GMM, we assess identification under different levels of shock persistence. Note that the augmented model includes three shocks instead of two, and therefore the conditions for GMM identification in this model differ from those discussed in the previous section. In the extended version with three shocks, the requirement for identification is that at least two of the shocks exhibit some persistence. Therefore, to leave identification problems aside and fairly compare results obtained by OLS and GMM, in what follows we set $\rho_a = \rho_u = 0.8$, and entertain a range of values for ρ_v.

To compare OLS and GMM estimators of Taylor rule coefficients, we simulate economies over 80 quarters.[6] For GMM, we use a *k-step* estimator, which is frequently used in the empirical

[6] Lessons based on model simulations with samples of 150 or 500 observations are essentially unchanged.

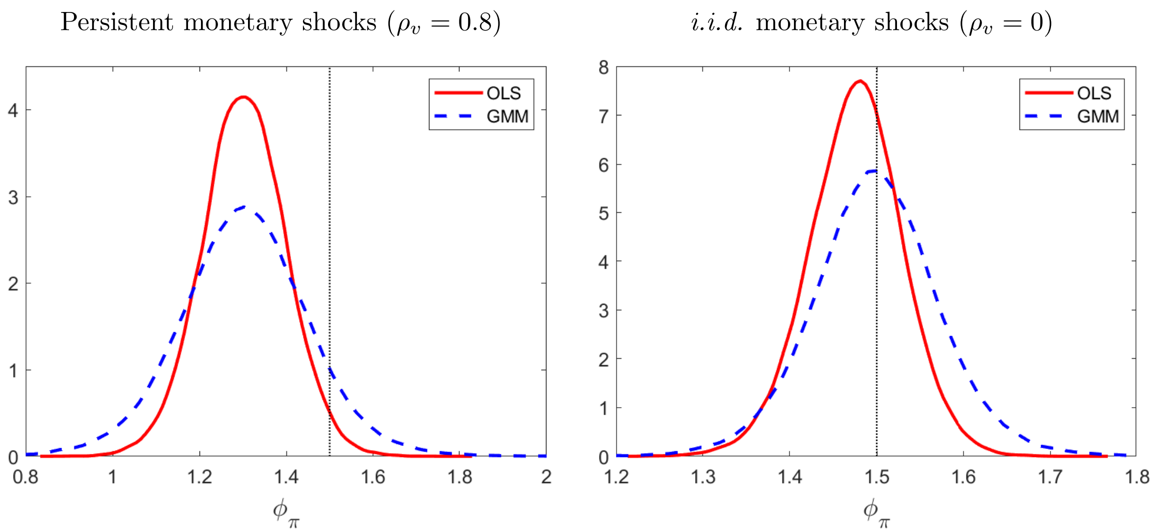

Figure 1: Distributions of OLS and GMM estimates for different levels of monetary shock persistence.

Note: OLS distributions in red (solid) lines, GMM distributions in blue (dashed) lines. Vertical black dotted line corresponds to the true parameter value. Distributions obtained from 50,000 Monte Carlo simulations of the model with sample size $T = 80$.

literature (*e.g.*, Clarida, Galí and Gertler, 2000).[7] We exclude simulations that fail Sargan-Hansen's test for the validity of overidentified restrictions (Sargan, 1958). This is an attempt to favor GMM, which we view as "conservative" given our findings that OLS is preferable. We use two lags of inflation and output gap as instruments, and rely on Newey-West standard errors.[8] Results are based on 50,000 simulations, with Gaussian innovations.

Initially, we assume away policy smoothing ($\rho_i = 0$). The first set of results are reported in Figure 1, which displays kernel densities of the estimated Taylor rule coefficients by OLS (solid red line) and GMM (dashed blue line). The left panel reports results when the model is calibrated with persistent monetary shocks ($\rho_v = 0.8$), while the right panel reports results with *i.i.d.* monetary shocks. Vertical black dotted lines indicate the true parameter value ($\phi_\pi = 1.5$).

The left panel of Figure 1 shows that, when $\rho_v = 0.8$, OLS and GMM estimates have virtually identical central tendencies, around $\hat{\phi}_\pi = 1.3$. The bias in GMM estimates in this case is not all that surprising. As mentioned before, when monetary shocks are persistent, lagged dependent variables are correlated with the shock, and therefore are not suitable instruments. As pointed out by de Vries and Li (2013), the literature on estimation of monetary policy rules

Results available upon request.

[7] Alternatively, we considered the continuous updating estimator proposed by Hansen, Heaton and Yaron (1996). The method resulted in too many "extreme" estimates. As discussed in that paper, this may be the case because the continuous-updating criterion can make the numerical search for the minimizer difficult.

[8] The Newey-West estimator allows population moment conditions to be serially correlated. We use Bartlett kernel function with two lags. While numerical results may differ depending on the choice of kernel and the bandwidth parameter, they are qualitatively unchanged when we consider additional lags.

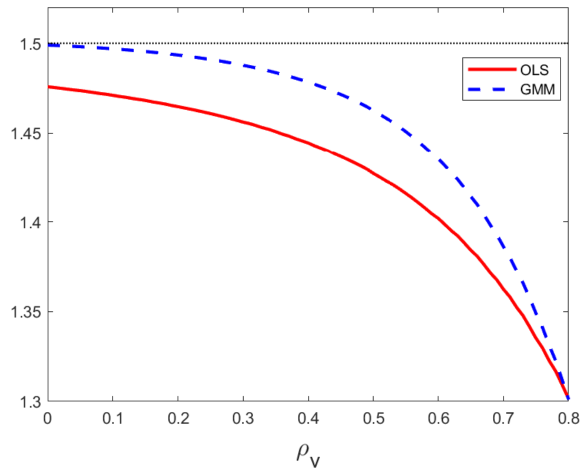

Figure 2: Mean OLS and GMM point estimates for ranging levels of monetary shock persistence.

Note: OLS estimates in red (solid) line, GMM estimates in blue (dashed) line. Mean point estimates obtained from 50,000 Monte Carlo simulations of the model with sample size $T = 80$.

often ignores this specific endogeneity problem, resulting in GMM estimates that are potentially as biased as OLS'.[9]

The right panel of Figure 1 reinforces this assessment by reporting results when monetary shocks are *i.i.d.* (*i.e.*, $\rho_v = 0$). In this case, GMM is unbiased. OLS does a slightly worse job in terms of bias, but mean point estimates under both methods are quite close to each other. Note that the set of instruments used in the estimation is the same in both panels – two lags of inflation and output gap. When $\rho_v = 0$, however, these are valid instruments. Finally, in both panels, the distribution of OLS estimates is less dispersed than GMM's.[10]

Figure 2 complements Figure 1 by reporting mean OLS (solid red line) and GMM (dashed blue line) point estimates of ϕ_π for ranging levels of monetary shock persistence (ρ_v). When shock persistence is not too high, GMM and OLS mean point estimates are quite close to each other and close to the true parameter value ($\phi_\pi = 1.5$). As the persistence of monetary policy shock increases, GMM and OLS mean point estimates approach each other.

We now turn to the analytical insight on how the OLS bias varies with the intensity of monetary policy's response to inflation. Figure 3 shows the distributions of OLS and GMM estimates of the policy parameter for two different values for ϕ_π, assuming *i.i.d.* monetary policy shocks.[11] When $\phi_\pi = 4$ (left panel), endogeneity poses a bigger problem for OLS. In

[9]Following a suggestion by our referee, we also analyzed GMM performance when the true structural technology and cost-push shocks are used as instruments. This is the ideal setting for GMM, when the econometrician has as instruments the true exogenous drivers of the endogenous variables that enter the Taylor Rule. As expected, in that case GMM performs very well.

[10]Appendix Table A1 reports additional moments and statistics.

[11]In these simulations we hold the scale of monetary policy shocks (σ_v) constant at the value used in the baseline parameterization.

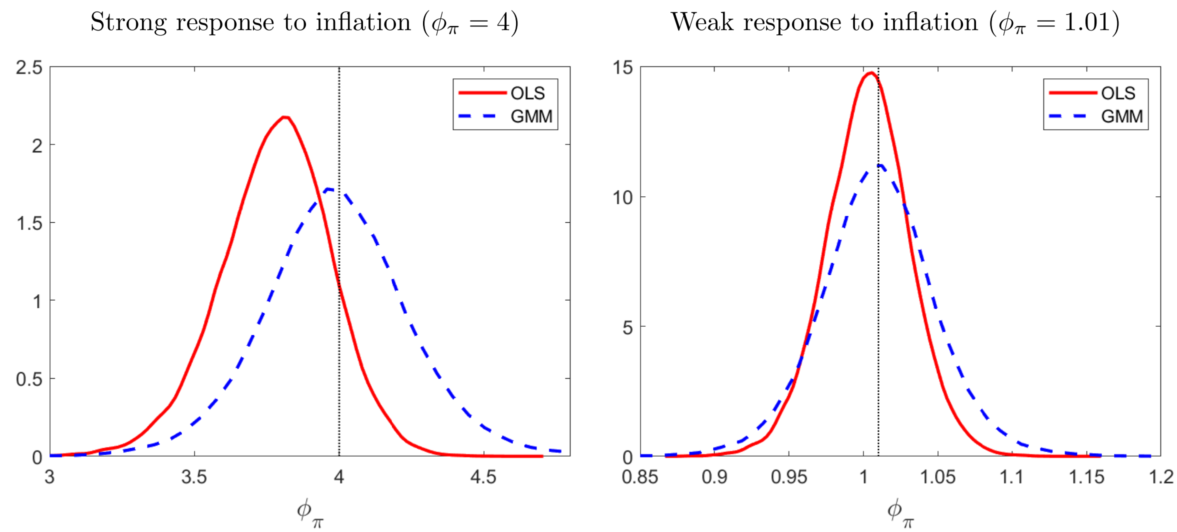

Figure 3: Distributions of OLS and GMM estimates for different policy responses to inflation.

Note: OLS estimates in red (solid) lines, GMM estimates in blue (dashed) lines. Vertical black dotted line corresponds to the true parameter value. Distributions are obtained from 50,000 Monte Carlo simulations of the model assuming a with sample size $T = 80$.

turn, the right panel shows that the bias almost disappears when we set the policy parameter close to the limit imposed by the Taylor principle ($\phi_\pi = 1.01$).

If one is interested in testing whether monetary policy satisfies the Taylor principle ($\phi_\pi > 1$), results in Figures 1 and 3 illustrate that biases in OLS and GMM estimates are unlikely to pose a problem if the policy response to inflation is strong enough. For example, when $\phi_\pi = 1.5$ (Figure 1) or $\phi_\pi = 4$ (Figure 3, left panel), nearly all estimates exceed unit. In other words, one is quite unlikely to reject the Taylor principle in these cases. When the policy response to inflation is weak ($\phi_\pi = 1.01$, right panel of Figure 3), however, both GMM and OLS wrongly reject the Taylor principle more than half of the time.[12]

Finally, we illustrate how OLS and GMM biases behave when persistence in the Taylor rule stems from policy smoothing rather than persistent policy shocks. This case is not covered by the analytical results presented in Section 2.1. For this exercise, we set $\rho_i = 0.8$ and assume *i.i.d.* monetary policy shocks. Mirroring Figure 1, Figure 4 displays kernel densities of the estimated Taylor rule coefficients by OLS (solid red line) and GMM (dashed blue line). The left panel reports estimates of the policy smoothing parameter (ρ_i), while the right panel presents estimates of the coefficient on inflation ($(1-\rho_i)\phi_\pi$). Vertical black dotted lines indicate the true parameter values ($\rho_i = 0.8$, $(1-\rho_i)\phi_\pi = 0.3$). As in the case without policy smoothing, GMM outperforms OLS in terms of bias when monetary policy shocks are *i.i.d.* (Figure 1, right panel).[13]

[12] The frequencies of undue rejections of the Taylor principle are obtained by computing the area under the kernel densities to the left of unit.

[13] Appendix Table A2 reports additional moments and statistics.

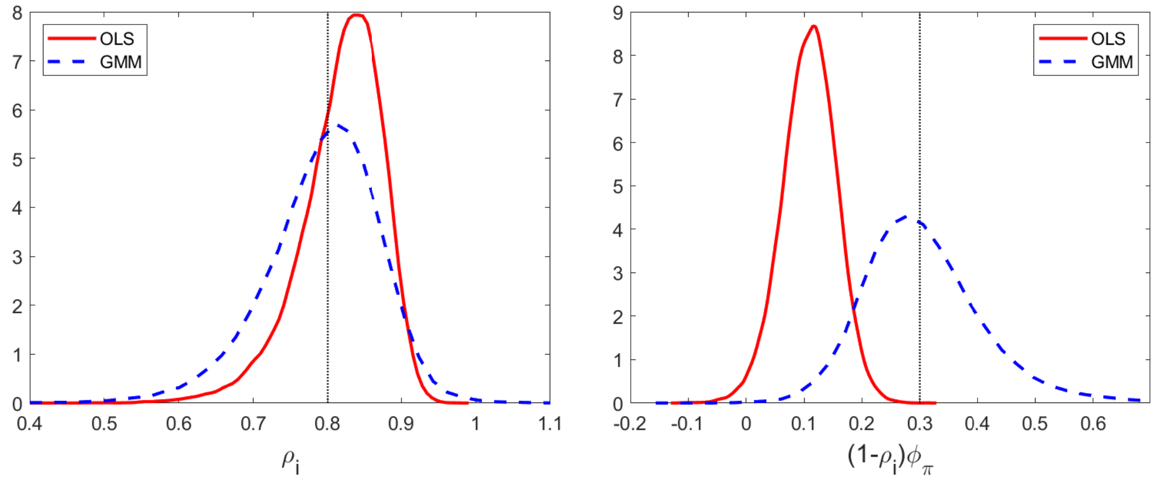

Figure 4: Distributions of OLS and GMM estimates with interest rate smoothing ($\rho_i = 0.8$) and *i.i.d.* monetary shocks.

Note: GMM estimates in blue (dashed) line, OLS estimates in red (solid) line. Vertical black dotted line corresponds to the true parameter value. Distributions are obtained from 50,000 Monte Carlo simulations of the model assuming a with sample size $T = 80$.

In conclusion, while the simple New Keynesian model studied in Section 2.1 allows interesting analytical insights, the simulations presented in this section illustrate that the relative merits of OLS versus GMM may depend on various features of the economic environment. Hence, assessing their relative performance in quantitative terms requires a richer model. In the next section, we pursue this route and study OLS and GMM performance using the Smets and Wouters (2007) model as a laboratory.

3 Assessing estimation bias with a quantitative DSGE model

The three-equation NK models considered in Section 2 are very simple. In more quantitative representations of the macroeconomy, there are at least a few endogenous state variables and the economy is driven by a larger set of disturbances.

A (log-linearized) medium-scale NK model can be cast in the form:[14]

$$\Gamma_0 y_t = C + \Gamma_1 y_{t-1} + \Psi z_t + \Pi \eta_t,$$

where y_t is a vector of state variables, C is a vector of constants, z_t is vector of exogenous variables, and η_t is a vector of expectational errors, satisfying $E_t[\eta_{t+1}] = 0$ for all t. The interest rate is included in the vector y_t, while monetary shock innovations are included in the

[14] We follow the notation in Sims (2002).

vector z_t.

The solution of the linear rational expectation model can be cast in the following reduced form:

$$y_t = A_c(\theta) + A_y(\theta)y_{t-1} + A_z(\theta)z_t, \tag{7}$$

i.e., a vector autoregressive representation, where θ collects the model's structural parameters. The solution described in equation (7) shows that all endogenous variables may be affected by monetary shocks, raising concerns of endogeneity biases if one estimates the interest rate rule coefficients by OLS. Furthermore, as discussed before, when monetary shocks are persistent, lagged endogenous variables may be correlated with monetary shocks as well, and hence they may be invalid instruments for a single-equation GMM estimation approach. The set of valid instruments will depend on details of the model.

To study the performance of OLS and GMM under a richer data generating process, we rely on the workhorse model of Smets and Wouters (2007), who estimate a fully-specified medium-scale DSGE model for the U.S. economy that includes several types of real and nominal frictions and a good number of structural shocks. The model is estimated using a Bayesian approach and is used to investigate the relative importance of its various frictions and shocks for the U.S. business cycle.[15]

We proceed as in Section 2.2, and generate artificial data using the Smets and Wouters (2007) model, with parameters at the mode of the joint posterior distribution estimated by the authors. We generate $50,000$ simulations of the model over $T = 80$ quarters (results with $T = 150$ in Appendix B) with Gaussian innovations, and use model-simulated data to estimate the model's policy rule, which is given by:

$$r_t = \rho r_{t-1} + (1-\rho)\left[\phi_\pi \pi_t + r_y(y_t - y_t^p)\right] + r_\Delta\left[(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)\right] + \varepsilon_t^r, \tag{8}$$

where the monetary shock, ε_t^r, follows an autoregressive process ($\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r$).

The policy rule described in equation (8) follows the same notation as in Smets and Wouters (2007). It allows for both persistent shocks and policy inertia, thus speaking to the debate on which of these two factors helps explain the persistence in policy rates (*e.g.*, Rudebusch, 2002; Coibion and Gorodnichenko, 2012). The posterior mode values estimated by Smets and Wouters (2007) imply a relatively high degree of policy inertia ($\rho = 0.81$), a strong policy response to inflation ($\phi_\pi = 2.03$), some response to the output gap ($r_y = 0.08$) and to output-gap growth ($r_\Delta = 0.22$), a low degree of monetary shock persistence ($\rho_r = 0.12$), and relatively small monetary shocks (standard deviation of monetary policy innovations $\sigma_r = 0.24$).

The low degree of persistence in monetary policy shocks is beneficial for GMM estimation with lagged regressors as instruments. Nevertheless, to give GMM its best chance of outper-

[15] Iskrev (2010) finds that this model is locally identified, which allows for consistent estimation of structural parameters. Local identification also guarantees the usual asymptotic properties of estimators.

forming OLS, we refrain from using lagged regressors as instruments, and use current values and three lags of marginal cost and wages instead.[16] As in our analysis of three-equation model, we rely on Newey-West standard errors. Finally, we discard samples that yield "extreme" GMM estimates, keeping estimates only when the model is not rejected by Sargan-Hansen's test for the validity of overidentification restrictions (Sargan, 1958).

Note that equation (8) can be rewritten as:

$$r_t = \theta_1 r_{t-1} + \theta_2 \pi_t + \theta_3 \tilde{y}_t + \theta_4 \tilde{y}_{t-1} + \epsilon_t^r, \tag{9}$$

where $\theta_1 = \rho$, $\theta_2 = (1-\rho)\phi_\pi$, $\theta_3 = (1-\rho)r_y + r_{\Delta y}$, and $\theta_4 = -r_{\Delta y}$. $\tilde{y}_t = y_t - y_t^p$ is the output gap. Hence, in practice, we estimate equation (9) and recover estimates of structural parameters from estimates of reduced-form parameters $\{\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4\}$.

With parameters at the estimated posterior mode, the Smets and Wouters (2007) model implies that monetary shocks account for 4.88% and 2.73% of the variance of inflation and output, respectively. Our analytical results suggest this should lead to only a small endogeneity bias, and thus single-equation OLS estimation should perform well.

Figure 5 reports the distribution of parameters estimated by OLS (solid red line) and GMM (dashed blue line). The horizontal dotted black lines report the true Taylor rule parameters. Similarly to the findings based on the three-equation model, the central tendencies of OLS and GMM estimates are close to one another and, importantly, somewhat close to the true parameter values. In addition, the dispersion in estimated coefficients is much larger for GMM than for OLS. These findings also hold when we consider larger samples (see Appendix Figure A2).

Our analytical results of Section 2.1 suggest the OLS endogeneity bias should be related to the fraction of the variance of Taylor rule regressors that is due to monetary shocks. To assess whether this result also holds under a richer data generating process, Figure 6 shows how OLS and GMM mean point estimates behave as we vary monetary shock volatility (σ_r) between 0.05 and 0.7. Dotted vertical lines in Figure 6 report the "true" (estimated posterior mode) volatility of monetary policy shocks, $\sigma_r = 0.24$, while dotted horizontal lines report the "true" Taylor rule parameters. In line with our analysis of Section 2, OLS estimates become more biased as σ_r increases. GMM estimates also become more biased as σ_r increases, but less so.

So far we have focused on estimates of individual parameters. This is useful and informative if the focus is on their specific estimated values. Frequently, however, one is interested in the dynamics of the model as a whole, which depend jointly on all estimated parameters. These dynamics are commonly summarized by impulse response functions (IRFs) that yield the path

[16] We also considered other sets of instruments, such as lags of output and inflation, but those were less favorable to GMM.

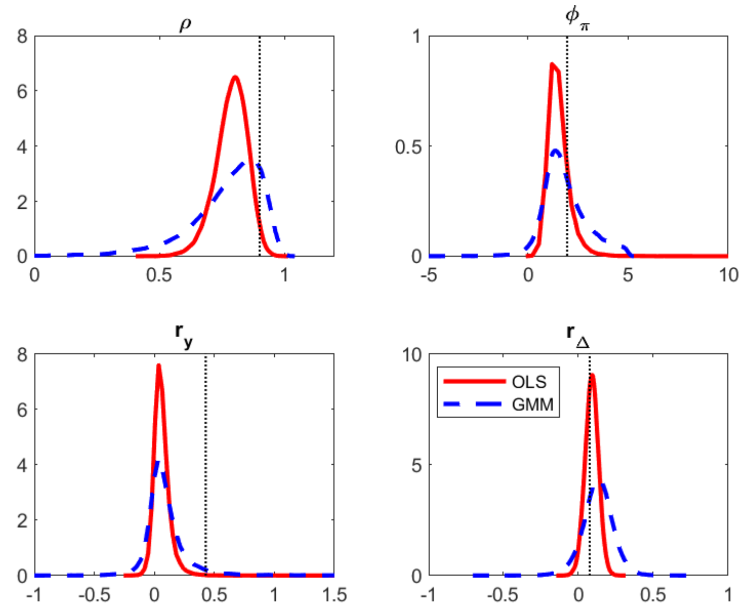

Figure 5: Distributions of OLS and GMM estimates, Smets and Wouters (2007) model.

Note: OLS distributions in red (solid) lines, GMM distributions in blue (dashed) lines. Vertical black dotted lines correspond to the true parameter values. Simulations are based on the Smets and Wouters (2007) model with parameters set at their posterior modes. Distributions obtained from 50,000 Monte Carlo simulations of the model with sample size $T = 80$.

followed by endogenous variables in response to an unexpected shock. Hence, we now report the IRFs that obtain when we replace the model's true Taylor rule parameters with estimates obtained through single-equation estimation methods.

Figure 7 reports IRFs of output and inflation to a monetary policy shock. The black lines are the "true" IRFs of output and inflation – *i.e.*, the responses that obtain with the true Taylor rule parameters. The figure also reports the responses obtained when the Taylor rule is parameterized using mean OLS (red line) and GMM (blue line) point estimates.[17] Finally, the panels in Figure 7 include shaded areas that report, for each time horizon, the range between the 5^{th} and 95^{th} percentiles of the distribution of IRFs produced by our Monte Carlo exercise. Each IRF underlying this range is obtained by plugging into the DSGE model a Taylor rule estimated through single-equation methods on one of the artificial samples. Repeating this procedure for all artificial samples gives rise to the distribution underlying the reported IRF range.

As shown in Figure 7, IRFs obtained under Taylor rules estimated by both OLS and GMM are close to the true IRFs. In other words, OLS endogeneity bias need not induce meaningful biases in the dynamics implied by the model. Shaded areas in Figure 7 reinforce the higher precision of OLS estimates. While mean point estimates from OLS and GMM yield IRFs that

[17]Appendix Table A3 reports mean OLS and GMM point estimates, as well as true parameter values, used to construct the exhibits in Figure 7. Results for a larger sample size are reported in Appendix Figure A3.

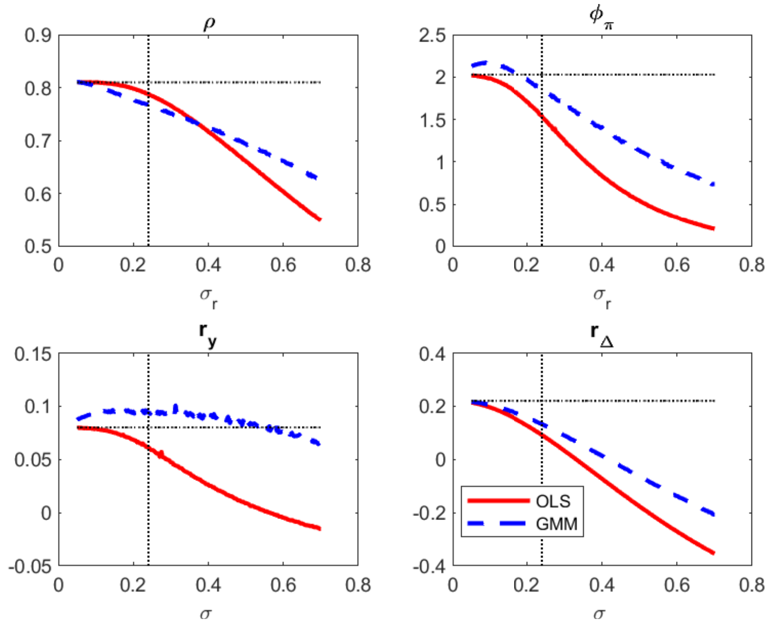

Figure 6: Mean OLS and GMM point estimates for varying monetary shock volatility, Smets and Wouters (2007) model.

Note: OLS estimates in red (solid) line, GMM estimates in blue (dashed) line. Vertical dotted black lines correspond to the calibrated standard deviation of the monetary shock, while horizontal dotted black line reports the "true" Taylor rule parameters. Simulations are based on the Smets and Wouters (2007) model with all other parameters set at their posterior modes. Mean point estimates obtained from 50,000 Monte Carlo simulations of the model with sample size $T = 80$.

are close to one another and to the true one, narrower shaded areas in the left-hand-side panels show that OLS yields more precise estimates than GMM.

In sum, using the Smets and Wouters (2007) model as a laboratory, we find the OLS estimation bias to be small. More importantly, OLS estimates imply model dynamics that are remarkably close to the true ones, with higher precision than dynamics implied by GMM estimates.

4 An empirical analysis

In the previous section we relied on a quantitative DSGE model as a laboratory to quantify and study the endogeneity bias produced by different single-equation estimation methods. One key advantage of that approach is that, in such an exercise, we know the data-generating processes, the true parameters, and the interest rate rule specification to compare our estimates to.

Having made the case for OLS estimation using known data-generating processes, it is only natural that we take our findings to actual data. We compare OLS and IV estimates of an interest rate rule similar to the one analyzed in Clarida, Galí and Gertler (2000).

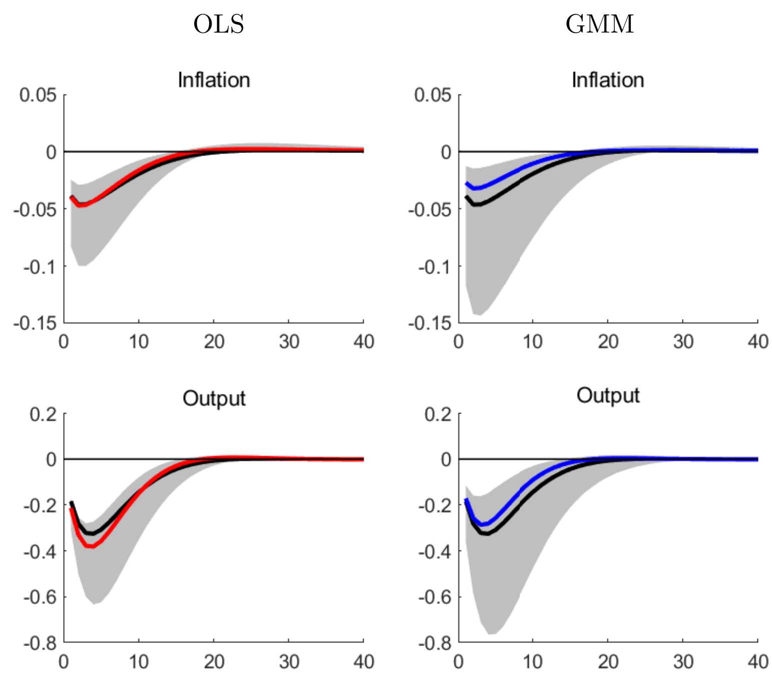

Figure 7: Output and inflation responses to monetary shock in Smets and Wouters (2007) model for different Taylor rule parameters.

Note: Black lines are IRFs implied by the Smets and Wouters (2007) model with parameters at the estimated posterior mode. Red (blue) lines are IRFs with mean OLS (GMM) estimates of the Taylor rule parameters. Shaded areas cover, for each point in time, range between 5^{th} and 95^{th} percentiles of the distribution of IRFs implied by the different OLS (or GMM) point estimates of the Taylor rule. Remaining model parameters fixed at the posterior mode estimated by Smets and Wouters (2007). Estimates obtained from 50,000 Monte Carlo simulations of the model with sample size $T = 80$.

Using instrumental variables, Clarida, Galí and Gertler (2000) estimate:

$$E\{[r_t - \rho_1 r_{t-1} - \rho_2 r_{t-2} - (1 - \rho_1 - \rho_2)(\beta\pi_{t+1} + \gamma x_t + rr^* + (1 - \beta)\pi^*)]z_t\} = 0, \tag{10}$$

where r_t is the fed funds rate, rr^* is the equilibrium real interest rate, π^* is the inflation target, π_{t+1} denotes the percentage change in the price level between t and $t + 1$, and x_t denotes the average output gap at t. The set of instruments z_t includes four lags of inflation, output gap, the fed funds rate, money growth, the spread between long and short term bond rates, and commodity price inflation. For additional details, we refer the reader to Clarida, Galí and Gertler (2000).

Because our goal is to compare OLS and IV estimates, we consider an interest rate rule with current inflation, instead of the forward-looking specification in equation (10). In particular, we estimate:

$$r_t = \alpha_{aux} + \rho_{1,aux} r_{t-1} + \rho_{2,aux} r_{t-2} + \beta_{aux}\pi_t + \gamma_{aux} x_t + \epsilon_t, \tag{11}$$

to obtain $\hat{\alpha}_{aux}$, $\hat{\rho}_{1,aux}$, $\hat{\rho}_{2,aux}$, $\hat{\beta}_{aux}$, and $\hat{\gamma}_{aux}$, from which we can back out $\hat{\rho} = \hat{\rho}_{1,aux} + \hat{\rho}_{2,aux}$,

$\hat{\gamma} = \frac{\hat{\gamma}_{aux}}{1-\hat{\rho}}$, $\hat{\beta} = \frac{\hat{\beta}_{aux}}{1-\hat{\rho}}$, and $\hat{\pi}^* = \frac{\hat{\alpha}_{aux}-(1-\hat{\rho})\hat{rr}^*}{(1-\hat{\rho})(1-\hat{\beta})}$.[18] To estimate the equilibrium real rate, we follow Clarida, Galí and Gertler (2000) and set $\hat{rr}^*$ to the sample average of the ex-post real rate, $r_t - \pi_{t+1}$, for each subsample.

We use real-time quarterly data, from 1960Q1 to 2007Q4.[19] Interest rate is the federal funds rate, inflation is the year-on-year rate of change in core PCE, output gap is constructed using the Congressional Budget Office estimate of potential GDP, money growth is the percentage change in M2, commodity prices are a composite of commodity goods (including oil, gold and food items), and long- and short-term bond yields are the 10-year and 3-month Treasury bill rates, respectively. We rely on the same set of instruments as Clarida, Galí and Gertler (2000).

We consider four subsamples:[20] (1) "Pre-Volcker" from 1960Q1 to 1979Q2, (2) "Volcker-Greenspan" from 1979Q3 to 2005Q4, (3) "Greenspan-Bernanke" from 1987Q3 to 2007Q4, and (4) "Post-Volcker" from 1979Q3 to 2007Q4. Because real-time data on core PCE inflation and M2 growth are only available starting in 1959Q2 and we use four lags as instruments, our first subsample has 77 data points.

Table 1 reports the results. The top panel shows IV estimates and the bottom panel reports OLS estimates. While there are some differences in estimated coefficients, the two panels show that, in general, OLS and IV estimates are close to one another and that the former tends to be more precise.[21]

To further assess the properties of OLS and IV estimates, we consider three approaches. First, we take the residuals from the regressions above and estimate local projections as introduced by Jordà (2005) to obtain impulse response functions of inflation and output to estimated OLS and IV residuals. Next, we undertake an alternative estimation in which we use Greenbook forecasts as regressors. This strategy was introduced by Romer and Romer (2004) in order to address endogeneity and forward-looking Federal Reserve behavior. It was later used by Coibion and Gorodnichenko (2011) to study time-variation in the Taylor rule. Finally, we directly assess some of the properties of the estimated OLS and IV residuals by comparing them to estimates of structural monetary policy shocks available in the literature.

[18] We took a similar two-step approach in Section 3. Alternatively, one could directly estimate a nonlinear regression similar to equation (10). Results for the latter approach yield the same conclusions and are available upon request.

[19] Sample vintages are reported in the Appendix Table A4. Results based on the latest vintage of data are qualitatively similar and are available upon request.

[20] In their benchmark specification, Clarida, Galí and Gertler (2000) consider two sample periods: "Pre-Volcker", which runs from 1960Q2 to 1979Q2, and "Volcker-Greenspan". which runs from 1979Q3 to 1996Q4.

[21] The estimates reported in columns (1) and (2) of Table 1 do not exactly replicate the findings reported in Clarida, Galí and Gertler (2000, Table II). The reasons for this are fourfold. First, we estimate an interest rate rule with current, instead of expected inflation. Second, we rely on core PCE as a measure of inflation, while those authors use, alternatively, GDP deflator or CPI. Third, we expand the Volcker-Greenspan sample up to the last quarter of Greenspan's term, instead of stopping that subsample in 1996Q4 (the latest data point used by those authors). Fourth, we use real-time data (real-time core CPI data yields similar results as real-time core PCE data). When we use the same subsamples, dataset, and interest rate rule specification, our results nearly perfectly match those of Clarida, Galí and Gertler (2000). We thank the authors for kindly sharing their data.

Table 1: IV and OLS estimates

	IV estimates			
	Pre-Volcker 1960Q1-1979Q2	Volcker-Greenspan 1979Q3-2005Q4	Greenspan-Bernanke 1987Q3-2007Q4	Post-Volcker 1979Q3-2007Q4
β	0.86***	1.97***	1.40***	1.97***
	(0.22)	(0.41)	(0.32)	(0.43)
γ	0.75**	0.65***	0.95***	0.70***
	(0.32)	(0.21)	(0.20)	(0.24)
ρ	0.81***	0.63***	0.83***	0.64***
	(0.10)	(0.11)	(0.05)	(0.10)
π^*	4.24	2.86***	1.42	2.75**
	(5.28)	(1.05)	(2.04)	(1.08)
N	77	106	82	114
R^2	0.89	0.86	0.96	0.86
RMSE	0.82	1.42	0.41	1.38
	OLS estimates			
	Pre-Volcker 1960Q1-1979Q2	Volcker-Greenspan 1979Q3-2005Q4	Greenspan-Bernanke 1987Q3-2007Q4	Post-Volcker 1979Q3-2007Q4
β	0.90***	1.99***	1.39***	2.00***
	(0.22)	(0.30)	(0.29)	(0.31)
γ	0.79***	0.75***	1.01***	0.81***
	(0.29)	(0.15)	(0.19)	(0.16)
ρ	0.80***	0.55***	0.82***	0.56***
	(0.08)	(0.12)	(0.04)	(0.12)
π^*	4.34	2.75***	1.29	2.63***
	(7.14)	(0.80)	(1.92)	(0.81)
N	77	106	82	114
R^2	0.89	0.87	0.96	0.87
RMSE	0.84	1.42	0.42	1.37

Note: The table reports estimates of equation (11) by OLS and IV. The set of instruments includes four lags of inflation, output gap, the fed funds rate, money growth (M2), the spread between long and short term bond rates, and commodity price inflation. Statistical significance at the 90/95/99% confidence level indicated with */**/***, respectively. Data vintages are reported in the Appendix Table A4. Robust standard errors are reported in parenthesis.

Turning to our first approach, we resort to local projections, as introduced by Jordà (2005), and estimate:

$$\begin{aligned} \pi_{t+h} &= \mu_{\pi,h} + \delta_{\pi,h}\hat{\epsilon}_{m,t} + \lambda_{\pi,i}\mathbf{c}_{t-i}, \\ x_{t+h} &= \mu_{x,h} + \delta_{y,h}\hat{\epsilon}_{m,t} + \lambda_{x,i}\mathbf{c}_{t-i}, \end{aligned} \tag{12}$$

where $\hat{\epsilon}_{m,t}$ are the residuals from the OLS and IV regressions from Table 1, *i.e.*, $m = \{OLS, IV\}$. $\mathbf{c}_{t-i}$ is a matrix of control variables that includes four lags ($i = 1, ..., 4$) of r_t, π_t, and x_t. The impulse responses of inflation and output are given by the estimated coefficients $\delta_{\pi,h}$ and $\delta_{x,h}$,

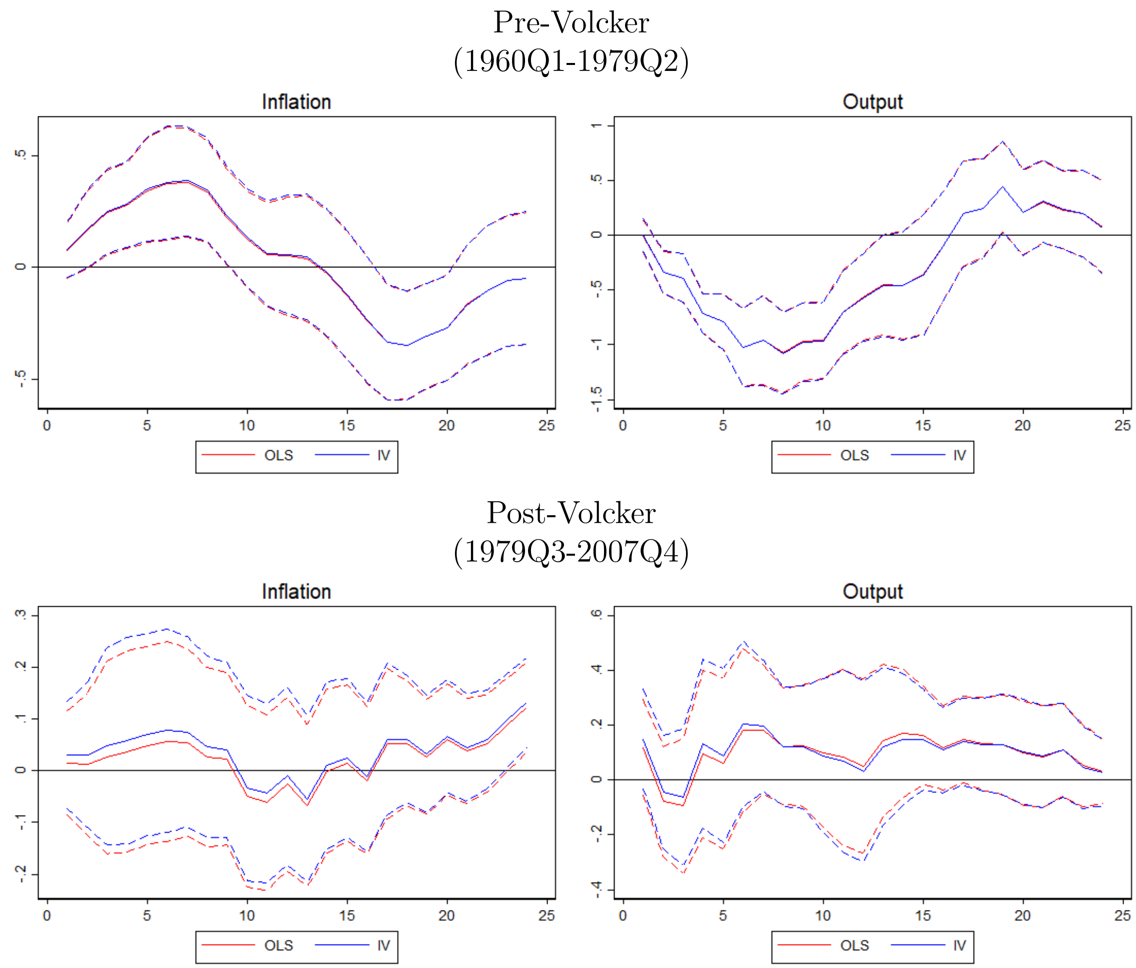

Figure 8: OLS and IV empirical impulse response functions.

Note: The red line shows the response implied by OLS estimates. The blue line reports the response implied by IV estimates. The dashed lines corresponds to 90% confidence bands. IRFs are constructed by local projections of inflation and output on OLS and IV residuals, as described in equation (12).

respectively, for horizons $h = 1, ..., 24$.

Figure 8 reports the resulting impulse response functions of inflation and output gap to a one-standard-deviation shock. We focus on the pre- and post-Volcker periods to achieve reasonable sample sizes. All four panels of Figure 8 yield the conclusion that the OLS- and IV-based IRFs are close to each other.[22] In addition, the IRFs reported in Figure 8 show patterns that accord with findings of the empirical literature on the effects of monetary policy shocks on inflation and output. In particular, the response of inflation during the pre-Volcker period shows a statistically significant "price puzzle" (Sims, 1992 and Eichenbaum, 1992) that becomes insignificant during the post-Volcker sample (Castelnuovo and Surico, 2010 and Baumeister, Liu and Mumtaz, 2013).[23] Moreover, the response of economic slack supports the evidence that

[22]Estimates of equation (12) that do not include the control variable matrix $\mathbf{c}_{t-i}$ also yield OLS- and IV-based IRFs that are close to each other.

[23]An attentive reader may be initially troubled by a price puzzle in the first part of the sample. Note,

monetary policy has become more stabilizing in the later part of the sample (*e.g.*, Boivin and Giannoni, 2006, Castelnuovo and Surico, 2010 and Baumeister, Liu and Mumtaz, 2013).

Turning to our second approach, we estimate a version of equation (11) in which we replace inflation and output gap by their one-quarter-ahead Greenbook forecasts. More specifically, we estimate:

$$r_t = \alpha_{aux} + \rho_{1,aux} r_{t-1} + \rho_{2,aux} r_{t-2} + \beta_{aux} E_t[\pi_{t+1}] + \gamma_{aux} E_t[x_{t+1}] + u_t, \tag{13}$$

where $E_t[x_{t+1}]$ and $E_t[\pi_{t+1}]$ are one-quarter-ahead Greenbook forecasts for the output gap and inflation, respectively. Because Greenbook forecasts for core PCE are only available starting in 2000, we rely instead on Greenbook forecasts for core CPI inflation, which are available since 1986Q1.[24] To make results more comparable, we also switch to real-time core CPI data when estimating the Taylor rule by OLS.

Table 2 compares OLS estimates using real-time data and one-quarter-ahead Greenbook forecasts.[25] Most point estimates using real-time data and Greenbook forecasts are not too far from one another. The larger estimated R^2 and smaller RMSEs in regressions using Greenbook forecasts suggest this approach would be preferable with respect to OLS. One important drawback of the former, however, is that Greenbook forecasts are only available with a 5 year delay – a meaningful impediment to more timely analyses. Finally, differences in point estimates are to be expected, given the much broader information set underlying Greenbook forecasts (see footnote 23).

Finally, we compare OLS and IV residuals to monetary shocks estimated by Tenreyro and Thwaites (2016). The authors apply the well-known methodology of Romer and Romer (2004) to extend the series of shocks based on information in the Fed's Greenbook releases.

For ease of exposition, denote the regression-based estimated residuals by ε_t^i, where $i \in \{OLS, IV\}$, and the Greenbook-based shocks, from Tenreyro and Thwaites (2016), by ε_t^{GB}. Table 3 reports the correlations $\sigma(\varepsilon_t^{OLS}, \varepsilon_t^{GB})$, $\sigma(\varepsilon_t^{IV}, \varepsilon_t^{GB})$ and $\sigma(\varepsilon_t^{OLS}, \varepsilon_t^{IV})$, as well as the standard deviations $\sigma(\varepsilon_t^{OLS})$, $\sigma(\varepsilon_t^{IV})$ and $\sigma(\varepsilon_t^{GB})$ for each subsample. The correlations $\sigma(\varepsilon_t^{OLS}, \varepsilon_t^{IV})$ are close to one in all samples. This underscores the key point of this paper, that OLS and

however, that the Taylor rules we estimate feature a very limited information set – in fact, the same information set as in canonical 3-variable VARs that have been extensively used in the literature on the effects of monetary policy shocks. Hence, a price puzzle is actually to be expected. The point of this paper is not to argue that one can recover true structural monetary shocks with a simple Taylor rule estimated with a narrow information set. Rather, our point is that OLS endogeneity bias is not such a material problem. This is established through the comparisons with IV estimates that we have explored throughout the paper. Estimating proper monetary policy shocks still requires getting the central bank's information set right (see, *e.g.*, Bernanke, Boivin and Eliasz, 2005).

[24] GDP deflator forecasts are available since the 1960s. Output gap estimates and forecasts, however, are also only available since 1986Q1, and hence these are the data that restrict our sample period.

[25] We also estimated specifications with four-quarter-ahead forecasts and results are qualitatively similar (available upon request). Forecasts are obtained from `https://www.philadelphiafed.org/research-and-data/real-time-center/greenbook-data`.

Table 2: Estimates using Greenbook forecasts for output and core CPI inflation

	OLS estimates with Greenbook forecasts			
	Pre-Volcker 1960Q1-1979Q2	Volcker-Greenspan 1986Q1-2005Q4	Greenspan-Bernanke 1987Q3-2007Q4	Post-Volcker 1986Q1-2007Q4
β	-	1.76***	1.81***	1.70***
	-	(0.22)	(0.23)	(0.23)
γ	-	0.80***	0.79***	0.80***
	-	(0.10)	(0.11)	(0.11)
ρ	-	0.80***	0.82***	0.81***
	-	(0.03)	(0.03)	(0.03)
π^*	-	2.86***	2.70***	2.81***
	-	(0.78)	(0.75)	(0.86)
N	-	80	82	88
R^2	-	0.98	0.98	0.98
RMSE	-	0.36	0.31	0.35
	OLS estimates with real-time core CPI data			
	Pre-Volcker 1960Q1-1979Q2	Volcker-Greenspan 1986Q1-2005Q4	Greenspan-Bernanke 1987Q3-2007Q4	Post-Volcker 1986Q1-2007Q4
β	-	1.32***	1.19***	1.24***
	-	(0.20)	(0.32)	(0.28)
γ	-	0.99***	0.91***	1.06***
	-	(0.10)	(0.21)	(0.15)
ρ	-	0.64***	0.82***	0.74***
	-	(0.10)	(0.05)	(0.07)
π^*	-	1.73	0.81	1.08
	-	(1.86)	(5.09)	(3.35)
N	-	80	82	88
R^2	-	0.91	0.96	0.91
RMSE	-	0.68	0.43	0.67

Note: The table reports estimates of equation (11) using real-time data and equation (13) using Greenbook forecasts for output gap and core CPI inflation. Subsample periods differ from those used earlier due to data availability. Statistical significance at the 90/95/99% confidence level indicated with $^{*/**/***}$, respectively. Data vintages are reported in the Appendix Table A4. Robust standard errors are reported in parenthesis.

IV estimation of Taylor rules deliver very similar results. The table also provides correlations between OLS and IV estimated residuals and the Tenreyro and Thwaites (2016)'s Greenbook-based shocks. Not surprisingly, given the near-perfect correlation between OLS and IV residuals, the correlations of OLS and IV residuals with Greenbook-based shocks are very close to one another. The Volcker-Greenspan sample shows the highest correlations between Greenbook-based shocks and our estimated residuals. Other subsamples yield lower, but still meaningful correlations (*e.g.*, the Greenspan-Bernanke period). When it comes to standard deviations,

Table 3: Residual correlations

	Correlations			Standard Deviations		
	$\sigma(\varepsilon_t^{IV}, \varepsilon_t^{GB})$	$\sigma(\varepsilon_t^{OLS}, \varepsilon_t^{GB})$	$\sigma(\varepsilon_t^{OLS}, \varepsilon_t^{IV})$	$\sigma(\varepsilon^{IV})$	$\sigma(\varepsilon^{OLS})$	$\sigma(\varepsilon^{GB})$
Pre-Volcker (1960Q1 – 1979Q2)	0.76	0.76	0.99	0.82	0.82	0.53
Volcker-Greenspan (1979Q3 – 2005Q4)	0.76	0.68	0.97	1.43	1.39	0.61
Greenspan-Bernanke (1987Q3 – 2007Q4)	0.44	0.47	1	0.41	0.41	0.3
Post-Volcker (1979Q3 – 2007Q4)	0.76	0.68	0.97	1.38	1.34	0.59

Note: Real time data pulled from Archival FRED on April 25, 2019. Greenbook-based monetary shocks are obtained from Tenreyro and Thwaites (2016).

Greenbook-based shocks clearly tend to be less volatile than OLS and IV residuals.

Differences between Greenbook-based shocks and OLS or IV residuals are to be expected for at least two reasons. First, this is a common finding when one compares different estimates of "structural shocks" obtained through arguably valid – but different – identification strategies. Thus, it is not surprising that a given series of structural shocks and OLS or IV residuals do not line up perfectly. Second, reinforcing earlier remarks, the information set underlying shocks identified from Greenbook releases is certainly much broader than the information contained in the time series of inflation and the output gap (see footnote 23).

5 Conclusion

This paper argues in favor of estimation of Taylor rule parameters by OLS. We show analytically, in a three-equation NK model, that the OLS asymptotic bias is a function of the fraction of the variance of inflation due to monetary policy shocks. This suggests the endogeneity bias in OLS estimates may be limited, given that monetary policy shocks appear to explain only a small fraction of the variance of endogenous variables to which the monetary authority responds, such as inflation and the output gap (*e.g.*, Leeper, Sims and Zha, 1996, Christiano, Eichenbaum and Evans, 1999).

To quantify the estimation bias, we resort to Monte Carlo simulations of the well-established Smets and Wouters (2007) model. We generate artificial data from the model and estimate its interest rate rule by OLS and GMM. For realistic sample sizes, OLS and GMM estimates are close to one another and close to the true parameter values. This arises because monetary policy shocks play a limited role in explaining inflation and output gap variation in the model. OLS estimates are, however, more precise. More importantly, the dynamic properties of the model are essentially unaffected by the OLS estimation bias. More specifically, impulse response functions produced by the DSGE model when using Taylor rules estimated by single-equation OLS in place of the true one are close to the model's IRF with the true Taylor rule.

The insight we exploit to establish the benefits of OLS estimation of Taylor rules may be useful in the context of single-equation estimation of other equations that are of interest in macroeconomics. The key is that the structural shock that enters the equation of interest be relatively unimportant in the variance decomposition of endogenous regressors. If this is the case, then the latter are "not too endogenous" to the shock that shifts the equation of interest, and OLS estimates are likely to have good properties.

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Appendix

A The basic New Keynesian model

This section presents a version of the basic NK model, building on Galí (2008, chapter 3).

A.1 Households

A representative infinitely-lived household maximizes:

$$E_o \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right],$$

where consumption C_t is given by:

$$C_t \equiv \left(\int_0^1 C_t\left(i\right)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

and $C_t\left(i\right)$ represents the quantity of good i consumed by the household in period t.

The budget constraint takes the form:

$$\int_0^1 P_t\left(i\right) C_t\left(i\right) di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t,$$

for $t = 0, 1, 2, ...,$ where $P_t\left(i\right)$ is the price of good i, N_t denotes hours of work, W_t is the nominal wage, B_t represents purchases of one-period bonds at price Q_t, and T_t is lump-sum income.

Optimality conditions yield the household's demand equations:

$$C_t\left(i\right) = \left(\frac{P_t\left(i\right)}{P_t} \right)^{-\varepsilon} C_t,$$

for all $i \in [0, 1]$, where $P_t \equiv \left(\int_0^1 P_t\left(i\right)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$ is the aggregate price index.

A.2 Firms

Firms produce differentiated varieties with the following production function:

$$Y_t\left(i\right) = A_t N_t\left(i\right)^{1-\alpha},$$

where A_t is the level of technology.

They adjust prices as in Calvo (1983), where each firm resets its price with probability $(1-\theta)$ in any given period.

A firm optimizing in period t chooses a price P_t^* to solve:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left[P_t^* Y_{t+k/t} - \Psi_{t+k} \left(Y_{t+k/t} \right) \right] \right\}$$
$$s.t.$$
$$Y_{t+k/t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} C_{t+k} \text{ for } k = \{0, 1, 2, \dots\},$$

where $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right)$ is the stochastic discount factor, $\Psi_t(.)$ is the cost function and $Y_{t+k/t}$ denotes output in period $t+k$ for a firm that last adjusted its price in period t.

A.3 The three-equation model

A log-linear approximation of the model around its zero-inflation steady state yields:

$$\begin{aligned} \pi_t &= \beta E_t \{\pi_{t+1}\} + \kappa \tilde{y}_t, \\ \tilde{y}_t &= E_t \{\tilde{y}_{t+1}\} - \frac{1}{\sigma} \left(i_t - E_t \{\pi_{t+1}\} - r_t^n \right), \end{aligned}$$

where $\tilde{y}_t = y_t - y_t^n$, $y_t^n = \psi_{ya}^n a_t$, with $\psi_{ya}^n \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$, $r_t^n \equiv \sigma \psi_{ya}^n E_t \{\Delta a_{t+1}\}$, and $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)(1-\alpha)}{\theta(1-\alpha+\alpha\varepsilon)} \left(\sigma + \frac{\varphi+\alpha}{1-\alpha} \right)$.

Closing the model requires specifying a monetary policy rule, which we assume to be:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\phi_\pi \pi_t + v_t,$$

where v_t is a monetary shock.

Assuming shocks follow AR(1) processes, and appending a cost-push shock to the Phillips curve, this simple NK economy is summarized by:

$$\begin{aligned} \pi_t &= \beta E_t \{\pi_{t+1}\} + \kappa \tilde{y}_t + u_t && \text{(A.1)} \\ \tilde{y}_t &= E_t \{\tilde{y}_{t+1}\} - \frac{1}{\sigma} \left(i_t - E_t \{\pi_{t+1}\} - r_t^n \right) && \text{(A.2)} \\ i_t &= \rho_i i_{t-1} + (1-\rho_i)\phi_\pi \pi_t + v_t && \text{(A.3)} \\ r_t^n &= \sigma \psi_{ya}^n E_t \{\Delta a_{t+1}\} && \text{(A.4)} \\ a_t &= \rho_a a_{t-1} + \varepsilon_t^a \quad \rho_a \in [0,1) && \text{(A.5)} \\ v_t &= \rho_v v_{t-1} + \varepsilon_t^v \quad \rho_v \in [0,1) && \text{(A.6)} \\ u_t &= \rho_u u_{t-1} + \varepsilon_t^u \quad \rho_u \in [0,1) && \text{(A.7)} \end{aligned}$$

The first three equations correspond to the Phillips curve, IS curve, and monetary policy rule, respectively. The fourth equation defines the natural rate of interest. The last three equations specify the dynamics of the technology, monetary, and cost-push shocks.

A.4 Solving the three-equation model analytically

We solve the model by the method of undetermined coefficients. For simplicity, we abstract from the shock to the Phillips curve. Obtaining the analytical solution with the three shocks is straightforward and does not generate any additional insight for our purposes.

To obtain the analytical results explored in Section 2.1, we set $\rho_i = 0$, and guess the solution takes the form:

$$\begin{aligned} \tilde{y}_t &= \psi_{yv} v_t + \psi_{ya} r_t^n, && \text{(A.8)} \\ \pi_t &= \psi_{\pi v} v_t + \psi_{\pi a} r_t^n, && \text{(A.9)} \\ i_t &= \phi_\pi \pi_t + v_t, \end{aligned}$$

where coefficients ψ_{yv}, ψ_{ya}, $\psi_{\pi v}$, and $\psi_{\pi a}$ are to be determined.

Because all shocks follow autoregressive processes:

$$\begin{aligned} r_t^n = & -\sigma\psi_{ya}^n (1-\rho_a)\, a_t && \Rightarrow E_t\left(r_{t+1}^n\right) = \rho_a r_t^n, \\ v_t = & \quad \rho_v v_{t-1} + \varepsilon_t^v && \Rightarrow E_t\left(v_{t+1}\right) = \rho_v v_t. \end{aligned}$$

Replacing equations (A.3), (A.8) and (A.9) in equation (A.2) and rearranging:

$$\begin{aligned} \tilde{y}_t &= E_t\left\{\tilde{y}_{t+1}\right\} - \frac{1}{\sigma}\left(i_t - E_t\left\{\pi_{t+1}\right\} - r_t^n\right) \\ &= \left\{-\frac{1}{\sigma}\phi_\pi\psi_{\pi v} + \frac{1}{\sigma}\psi_{\pi v}\rho_v + \psi_{yv}\rho_v - \frac{1}{\sigma}\right\} v_t \\ &\quad + \left\{\frac{1}{\sigma} - \phi_\pi\frac{1}{\sigma}\psi_{\pi a} + \frac{1}{\sigma}\psi_{\pi a}\rho_a + \psi_{ya}\rho_a\right\} r_t^n. \end{aligned}$$

Replacing equations (A.3), (A.8) and (A.9) in equation (A.1) and rearranging:

$$\begin{aligned} \pi_t &= \beta E_t\left\{\pi_{t+1}\right\} + \kappa\tilde{y}_t + u_t \\ &= \left\{\beta\psi_{\pi v}\rho_v + \kappa\psi_{yv}\right\} v_t + \left\{\beta\psi_{\pi a}\rho_a + \kappa\psi_{ya}\right\} r_t^n. \end{aligned}$$

Matching coefficients:

$$\begin{aligned}
\psi_{yv} &= \left\{-\frac{1}{\sigma}\phi_\pi\psi_{\pi v} + \frac{1}{\sigma}\psi_{\pi v}\rho_v + \psi_{yv}\rho_v - \frac{1}{\sigma}\right\}, \\
\psi_{ya} &= \left\{\frac{1}{\sigma} - \phi_\pi\frac{1}{\sigma}\psi_{\pi a} + \frac{1}{\sigma}\psi_{\pi a}\rho_a + \psi_{ya}\rho_a\right\}, \\
\psi_{\pi v} &= \left\{\beta\psi_{\pi v}\rho_v + \kappa\psi_{yv}\right\}, \\
\psi_{\pi a} &= \left\{\beta\psi_{\pi a}\rho_a + \kappa\psi_{ya}\right\}.
\end{aligned}$$

Solving for the coefficients:

$$\begin{aligned}
\psi_{yv} &= \frac{-\left(1-\beta\rho_v\right)}{\sigma\left(1-\rho_v\right)\left(1-\beta\rho_v\right)+\kappa\left(\phi_\pi-\rho_v\right)}, \\
\psi_{ya} &= \frac{\left(1-\beta\rho_a\right)}{\sigma\left(1-\rho_a\right)\left(1-\beta\rho_a\right)+\kappa\left(\phi_\pi-\rho_a\right)}, \\
\psi_{\pi v} &= \frac{-\kappa}{\sigma\left(1-\rho_v\right)\left(1-\beta\rho_v\right)+\kappa\left(\phi_\pi-\rho_v\right)}, \\
\psi_{\pi a} &= \frac{\kappa}{\sigma\left(1-\rho_a\right)\left(1-\beta\rho_a\right)+\kappa\left(\phi_\pi-\rho_a\right)}.
\end{aligned}$$

Define:

$$\Lambda_v \equiv \frac{1}{\sigma\left(1-\rho_v\right)\left(1-\beta\rho_v\right)+\kappa\left(\phi_\pi-\rho_v\right)}, \tag{A.10}$$

$$\Lambda_a \equiv \frac{1}{\sigma\left(1-\rho_a\right)\left(1-\beta\rho_a\right)+\kappa\left(\phi_\pi-\rho_a\right)}, \tag{A.11}$$

and we can rewrite:

$$\begin{aligned}
\psi_{yv} &= -\left(1-\beta\rho_v\right)\Lambda_v, \\
\psi_{ya} &= \left(1-\beta\rho_a\right)\Lambda_a, \\
\psi_{\pi v} &= -\kappa\Lambda_v, \\
\psi_{\pi a} &= \kappa\Lambda_a.
\end{aligned}$$

Hence, equilibrium inflation evolves according to

$$\pi_t = -\kappa\Lambda_v v_t - \sigma\psi_{ya}^n\left(1-\rho_a\right)\kappa\Lambda_a a_t. \tag{A.12}$$

The matrix representation of the system solution is:

$$\begin{aligned}
X_t &= A\Psi_t \\
\Psi_t &= \Xi\Psi_{t-1} + \varepsilon_t
\end{aligned}$$

where:

$$\begin{aligned} X_t &= \begin{pmatrix} \tilde{y}_t \\ \pi_t \end{pmatrix},\ A = \begin{pmatrix} \psi_{yv} & -\sigma\psi_{ya}^n\left(1-\rho_a\right)\psi_{ya} \\ \psi_{\pi v} & -\sigma\psi_{ya}^n\left(1-\rho_a\right)\psi_{\pi a} \end{pmatrix},\ \Psi_t = \begin{pmatrix} v_t \\ a_t \end{pmatrix}, \\ \Xi &= \begin{pmatrix} \rho_v & 0 \\ 0 & \rho_a \end{pmatrix},\ \text{and } \varepsilon_t = \begin{pmatrix} \varepsilon_t^v \\ \varepsilon_t^a \end{pmatrix}. \end{aligned}$$

$$\begin{aligned} X_t &= A\Psi_t \Rightarrow A'X_{t-1} = A'A\Psi_{t-1} \Rightarrow \Psi_{t-1} = \left(A'A\right)^{-1}A'X_{t-1} \\ X_t &= A\Psi_t = A\left(\Xi\Psi_{t-1}+\varepsilon_t\right) = A\Xi\Psi_{t-1} + A\varepsilon_t \Rightarrow E_{t-1}\left(X_t\right) = A\Xi\left(A'A\right)^{-1}A'X_{t-1}. \end{aligned}$$

A.5 Variance decomposition and OLS bias

Using equation (A.12) and the Taylor rule (equation (A.3)), one can obtain:

$$\begin{aligned} plim\ \hat{\phi}_\pi^{OLS} &= \frac{cov\left(i_t,\pi_t\right)}{var\left(\pi_t\right)} \\ &= \frac{cov\left(\phi_\pi\pi_t+v_t,\pi_t\right)}{var\left(\pi_t\right)} \\ &= \phi_\pi + \frac{cov\left(v_t,\pi_t\right)}{var\left(\pi_t\right)} \\ &= \phi_\pi + \frac{cov\left(v_t,\left\{-\kappa\Lambda_v v_t - \sigma\psi_{ya}^n\left(1-\rho_a\right)\kappa\Lambda_a a_t\right\}\right)}{var\left(\left\{-\kappa\Lambda_v v_t - \sigma\psi_{ya}^n\left(1-\rho_a\right)\kappa\Lambda_a a_t\right\}\right)} \\ &= \phi_\pi - \frac{\kappa\Lambda_v var\left(v_t\right)}{\left(\kappa\Lambda_v\right)^2 var\left(v_t\right) + \left(\sigma\psi_{ya}^n\left(1-\rho_a\right)\kappa\Lambda_a\right)^2 var\left(a_t\right)}. \end{aligned}$$

More compactly, we can express the OLS bias as:

$$Bias^{OLS} \equiv plim\ \hat{\phi}_\pi^{OLS} - \phi_\pi = -\frac{1}{\kappa\Lambda_v}\gamma_v, \tag{A.13}$$

where

$$\gamma_v = \frac{\left(\kappa\Lambda_v\right)^2 var\left(v_t\right)}{\left(\kappa\Lambda_v\right)^2 var\left(v_t\right) + \left(\sigma\psi_{ya}^n\left(1-\rho_a\right)\kappa\Lambda_a\right)^2 var\left(a_t\right)}$$

is the fraction of the variance of π_t due to monetary policy shocks. Λ_v and Λ_a are defined in equations (A.10) and (A.11), respectively.

A.6 OLS bias as a function of policy response to inflation

The simple three-equation model also yields some analytical insights relating the size of the bias given by equation (A.13) to the strength of the policy response to inflation (ϕ_π).

From equation (A.13), the OLS bias is given by:

$$Bias^{OLS} = -\frac{\kappa\Lambda_v var(v_t)}{(\kappa\Lambda_v)^2 var(v_t) + \left(\sigma\psi_{ya}^n (1-\rho_a)\kappa\Lambda_a\right)^2 var(a_t)} \equiv -\frac{f(\phi_\pi)}{g(\phi_\pi)},$$

with Λ_v and Λ_a defined in equations (A.10) and (A.11), respectively.

One can obtain:

$$\begin{aligned}
\frac{\partial\Lambda_v}{\partial\phi_\pi} &= -\frac{\kappa}{\left[\sigma(1-\rho_v)(1-\beta\rho_v)+\kappa(\phi_\pi-\rho_v)\right]^2} = -\kappa\Lambda_v^2 \\
\frac{\partial\Lambda_a}{\partial\phi_\pi} &= -\frac{\kappa}{\left[\sigma(1-\rho_a)(1-\beta\rho_a)+\kappa(\phi_\pi-\rho_a)\right]^2} = -\kappa\Lambda_a^2 \\
\frac{\partial f}{\partial\phi_\pi} &= \kappa\left(\frac{\partial\Lambda_v}{\partial\phi_\pi}\right) var(v_t) = -(\kappa\Lambda_v)^2 var(v_t) \\
\frac{\partial g}{\partial\phi_\pi} &= 2\kappa^2\Lambda_v var(v_t)\left(\frac{\partial\Lambda_v}{\partial\phi_\pi}\right) + 2\left(\sigma\psi_{ya}^n(1-\rho_a)\kappa\right)^2\Lambda_a var(a_t)\left(\frac{\partial\Lambda_a}{\partial\phi_\pi}\right) \\
&= -2(\kappa\Lambda_v)^3 var(v_t) - 2\left(\sigma\psi_{ya}^n(1-\rho_a)\right)^2(\kappa\Lambda_a)^3 var(a_t).
\end{aligned}$$

Therefore:

$$\frac{\partial Bias^{OLS}}{\partial\phi_\pi} = -\frac{\overbrace{-(\kappa\Lambda_v)^2 var(v_t) g(\phi_\pi) + 2f(\phi_\pi)\left[(\kappa\Lambda_v)^3 var(v_t) + \left(\sigma\psi_{ya}^n(1-\rho_a)\right)^2(\kappa\Lambda_a)^3 var(a_t)\right]}^{\mathcal{N}}}{g(\phi_\pi)^2}.$$

As the denominator of the above expression is always positive, the sign of $\frac{\partial\gamma_v}{\partial\phi_\pi}$ depends solely on the sign of the numerator ($\mathcal{N}$), which can be rearranged as follows:

$$\begin{aligned}
\mathcal{N} &= \overbrace{(f(\phi_\pi)(\kappa\Lambda_v))}^{>0}\overbrace{\left[2\left((\kappa\Lambda_v)^2 var(v_t) + \left(\sigma\psi_{ya}^n(1-\rho_a)\kappa\Lambda_a\right)^2 var(a_t)\frac{\Lambda_a}{\Lambda_v}\right) - g(\phi_\pi)\right]}^{\gtrless 0} \\
&= (f(\phi_\pi)(\kappa\Lambda_v))\left[(\kappa\Lambda_v)^2 var(v_t) + \left(\sigma\psi_{ya}^n(1-\rho_a)\kappa\Lambda_a\right)^2 var(a_t)\underbrace{\left[2\left(\frac{\Lambda_a}{\Lambda_v}\right)-1\right]}_{\gtrless 0}\right].
\end{aligned}$$

This implies two possible results that hinge on whether $\rho_v \gtrless \rho_a$.

If $\rho_v \leq \rho_a$, $\frac{\Lambda_a}{\Lambda_v} \geq 1$, and hence, $\frac{\partial Bias^{OLS}}{\partial\phi_\pi} \leq 0$. In this case, the OLS bias increases in magnitude – *i.e.*, becomes more negative – as ϕ_π increases.

If $\rho_v > \rho_a$, $\left[2\left(\frac{\Lambda_a}{\Lambda_v}\right)-1\right]$ can be either positive or negative, depending on the relative magnitude of the shocks' variances. Note that, in this case, if we further assume that

$$i. \quad \left(\frac{\Lambda_a}{\Lambda_v}\right) \quad < \quad \frac{1}{2}, \text{ and} \tag{A.14}$$

$$ii. \quad \frac{var(v_t)}{var(a_t)} \quad < \quad \left(\sigma\psi_{ya}^n (1-\rho_a)\right)^2 \left(\frac{\Lambda_a}{\Lambda_v}\right)^2 \left[1 - 2\left(\frac{\Lambda_a}{\Lambda_a}\right)\right], \tag{A.15}$$

then $\frac{\partial Bias^{OLS}}{\partial \phi_\pi} > 0$.

In sum, when $\rho_v > \rho_a$ and conditions (A.14) and (A.15) hold, $\frac{\partial Bias^{OLS}}{\partial \phi_\pi} > 0$. Under the more plausible case in which $\rho_v \leq \rho_a$, $\frac{\partial Bias^{OLS}}{\partial \phi_\pi} \leq 0$.

A.7 Model parameterization for results in Section 2.2

For the numerical simulations in Section 2.2, we set the model's structural parameters to standard values used in the literature. In particular, we set $\beta = 0.99$, $\sigma = 1$, $\alpha = 1/3$, $\varepsilon = 6$, $\varphi = 1$, $\theta = 2/3$, and $\phi_\pi = 1.5$. The relative scales of nonpolicy (technology and cost-push) shocks do not affect the OLS bias, and hence we fix them at arbitrary values ($\sigma_a = \sigma_u = 0.1$).

To calibrate the scale of monetary policy shocks, it is useful to rewrite the OLS bias as follows:

$$Bias^{OLS} \quad = \quad -\kappa\Lambda_v \frac{\sigma_v^2}{\sigma_\pi^2} = \underbrace{-\kappa\Lambda_v \frac{\sigma_v}{\sigma_\pi}}_{corr(v_t, \pi_t)} \frac{\sigma_v}{\sigma_\pi},$$

where σ_v and σ_π are the standard deviations of monetary policy shocks (v_t) and equilibrium inflation (π_t), respectively. We then pick σ_v so that its ratio to σ_π, in equilibrium, matches the estimates from the Smets and Wouters (2007) model ($i.e.$, $\frac{\sigma_v}{\sigma_\pi} = 0.45$).

B Additional results

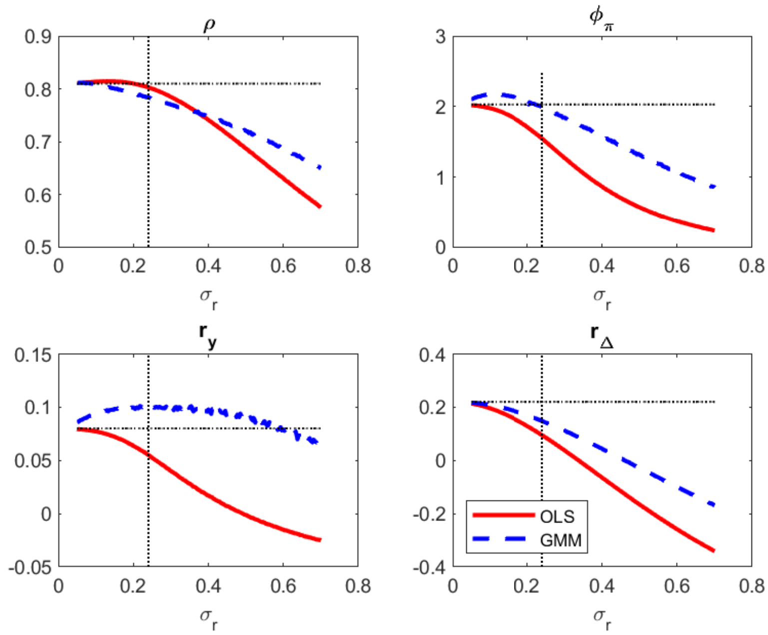

Figure A1: Mean OLS and GMM point estimates for varying monetary shock volatility, Smets and Wouters (2007) model with sample size $T = 150$ quarters.

Note: OLS estimates in red (solid) line, GMM estimates in blue (dashed) line. Vertical dotted black lines correspond to the calibrated standard deviation of the monetary shock, while horizontal dotted black line reports the "true" Taylor rule parameters. Simulations are based on the Smets and Wouters (2007) model with all other parameters set at their posterior modes. Mean point estimates obtained from 50,000 Monte Carlo simulations of the model with sample size $T = 150$.

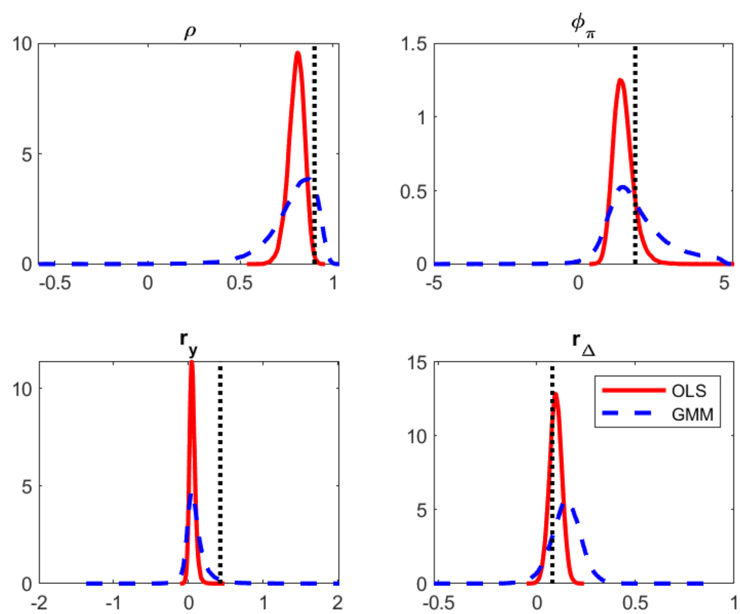

Figure A2: OLS and GMM estimate distributions in a medium-scale DSGE model with sample size $T = 150$ quarters.

Note: OLS distributions in red solid lines, GMM distributions in blue dashed lines. Vertical black dotted lines correspond to the true parameter values. Simulations are based on the Smets and Wouters (2007) model with parameters set at their posterior modes. Distributions obtained from 50,000 Monte Carlo simulations of the model with sample size $T = 150$.

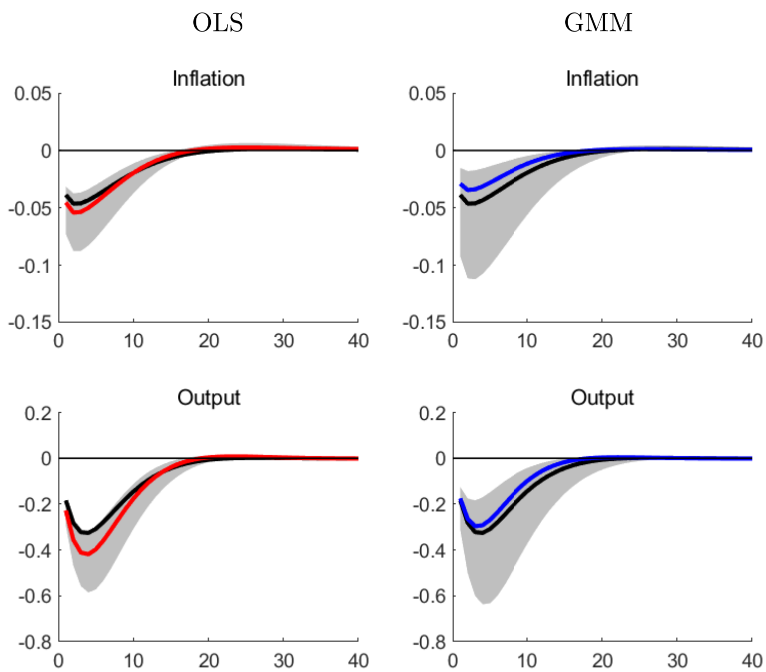

Figure A3: Output and inflation responses to monetary shock in Smets and Wouters (2007) model for different Taylor rule parameters with sample size $T = 150$ quarters.

Note: Black lines are IRFs implied by the Smets and Wouters (2007) model with parameters set at their posterior modes. Red and blue lines are IRFs implied by mean OLS and GMM estimates of the Taylor rule parameters, respectively. Shaded areas correspond to the 5^{th} and 95^{th} percentiles of the distribution of IRFs implied by OLS and GMM. Estimates obtained from 50,000 Monte Carlo simulations of the model with sample size $T = 150$.

Table A1: OLS and GMM estimates of ϕ_π in the three-equation NK model

	$\rho_v = 0$						
	True values	OLS	OLS bias	GMM	GMM bias	Relative bias	Relative MSE
ϕ_π	1.5	1.4758	-0.0242	1.4990	-0.0010	-0.0154	1.5684
	$\rho_v = 0.8$						
	True values	OLS	OLS bias	GMM	GMM bias	Relative bias	Relative MSE
ϕ_π	1.5	1.3031	-0.1969	1.3028	-0.1972	0.0002	1.2786

Note: For a given parameter β, relative bias is given by $\frac{|\hat{\beta}^{GMM}-\beta|-|\hat{\beta}^{OLS}-\beta|}{\beta}$, and relative mean squared error (MSE) equals $\frac{MSE(\hat{\beta}^{GMM})}{MSE(\hat{\beta}^{OLS})}$. A negative relative bias indicates that GMM outperforms OLS in terms of mean point estimates. A relative MSE below unit indicates GMM is more precise than OLS.

Table A2: OLS and GMM estimates of ρ_i and $(1-\rho_i)\phi_\pi$ in the three-equation NK model, with *i.i.d.* monetary shocks

	True values	OLS	OLS bias	GMM	GMM bias	Relative bias	Relative MSE
ρ_i	0.8	0.8189	0.0189	0.7934	-0.0066	-0.0082	0.5715
$(1-\rho_i)\phi_\pi$	0.30	0.1093	-0.1907	0.3069	0.0069	-1.3785	0.2962

Note: For a given parameter β, relative bias is given by $\frac{|\hat{\beta}^{GMM}-\beta|-|\hat{\beta}^{OLS}-\beta|}{\beta}$, and relative mean squared error (MSE) equals $\frac{MSE(\hat{\beta}^{GMM})}{MSE(\hat{\beta}^{OLS})}$. A negative relative bias indicates that GMM outperforms OLS in terms of mean point estimates. A relative MSE below unit indicates GMM is more precise than OLS.

Table A3: OLS and GMM estimates in Smets and Wouters (2007) model

	T = 80						
	True values	OLS	OLS bias	GMM	GMM bias	Relative bias	Relative MSE
ρ	0.81	0.7883	-0.0217	0.7624	-0.0476	0.0319	5.9847
ϕ_π	2.03	1.5394	-0.4906	1.8447	-0.1853	-0.1504	2.0831
r_y	0.08	0.0609	-0.0191	0.0928	0.0128	-0.0795	6.4231
r_Δ	0.22	0.0922	-0.1278	0.1316	-0.0884	-0.1792	0.9876
	T = 150						
	True values	OLS	OLS bias	GMM	GMM bias	Relative bias	Relative MSE
ρ	0.81	0.8031	-0.0069	0.7812	-0.0288	0.0270	9.8193
ϕ_π	2.03	1.5485	-0.4815	1.9884	-0.0416	-0.2167	2.8368
r_y	0.08	0.0551	-0.0249	0.1000	0.0200	-0.0606	11.1449
r_Δ	0.22	0.0953	-0.1247	0.1471	-0.0729	-0.2355	0.6803

Note: For a given parameter β, relative bias is given by $\frac{|\hat{\beta}^{GMM}-\beta|-|\hat{\beta}^{OLS}-\beta|}{\beta}$, and relative mean squared error (MSE) equals $\frac{MSE(\hat{\beta}^{GMM})}{MSE(\hat{\beta}^{OLS})}$. A negative relative bias indicates that GMM outperforms OLS in terms of mean point estimates. A relative MSE below unit indicates GMM is more precise than OLS.

Table A4: Real-time data vintages

	Pre-Volcker 1960Q1 – 1979Q2	Volcker-Greenspan 1979Q3 – 2005Q4	Greenspan-Bernanke 1987Q3 – 2007Q4	Post-Volcker 1979Q3 – 2007Q4
Fed funds rate	12/9/1996	01/03/2006	12/31/2007	12/31/2007
Core PCE	10/30/1999 (Index 1992=100)	01/27/2006 (Index 2000=100)	01/30/2008 (Index 2000=100)	01/30/2008 (Index 2000=100)
Core CPI	12/12/1996 (Index 1984=100)	01/18/2006 (Index 1984=100)	01/16/2008 (Index 1984=100)	01/16/2008 (Index 1984=100)
Real GDP	01/29/1992 (Chained 1987 dollars)	01/27/2006 (Chained 2000 dollars)	01/30/2008 (Chained 2000 dollars)	01/30/2008 (Chained 2000 dollars)
Potential GDP	01/22/1992 (Chained 1987 dollars)	08/31/2005 (Chained 2000 dollars)	08/23/2007 (Chained 2000 dollars)	08/23/2007 (Chained 2000 dollars)
Money Stock (M2)	02/08/1980	01/12/2006	01/10/2008	01/10/2008

Note: Data pulled from Archival FRED on April 25, 2019.